Cross-Layer Optimization for Wireless Sensor Network with Multi-Packet Reception

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Abstract—**In this paper, we consider how to exploit multi-packet reception (MPR) to increase the capacity for a wireless sensor network. Since MPR behavior at the physical layer affects link layer scheduling, it is necessary to follow a cross-layer approach to obtain an optimal solution. Due to the complexity of cross-layer optimization, although MPR has great potential to increase capacity, optimal solutions are yet to be developed. We build constraints for the signal-to-noise-ratio requirement under MPR at the physical layer such that we can check the feasibility for a set of concurrent transmissions. We further develop an upper bound for the number of concurrent transmissions, which enables us to identify all feasible sets of concurrent transmissions in polynomial time. Then a capacity problem can be formulated as a linear program (LP) but with a large number of variables. We propose a concept of maximum feasible set to decrease the size of LP. Finally, by comparing optimal solutions with and without MPR, we show that network capacity can be increased about 100% by using MPR.**

Keywords - Multi-packet reception; successive interference cancellation; wireless sensor network; capacity; cross-layer optimization

I. INTRODUCTION

Recently, there have been considerable interests in wireless sensor networks (WSNs) (see e.g., [1], [2], [3], [4]) because WSNs have many important applications, such as military, agriculture monitoring, environmental monitoring, health care, industrial control, and traffic management. A fundamental problem for WSNs is capacity, especially for video monitoring applications. In this paper, we consider how to maximize data rates from all sensor nodes to a base station.

Traditionally, the base station can only receive one packet from a sensor node at any time. If there are multiple sensor nodes transmitting at the same time, then all the packets are collided and cannot be received. But if the base station has multi-packet reception (MPR) capability [5], [6], [7], then it may receive multiple packets simultaneously. MPR can be achieved by several techniques, e.g., successive interference cancellation (SIC) [8], [9], parallel interference cancellation (PIC) [9], [10], multiple input multiple output (MIMO) [11], [12]. In this paper, we consider SIC for MPR.

Since the base station can receive data from multiple sensor nodes at the same time by using MPR, the network capacity may be significantly increased. It has been shown that the asymptotic capacity bounds of wireless ad hoc networks can be increased by using MPR [13], [14]. However, due to the complex MPR behavior in a networking environment, the optimal solution for a given network is still unknown.

In this paper, we aim to design a cross-layer optimal solution on how to apply the MPR technology in a single hop WSN. Note that although MPR is a physical layer technology, the decision of concurrent transmitting nodes affects the allocation of time slots at the link layer, which in turn affects the network capacity. Thus, it is necessary to follow a cross-layer approach such that the optimal solution can be obtained. There are several challenges in this study. First, since there is no existing model to characterize the MPR behaviors in a network environment, we need to build constraints for the signal-to-noise-ratio requirement under MPR such that we can check the feasibility for a set of concurrent transmissions. Second, since the number of all possible sets of concurrent transmissions is exponential, we cannot check all possible sets. To obtain a solution procedure with polynomial time complexity, we prove that a feasible set cannot have more than l concurrent transmissions, where l is a constant. Then we only need to check polynomial number of sets to identify all feasible sets. Subsequently, we can formulate a capacity problem as a linear program (LP) [15]. Third, it turns out that the formulated LP has a large number of variables, each corresponding to one feasible set. Thus, we want to further decrease the complexity of our solution procedure. We propose a concept of maximum feasible set (MFS), which can be used to effectively reduce the number of variables and enable us to solve the problem optimally in low complexity.

To show the benefit of MPR, we compare the optimal solutions with and without MPR. Our results show that MPR can increase the network capacity about 100%.

The paper is organized as follows. In Section Ⅱ, we solve the network capacity problem for a WSN without MPR. In Section Ⅲ, we develop a cross-layer model for a WSN with MPR and formulate an optimization problem based on this model. We further show how to decrease the problem size by MFS and then solve it optimally in polynomial time. In Section Ⅳ, we give numerical results to show the significant performance improvement by using MPR.

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II. Problem and Optimazition For WSNs without MPR

We now solve a capacity problem for a WSN when MPR is not used. The result obtained in this section will be compared with the result for a WSN with MPR.

We consider a WSN consisting of n sensor nodes and a base station deployed over a two-dimensional area. We denote N as the set of sensor nodes. Now we build constraints to characterize behaviors at the physical and link layers and formulate the problem.

SNR at the Physical Layer. Each sensor node uses power P and bandwidth W to transmit data to the base station directly. When s_i is transmitting, the base station receives a signal with power $g_i P$, where g_i denotes the channel gain from s_i to base station. Denote d_i as the distance between s_i and base station and λ as the path loss index. We have

$$g_i = a \cdot d_i^{-\lambda}, \qquad (1)$$

where a is an antennas related constant. In order to simplify the problem, we can normalize a and let $a = 1$. Then the SNR (signal-to-noise-ratio) of s_i is

$$SNR_i = \frac{g_i P}{N_0}, \qquad (2)$$

where N_0 is the noise power. For a successful transmission, SNR should be larger than a threshold $\beta > 1$ [16] and the peak data rate is $W \log_2(1 + SNR_i)$.

Time Slot Based Scheduling. Since MPR is not used, there can be at most one sensor node transmitting at any time. For a time slot based scheduling, we use n time slots, where each time slot i is assigned to node s_i. The length of each time slot is denoted as $t_1, t_2, \ldots, t_n$. Denote T as the total length of these time slots, i.e., $T = \sum_{i=1\cdots n} t_i$.

To ensure that the signal from node s_i can be decoded in time slot i, the data rate in this time slot should be no more than the peak data rate $W \log_2(1 + SNR_i)$ and thus the average data rate over all n time slots is no more than $\frac{t_i W}{T} \log_2(1 + SNR_i)$.

Problem Formulation. In this paper, we consider the case that each sensor node s_i ($s_i \in N$) has a minimum rate requirement $r(i)$. We aim to maximize a common scaling factor K, such that each sensor node s_i can transmit data to base station with rate $Kr(i)$. We note that there are many other objectives that can also be used in this investigation, e.g., the sum of all rates, the sum of log utility of rates, etc. In general, we may consider an objective function in the form of the total utility of all nodes, with the utility of a node being a concave function of its rate. We emphasize that the same approach that we will develop regarding how to identifying all MFS is independent of and can be applied for all these objective settings.

To formulate the problem with linear constraints, we can normalize all t_i by T and then $\sum_{i=1\cdots n} t_i = 1$. We now have the following problem.

$$\text{Max } K$$

$$\text{s.t.} \begin{cases} Kr(i) \le t_i W \log_2(1 + SNR_i) \quad (1 \le i \le n) \\ \sum_{i=1\cdots n} t_i = 1, \end{cases} \qquad (3)$$

Where t_i and K are variables. The constant value of SNR_i can be calculated by (2). This is a linear problem (LP). Since the number of variables and constraints are both $n+1$, this LP can be solved optimally in polynomial time [15].

III. Wireless Sensor Network with MPR

In this section, we consider the same problem as that in Section II, but now for a WSN with MPR. By using MPR, multiple sensor nodes can transmit data simultaneously. Thus, the data rates from sensor nodes can be increased.

MPR can be achieved by SIC technique [8], [9] as follows. The base station first decodes the strongest signal if the corresponding SINR (signal-to-interference-and-noise-ratio) is at least β. Once the strongest signal is successfully decoded, the base station can remove the strongest signal from the received signal. By doing so, the SINR for the second strongest signal can be increased (because the strongest interference is removed). If this increased SINR is at least β, then the second strongest signal can be successfully decoded. This process can be repeated until all signals are decoded or a signal cannot be decoded.

A. *MPR Model and A Naive Approach*

A naive approach can be developed by following a similar approach as that in Section II. Note that when MPR is not used, the physical layer behavior is very simple and only one node can transmit at any time. But with MPR, the physical layer behavior is much more complex. Now a set of nodes may transmit at the same time. For a given set of concurrent transmitting nodes, if we can decode the data from all nodes in this set, then we call it a feasible set. Now we need to assign one time slot for each feasible set. But before that, we need to identify all feasible sets. This can be done by analyzing the signal-to-noise-ratio requirement under MPR.

SINR at the Physical Layer. Denote F as the set of all feasible sets. We now consider a feasible set k ($k \in F$). Denote constant

$$x_i^k = \begin{cases} 1 & \text{if node } i \text{ transmit data to the base station in time slot } k \\ 0 & \text{otherwire.} \end{cases}$$

Under MPR, when we decode the signal from node s_i in time slot k, we already decoded all stronger signals and removed them. Thus, the SINR of s_i is increased as

$$SINR_i^k = \frac{g_i \cdot x_i^k P}{N_0 + \sum_{g_j < g_i} g_j \cdot x_j^k P}. \qquad (4)$$

For a successful transmission, $SINR_i^k$ should be larger than or equal to the threshold β for each nodes s_i with $x_i^k = 1$ and the achieved peak data rate is $W\log_2(1+SINR_i^k)$.

Time Slot Based Scheduling. Since a node s_i may transmit in multiple time slots and the peak data rate in time slot *k* is $W\log_2(1+SINR_i^k)$,[1] its average data rate over all time slots is no more than $\sum_{k\in F} t_k W\log_2(1+SINR_i^k)$, where t_k is the normalized length for time slot *k*.

Problem Formulation. We now have the following problem.

$$\text{Max } K$$
$$\text{s.t.} \begin{cases} Kr(i) \le \sum_{k\in F} t_k W\log_2(1+SINR_i^k) \quad (1\le i\le n) \\ \sum_{k\in F} t_k = 1, \end{cases} \qquad (5)$$

where t_k and K are variables. The constant value of $SINR_i^k$ can be calculated by (4). This problem is also an LP.

Thus, a naive approach is as follows.

(i) Check the feasibility for all possible sets.

(ii) Solve LP (5) to obtain the optimal scheduling t_k for each feasible set.[2]

However, the complexity of this naive approach is prohibitive. For an *n*-node network, the total number of all possible sets is $2^n - 1$. Thus, the complexity of (i) in the naive approach is $\Theta(2^n)$. Moreover, LP (5) has a large number of variables. The complexity of (ii) in the naive approach is also very high.

B. From Exponential Complexity to Polynomial Complexity

To obtain a polynomial time algorithm, we should not check the feasibility for all possible sets. In this section, we analyze the MPR behavior and prove that the number of nodes in a feasible set is bounded by a constant *l*. Therefore, we only need to check $\sum_{k=1\cdots l} C(n,k) = \Theta(n^l)$ sets to find all feasible sets and thus, the number of feasible sets is $O(n^l)$. Therefore, the complexity of solving LP is also polynomial.

To develop an upper bound for the number of nodes in a feasible set, we sort nodes in this set by their distances to the base station and consider the distance ratio between two neighboring nodes s_i and s_j in the sorted feasible set. Without loss of generality, we assume $d_i \le d_j$. We have the following lemma.

Lemma 1 *If nodes s_i and s_j transmit data to the base station in a time slot k and $d_i \le d_j$, then $d_j \ge (\frac{g_i}{\beta} - \frac{N_0}{P})^{-\frac{1}{\lambda}}$.*

Proof. By (1) and $d_i \le d_j$, we have $g_i P \ge g_j P$. Thus, the signal from s_i is stronger than that from s_j and then the signal from s_i is decoded before the signal from s_j. By the requirement to decode the signal from s_i, we have

$$\beta \le SINR_i^k \le \frac{g_i P}{N_0 + g_j P}, \qquad (6)$$

where the second inequality holds because there may be interference from other nodes. By (6), we have $d_j \ge (\frac{g_i}{\beta} - \frac{N_0}{P})^{-\frac{1}{\lambda}}$. □

Based on Lemma 1, we have

$$\frac{d_j}{d_i} \ge \frac{(\frac{g_i}{\beta} - \frac{N_0}{P})^{-\frac{1}{\lambda}}}{d_i} > \frac{(\frac{d_i^{-\lambda}}{\beta})^{-\frac{1}{\lambda}}}{d_i} = \beta^{\frac{1}{\lambda}}. \qquad (7)$$

For a sorted feasible set with *h* nodes, suppose the distances from these *h* nodes to the base station are $\hat{d}_1 < \hat{d}_2 < \ldots < \hat{d}_h$. By (7), we have

$$\hat{d}_2 > \beta^{\frac{1}{\lambda}} \cdot \hat{d}_1,$$
$$\hat{d}_3 > \beta^{\frac{1}{\lambda}} \cdot \hat{d}_2 > \beta^{\frac{2}{\lambda}} \cdot \hat{d}_1,$$
$$\ldots,$$
$$\hat{d}_h > \beta^{\frac{1}{\lambda}} \cdot \hat{d}_{h-1} > \ldots > \beta^{\frac{h-1}{\lambda}} \cdot \hat{d}_1.$$

Thus, *h* is at most $\left\lfloor \log_{\beta^{\frac{1}{\lambda}}} \frac{d_h}{d_1} \right\rfloor + 1$. For a given WSN, we have $d_1 \ge d_{\min}$ and $d_h \le d_{\max}$, where constants $d_{\min}$ and $d_{\max}$ are the minimum and maximum distance from a node in the network to the base station, respectively. Thus, the number of nodes in a feasible set is no more than a constant $l = \left\lfloor \log_{\beta^{\frac{1}{\lambda}}} \frac{d_{\max}}{d_{\min}} \right\rfloor + 1$. This result is stated in the following theorem.

Theorem 1 *The number of nodes in a feasible set is no more than a constant $l = \left\lfloor \log_{\beta^{\frac{1}{\lambda}}} \frac{d_{\max}}{d_{\min}} \right\rfloor + 1$.*

By Theorem 1, a better approach is as follows:

[1] This holds even if node s_i does not transmit in time slot *k*. When node s_i does not transmit in time slot *k*, we have $SINR_i^k = 0$ by (4) and thus the computed peak data rate is 0.

[2] If t_k =0 in a solution, then the *k*-th feasible set is not used in this solution.

(i) Check the feasibility for all possible sets with no more than l nodes.

(ii) Solve LP (5) to obtain the optimal scheduling t_k for each feasible set.

We call this approach as the FS approach. The complexity of (i) in the FS approach is $\sum_{k=1\cdots l} C(n,k) = \Theta(n^l)$. Further, the number of feasible sets is no more than $\Theta(n^l)$, i.e., is $O(n^l)$. Once we determine all feasible sets, we can obtain an optimal solution by solving the LP in (5), with the number of variables being $O(n^l)$. Thus, the complexity of (ii) in the FS approach is also polynomial. Therefore, the total complexity of the FS approach is polynomial.

C. *Maximum Feasible Set*

We note that the number of feasible sets is still a large number. To further decrease the complexity, we define a concept of maximum feasible set (MFS) and show that we only need to consider all MFS.

The following lemma considers two feasible sets A_1 and A_2 and data rates from nodes in these sets.

Lemma 2 *For two feasible sets A_1 and A_2, if $A_2 \subset A_1$ and nodes in the set A_1-A_2 are closer to the base station than nodes in the set A_2, then: (i) the data rate from each node in A_1-A_2 is positive in A_1's time slot and is zero in A_2's time slot; (ii) the data rate from each node in A_2 is the same in both A_1's and A_2's time slots.*

Proof. Because A_1 is a feasible set, for each node in A_1, its data rate is positive in A_1's time slot. In particular, for each node in A_1-A_2, its data rate is positive. On the other hand, such a node has a zero rate in A_2's time slot since it is not in A_2. Thus, (i) is proved.

By SIC, stronger signals are decoded and cancelled before decoding weaker signals. Since nodes in A_1-A_2 are closer to the base station than nodes in A_2, signals from nodes in A_1-A_2 are stronger than signals from nodes in A_2. Therefore, in A_1's time slot, before signals from nodes in A_2 are decoded, signals from nodes in A_1-A_2 are all cancelled. Thus, the decoding process for node in A_2 are the same in both A_1's and A_2's time slots, yielding the same data rates. (ii) is also proved. □

By results in Lemma 2, it is easy to see that if we replace A_2 by A_1 in a solution, the new solution will have a better objective value. Thus, we have the following definition.

Definition 1 *A feasible set A_1 is better than another feasible set A_2 if $A_2 \subset A_1$ and nodes in the set A_1-A_2 are closer than nodes in the set A_2.*

We can further define a concept of MFS.

Definition 2 *A feasible set A is a maximum feasible set (MFS) if for any node s that is closer to the base station than all nodes in A, $A \cup \{s\}$ is not a feasible set.*

TABLE 1 Optimal Objective Value with and without MPR

n	K (without MPR)	K (with MPR)	Improvement Ratio
20	11.62	20.83	79.19%
25	9.31	16.95	81.99%
30	7.18	14.67	104.28%
35	6.21	13.02	109.78%
40	5.25	10.84	106.42%
45	4.56	9.57	93.06%
50	4.50	8.69	93.03%

TABLE 2 Complexity Comparison

n	Complexity of the naive approach	Complexity of the FS approach	The size of LP in the MFS approach
20	1.04×10^6	9.1×10^5	1280.30
25	3.35×10^7	1.68×10^7	4067.75
30	1.07×10^9	1.94×10^8	11152.00
35	3.44×10^{10}	1.54×10^9	26722.25
40	1.10×10^{12}	2.55×10^9	55663.9
45	3.52×10^{13}	9.12×10^9	94613.55
50	1.13×10^{15}	1.72×10^{11}	167902.90

By Definition 2 and Lemma 2, for any non-MFS, we can always find a better feasible set. Thus, it is no need to consider non-MFS to obtain an optimal solution. So we only need to consider all MFS in the LP (5). We call this approach as the MFS approach. The MFS approach can further decrease the number of considered sets and the problem size. In the next section, we will show the benefit of MFS quantitatively.

IV. NUMERICAL RESULTS

In this section, we compare the performance of the optimal solutions with and without MPR to show the benefit of applying MPR. We also compare the complexity of each approach.

We consider WSNs with the base station at the center and *n*=20, 25, …, 50 sensor nodes within a disk with radio 250 meters. Each sensor has a minimum rate requirement within [10, 100] Kb/s. We randomly generated 20 different network instances for each *n*. The presented result for each *n* is the average value over all 20 instances. The transmission power is 1 W and the channel bandwidth is 22 MHz (bandwidth in 802.11). The path loss index is λ =4 and the noise is $N_0 = 10^{-10}$ W. The SINR threshold is β =3 [17].

We present optimal objective (K) value in Table 1. We can see that K value is increased significantly (about 100%) by using MPR for all different sized networks.

We show the complexity of each approach in Table 2. The complexity of the naive approach is $2^n - 1$, which is shown in column 2. The complexity of the FS approach is $\sum_{k=1\cdots l} C(n,k)$, which is shown in column 3. Finally, in the MFS approach, we further decrease the size of LP, which is shown in column 4. From Table 2, we can see that the improvement by bounding the maximum number of nodes in a feasible set and removing all non-MFS is significant.

In order to analyze how the actual size of the LP (the results in column 4) changes for different *n*, we do polynomial fitting for results in column 4, and obtain the curve in Fig. 1.

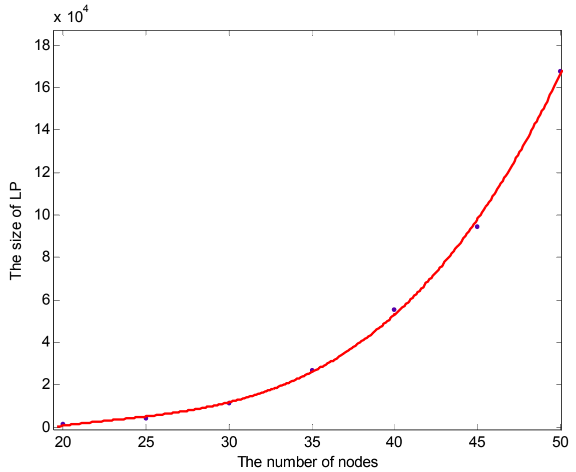

Fig.1. Fitting Curve for the number of nodes and the size of LP

The fitting formula:

$$y = 7.014x^3 - 480x^2 + 11780x - 98990.$$

This is a cube polynomial fitting curve, which shows that the relationship between the size of the LP and the number of nodes is $\Theta(n^3)$ approximately.

V. Conclusions

In this paper, we developed a cross-layer model for a single hop wireless sensor network (WSN) with MPR and applied this model to solve a network capacity problem. This problem can be formulated as a linear program (LP) with a large number of variables, each corresponding to a feasible set of concurrent transmitting nodes. The key challenge is to determine all the feasible sets. A naive approach to check all possible sets yields an exponential complexity. To develop a polynomial time algorithm, we analyzed the MPR behavior and proved that the number of nodes in a feasible set is bounded by a constant. Our approach based on this result has a polynomial time complexity. We also proposed a concept of maximum feasible set to further decrease the LP size. Our optimal solutions showed the benefit of applying MPR: the capacity of a WSN can be increased about 100% by using MPR. The developed MPR model can be extended for multi-hop networks. Optimal solutions for multi-hop WSNs will be addressed in our future work.

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