

# A two array spectrum sensing algorithm based on Duffing oscillator with the parameters adjustable

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**Abstract**—Due to the development of radio service and the limitation of the frequency spectrum, spectrum multiplexing has been attached more and more attention. Spectrum Detection Technology is particularly important. This article has designed a weak signal detection system based on the Duffing oscillator which uses a two-level array detection algorithm to do fast detection. Then the article proposes a new state determination algorithm—TICTACTOEMIDLET algorithm. It uses the distribution character of the sample value of the phase trace to determine the state of the system. The simulation result shows the algorithm can detect the spectrum accurately.

**Keywords:** Spectrum sensing; Weak signal detection; Duffing oscillator; TICTACTOEMIDLET algorithm

## I. INTRODUCTION

Due to the limitation of the frequency spectrum, Spectrum Detection Technology has been a key technique in radio area. As the foundation of the cognitive radio technology, Spectrum Detection Technology is particularly important. However, how to detect the weak signal is always a major problem. In the area of weak signal detection, as a new detecting method, Chaos has developed rapidly since 1990 and many algorithms have been proposed.<sup>[1]</sup> We may use the principle of chaos to detect signal. It can be divided into two categories. One is based on the background of chaos, which identifies and detects signal assuming background noises have the characteristic of chaos. Then the parameters of the noise can be inferred and the useful signal can be detected by

suppressing the noise. The other is based on the sensitive character to initial value of the chaos system. In the system of chaos, a tiny signal can cause the change of whole system. The weak signal can be detected based on the states of the system.

At present, the most common used chaotic model for detection weak signals is Duffing oscillator model. The idea is: the test signal is considered as a complement to specific parameters of chaotic systems, according to the system phase change from chaos to order, the presence of weak signals can be tested.

## II. THE SPECTRUM DETECTION SYSTEM WITH 2 DUFFING OSCILLATOR ARRAY

Duffing oscillator is a Holmes-type oscillator,<sup>[3][6]</sup> the system equation is

$$\ddot{x}(t) = -wk \dot{x}(t) + w^2(ax(t) + bx^3(t) + gs(t)) \quad (1)$$

Its more general expression is

$$\ddot{x}(t) + k \dot{x}(t) - x(t) + x^3(t) = gs(t) \quad (2)$$

Where  $k$  is damping ratio,  $x^3(t) - x(t)$  is non-linear restoring force,  $gs(t)$  is cycle driving motivation and it can be any periodic signal with any frequency.

In practice, this equation can be rewritten as follows:

$$\begin{cases} \dot{x} = y \\ \dot{y} = -ky + x - x^3 + gs(t) + r(t) + n(t) \end{cases} \quad (3)$$

Considering the signal under test  $r(t) + n(t)$  as the input of the chaos system, we can make the states of the system which have a phase change from chaos to order. According to the changes of the states, we can figure out the frequency characters of the signal under test.

When using the state changes of the chaotic systems to detect the signal, we make it by mainly using its transition from the chaotic state to large-scale periodic state. As for the parameter settings of the chaotic system states, there is not a precise formula, mostly by some formula to approximate the critical value. Using Melnikov's method we can obtain a lower limit of its critical value

As for periodic function,  $s(t) = \cos(\omega t)$ , we can set parameters<sup>[4]</sup>

$$g/k > \frac{4 \cosh(\frac{\pi W}{2})}{3\sqrt{2}\pi W} = \frac{\sqrt{2}(e^{\frac{\pi W}{2}} + e^{-\frac{\pi W}{2}})}{3\pi W} \quad (4)$$

when damping ratio  $k$  is fixed, we can obtain the threshold  $G$  corresponding with  $g$  of  $W$ . System changes from chaotic state to large-scale periodic state. In order to make the sensitivity adjustable, the system adds a local oscillator with the frequency  $\omega$ , which amplitude is  $G_0 < G$ , according the difference of  $G_0$ , and the system can detect different input signal.

frequency. After the filter, the signal only has the part in which band needed be perceived. Then the signal is input to the chaotic circuit. By looking up the table, the system can obtain the amplitude of the local oscillator and make it in the chaotic state in advance. If the input has the signal needed to be perceived and the amplitude reaches the threshold of the chaotic circuit, then the state of the chaotic circuit changes from chaotic state to large-scale periodic state. Or the system keeps its former state. The determination of the state changing is done by the state determination unit. It uses TICTACTOEMIDLET algorithm to judge and then obtain the result.

The so-called two array detection algorithm is for spectrum detection requirements in cognitive radio technology. It uses two-level detection to raise the detection accuracy. The realization of the two-level chaotic array is based on the multiplexing of the same chaotic array. After the first cognition, the result processing unit collects cognition result and analyzes it. For the bands which are needed another detection, it generates feedback information, including filter bandwidth, center frequency setting information and information of its local oscillator in the Duffing circuit and its threshold. Then the feedback information returns to the filter, chaotic circuit of the chaotic cognitive unit via setting the parameter and a second cognition will be done. At last, the cognition result will be sent to the processing unit of the next level.

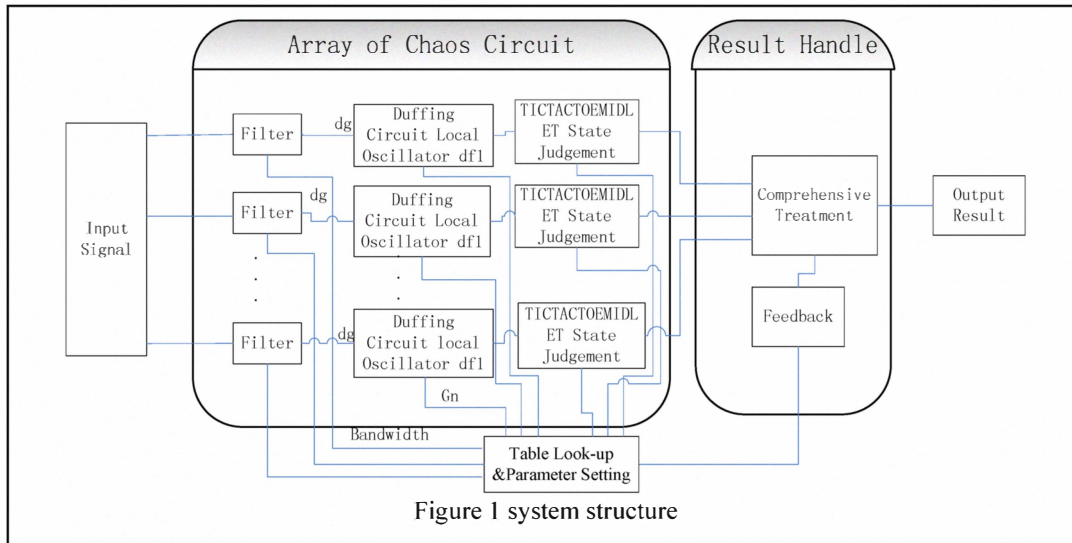


Figure 1 system structure

Based on the principle of Duffing oscillator before, this article has designed a detecting system in figure 1. The system includes several parts as below: Signal acquisition, chaotic circuit array, the result processing units and the threshold table storage unit. The chaotic circuit array includes  $n$  chaotic detecting circuits which is parameter adjustable. Each chaotic detecting circuit includes a narrowband filter, Duffing circuit and the state determine unit. Signal acquisition part put the digital signal into  $n$  narrowband filters before A/D transformation. The center frequency of each narrowband filter and bandwidth is corresponding with the Duffing circuit

In order to raise the rate of detection, a two-level cognition

is adopted which means to multiplex the chaotic array mentioned above. The first level makes a rough detection to the signal. In this step, the frequency bandwidth of each chaotic circuit is wide and the result is the preliminary judging of the spectrum pool. By determining the chaotic state, signals of some band can be eliminated. According to the cognition result, the processing unit generates feedback information, including filter bandwidth and

center frequency setting information and the local oscillator information and threshold of the corresponding Duffing circuit. Then by sending the feedback information via parameter setting unit to the filter, chaotic circuit of the chaotic cognitive unit and carry out second-level cognition. At last, the processing unit outputs the cognition result.

During the second-level cognition, we use variable step methods of secondary cognition: the first step adopts rough perception to eliminate the band with no signals. The second step adopts fine perception to make sure whether there are

signal inputs to the bands which are not eliminated by determining whether the system steps into the large-scale periodic state. The number of the filters in the two-level detection can be set according to the practice. Through this two-level chaotic array we can make an accurate determination of the current special frequency spectrum. The state judging module is used to determine the output of the chaotic array. The proposal in this article mainly uses TICTACTOEMIDLET algorithm to determine the state.

### III. DISCRIMINANT ALGORITHM

The key of spectrum detection using chaotic system is to judge the state of the chaotic system. The input signal spectrum holes can be detected by analyzing state changes of the chaotic system. Now, the discriminant algorithms include direct observation of phase, the Euler Number of the figure and Lyapunov Exponent etc. But there are still some disadvantages which might need the manual intervention or high computational complexity or long time of calculation. A new algorithm is proposed in this paper which cuts the phase trace trajectory into pieces first and then determines the coordinates of the sample of phase trace trajectory and calculate the frequencies of the appearing phase trace trajectory. And we can judge whether it belongs to the chaos state or large-scale periodic state. This algorithm is named TICTACTOEMIDLET algorithm for the region segmentation is alike the TICTACTOEMIDLET in mathematics. The details of the algorithm are described as followed:

We can divide the stimulated figures of both chaos state and periodic state according to a certain proportion. It is showed as followed by  $n*n$  small regions where  $n$  can be any integer and here we set  $n=4$ . In the small region or called cell, the value is set to be 1 if there is phase trace; otherwise the value is 0. If the chaotic system is in the state of chaos, the probability being value 1 of 8 surrounded cells to the one with value 1 is much bigger than that in the state of periodic state.

Chaos circuit always needs certain time to be into chaotic state. To avoid misjudging the state by TICTACTOEMIDLET algorithm, we will operate as followed in each judging process: 1) wait for certain time of the signal period 2) Multiple detection and determine on the average results. So we calculate the  $n*n$  ( $n>4$ ) small regions by the TICTACTOEMIDLET algorithm, and get the results of the cells with phase trace trajectory. The ratio of total numbers of small regions with phase trace  $N_1$  and all small regions  $n*n$  is  $\lambda_1=N_1/n^2$ , and the ratio of total numbers of small regions without phase trace  $N_0$  and all small regions  $n*n$  is  $\lambda_0=N_0/n^2$ . We will set a threshold  $i \in \{0,1\}$  to decide that ratio so to judge the phase state.

There are two ways to set the threshold  $\lambda_d$  of which one is rough perception and the other one is fine perception. The rough perception is used to exclude chaotic state, that is there is

no signal input when  $\lambda_1 > 0.98$ . Fine perception is used to detect the input signal. In the narrow band case, the system enter large-scale periodic state when input signal is bigger than the threshold. If  $n \rightarrow \infty$ , the square measure of regions without phase trace  $S_0$  is almost the same as that of the whole region  $S$ , that is  $S_0/S \rightarrow 1$ . Besides,  $N_0 \rightarrow (n-2)^2$ ,  $\lambda_0 \rightarrow (n-2)^2/n^2$ . According to numeric experiments, we can judge the chaotic state by setting  $\lambda_{0d}=0.8(n-2)^2/n^2$ . There is input signal in the narrow band if  $\lambda_0 > 0.8(n-2)^2/n^2$ , and there is no input signal if  $\lambda_0 > 0.8(n-2)^2/n^2$ . In order to improve the judging accuracy, fine perception would take more time than rough perception. In the detection system in last chapter, the first cascade takes the rough perception and in the second cascade detection will use rough perception to determine the spectrum of the chaos state and then use the fine perception to determine whether system is in large-scale periodic state.

### IV. SIMULATION

For 802.22 protocol, in the band range from 54MHz to 862MHz, the signals need to be perceived mainly include TV signal and wireless microphone signal. Wireless microphone transmission power is about 50mW and its coverage is about 100m and bandwidth is less than 200KHz. Because wireless microphone may appear and disappear suddenly, it can cause great challenge to cognitive technology. However, the method of chaos can guarantee that under a lower signal noise ratio environment it can still detect the signal accurately.<sup>[7]</sup>

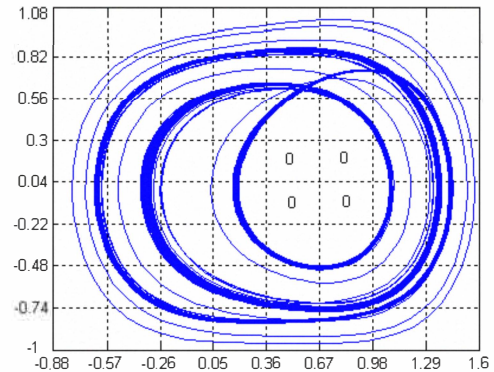


Figure 2 State of uncertainty

Now we simulate the algorithm by detecting whether there is weak signal from 97MHz to 103MHz occupied in 802.22 technique.

According to formula (4), we can obtain the threshold in the test band,  $G=0.6539205348745$ , the amplitude of the local oscillator  $G_0$  is set 0.65. The input is a single frequency signal with the frequency 101.17MHz and its amplitude 0.0040.

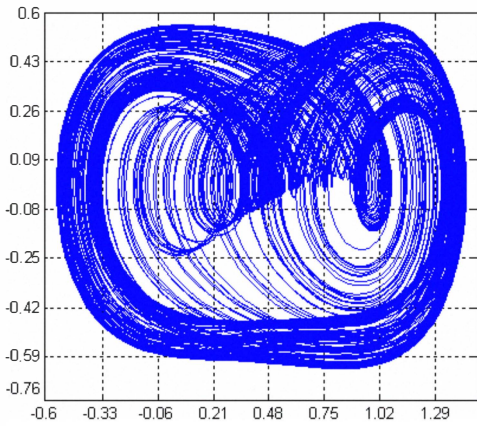


Figure 3 Chaotic

Set the number of filter to be 30 and the band width of each filter is 200KHz. The second cascade filter number is 16 with band width 12.5KHz. When in the rough perception, the TICTACTOEMIDLET is set  $8 \times 8$ , Detection time  $t=300T \approx 0.3mS$ . When in the fine perception, the TICTACTOEMIDLET is set  $16 \times 16$ , Detection time  $t=600T \approx 0.6mS$ . And the detection threshold is  $\lambda_{od}=0.8 \cdot (16-2)^2/16^2=0.6125$ .

Input single with the frequency 101.17MHz and its amplitude 0.0040.

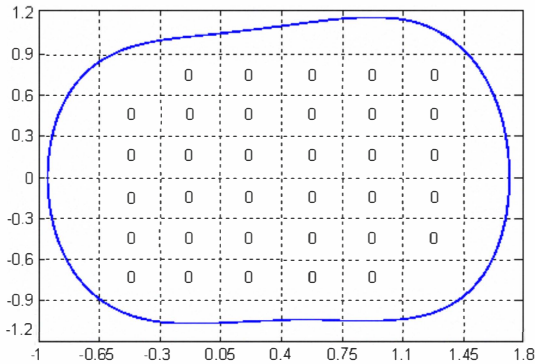


Figure 4 big-scale periodic state(rough cognition)

The first-level adopts rough perception. In the band from 101.0MHz to 101.2MHz,  $\lambda_1=0.9375 < 0.98$ , the simulation result is shown in figure 2. Then we can say the system steps into the large-scale periodic state. In the other band,  $\lambda_1$  equals 1, larger than 0.98, then we can say the system steps into the chaotic state. If we detect the band from 102.6MHz to 102.8MHz, the simulation result is shown in figure 3. Then we do the second-level detection. In this step, firstly a rough perception is done to 16 bands with the width 12.5kHz separately, the result shows in the band from 101.626MHz to 101.175MHz,  $\lambda_1=0.53125 < 0.98$ , the result is shown in figure 4. Then we can say the system steps into the large-scale periodic state and the other  $\lambda_1=1$  larger than 0.98, the system steps into the chaotic state, The simulation result is similar with

figure 3. Then we do fine perception in the band from 101.1625MHz to 101.175MHz, the center frequency is 101.168750Hz and the result is:  $\lambda_0=0.75 > 0.6125$  and the simulation result is shown in figure 5, then we can say the system is in the large-scale periodic state and we can determine there are signals input in these bands. So we can say it can detect the existence of signals.

## V. CONCLUSION

This article designs a two-level array detection system according to Duffing oscillator and proposes a new chaotic determination algorithm--TICTACTOEMIDLET algorithm. The simulation result shows that it can detect the existence of the weak signal accurately. This algorithm gets rid of the manual operation and has a lower computing complexity and fast computing speed. The algorithm has a reference value in practice in the weak signal detection area

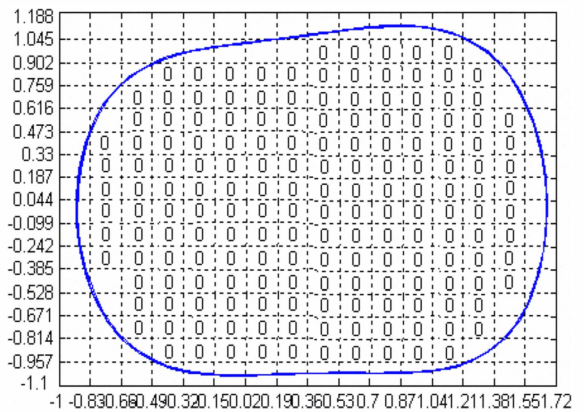


Figure 5 big-scale periodic state(fine cognition)

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