Multiple Two-way Relaying Channels: Precoding Design and Outage Performance Analysis

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Invited Paper

***Abstract*— For a scenario of two-way relaying channel where multiple pairs of source nodes wish to exchange information with their partners, the application of existing network coding protocols developed for a single pair of sources can be applicable by using the time sharing approach, and the number of the required time slots increases as the number of pairs. In this paper, we develop a new network coding transmission protocol, which can make all M pairs of source nodes accomplish information exchanging within two time slots. Particularly we focus on a scenario where all nodes are equipped with multiple antennas and the source nodes have less antennas than the relay. Outage probability and diversity-multiplexing tradeoff, which demonstrate that the proposed transmission protocol can achieve a larger multiplexing gain than the time sharing approach, are developed. Simulation results have been also provided to demonstrate the performance of the proposed network coding.**

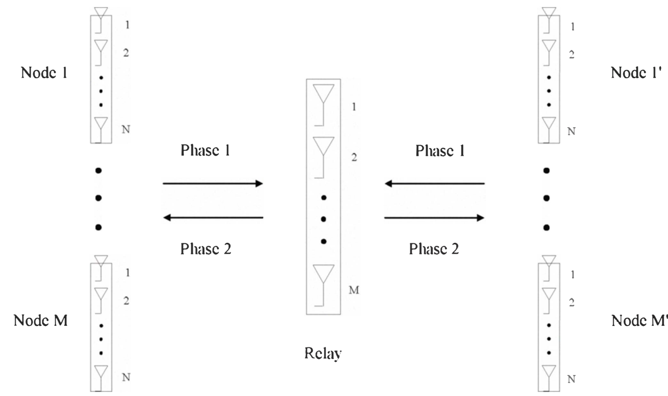

Fig. 1. A system diagram for the addressed communication scenario.

I. Introduction

Network coding was emerged in the context of wireline communications as a powerful mean to increase the overall system throughput [1], [2]. It employs intermediate relays to combine the messages from multiple sources and forward a mixture of these massages to multiple destinations. It is possible that only one relay transmission can serve more than one destinations if each of the destination nodes has a priori information about the mixture. Based on the broadcasting nature of radio propagation, there has recently been a lot of attention to the application of network coding to wireless communications [3]–[5].

For an efficient design of network coding, the type of addressed communication scenarios is important. The so-called butterfly structure has been focused on in [1] and the study of the application of network coding to multiple access channels has been provided in [5], [6]. As one of the communication scenarios which have been achieved most study, two-way relaying channel is increasingly important, especially its typical structure where two sources wish to exchange information with each other with the help of one relay. Such a communication scenario not only gives the best platform to show the benefit of network coding, but also acts as a common building block of wireless communication. The efficient design of network coding for the two-way relaying channel where all nodes are equipped with single antenna has been proposed in [3], [4]. In order to increase system reliability and throughput simultaneously, the application of relay selection to two-way relaying channels has been focused on in [7]. The impact of multiple antennas on two-way relaying channel where there is only one pair of source nodes has been studied in [8], [9].

In this paper, we studied a generalized scenario of two-way relaying channel that consists of one relay and multiple pairs of source nodes which wish to exchange information with their partners, while each source node is equipped with N antennas and the relay is equipped with M antennas. We focus on the scenario where $N \leq M \leq (2N-1)$. By using the time sharing approach, a straightforward application of existing protocols such as the ones in [8], [9] for such a particular scenario can be applicable. That is the existing network coding protocols developed for one pair of sources can be applied when each pair of source nodes is served in an orthogonal channel. Based on the max-flow min-cut theory, we can easily find that by using the time sharing approach, the multiplexing gain of each source is upper bounded $\frac{N}{M}$ which is less than one.

Our main contribution in this paper is to design a new network coding transmission protocol for such multiple two-way relaying channels to push the multiplexing gain of each source to one and provide the analytical results of outage probability and multiplexing gain. The proposed network coding transmission protocol is composed by two phases. All $2M$ source nodes transmit simultaneously during the multiple access phase. If we carefully design the precoders at each

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source, the two messages from the same pair can be aligned with each other at the relay. During the broadcasting phase, the relay broadcasts the M aligned mixtures. Before the second phase we carefully designed the precoding matrix at the relay to cancel cross-pair interference, so that each pair of sources can exchange information with each other without suffering interference from other nodes. When we randomly choose the source procoders, the diversity-multiplexing tradeoff achieved for all sources is $d(r) = 1 - r$. If the source precoders are carefully designed, a better diversity-multiplexing tradeoff, $d(r) = (2N - M)(1 - r)$, can be realized.

The impact of the precoder selection on the total transmission power for the proposed network coding protocol has been studied in detail [11], and the simulation results are provided in this paper.

This paper is organized as follows. The proposed network coding transmission protocol is described in in Section II. Since there are two ways to design the precoding vectors/matrices, two separate sections, Section III and Section IV, have been provided to show the corresponding outage performance and diversity-multiplexing tradeoff. Simulation results have been provided in Section V, where the proposed protocol is compared to existing network coding schemes. Finally, concluding remarks are given in Section VI.

II. Description of the Proposed Network Coding Protocol

Consider a network coding scenario with M pairs of source nodes and one relay. The aim of the designed protocol is to ensure reliable information exchanging between each pair with the help of the relay. The relay is equipped with M antennas, and all other nodes are equipped with N antennas. All wireless channels are assumed to be Rayleigh fading, and the half-duplexing constraint is imposed on all nodes. Time division duplexing is used for simplicity and to provide channel reciprocity. In this paper, we focus on the case of $N \leq M \leq (2N - 1)$. For such a scenario, existing protocols developed for a single pair of sources, such as the ones in [8], [9], can be applied by using the time sharing approach, which can only achieve the multiplexing gain N, $N \leq M$.

A. Protocol Description for the baseline scheme

To simplify notations, we denote the nodes at the left side in Fig. 1 as Node m, and the nodes at the right side in the figure as Node m'. The proposed network coding transmission protocol consists of two phases, one for multiple access and one for broadcasting respectively. During the multiple access phase, all nodes transmit simultaneity, and during the broadcasting phase, each node tries to decode its desirable message from the received mixture. The key idea is to align the messages from the same pair by designing precoders at the relay and sources.

1) Multiple access phase: During the first time slot, all source nodes transmit messages to the relay, and hence the relay observes

$$\mathbf{y}_R = \sum_{m=1}^{M} \mathbf{H}_m \mathbf{u}_m s_m + \sum_{m=1}^{M} \mathbf{H}_{m'} \mathbf{u}_{m'} s_{m'} + \mathbf{n}_R, \quad (1)$$

where s_m is the information symbol from Node m, $\mathbf{y}_R$ is the $M \times 1$ observation vector, $\mathbf{u}_m$ is the $N \times 1$ precoding vector at Node m, $\mathbf{H}_m$ is the $M \times N$ channel matrix for the link between Node m and the relay, and $\mathbf{n}_R$ is the white Gaussian noise vector. $s_{m'}$, $\mathbf{H}_{m'}$ and $\mathbf{u}_{m'}$ are defined similarly.

The key idea of the proposed network coding protocol is to ensure that the two messages from the same pair align to each other, which means

$$\begin{bmatrix}\mathbf{H}_m & -\mathbf{H}_{m'}\end{bmatrix} \begin{bmatrix}\mathbf{u}_m^T & \mathbf{u}_{m'}^T\end{bmatrix}^T = \mathbf{0}_{M\times 1}. \quad (2)$$

Note that $\begin{bmatrix}\mathbf{H}_m & -\mathbf{H}_{m'}\end{bmatrix}$ is a $M \times 2N$ matrix. So as long as $M \leq (2N - 1)$, such $\mathbf{u}_m$ and $\mathbf{u}_{m'}$ always exist. Since the dimension of the null space of $\begin{bmatrix}\mathbf{H}_m & -\mathbf{H}_{m'}\end{bmatrix}$ is $(2N - M)$, we can construct $(2N - M)$ different precoding vectors $\begin{bmatrix}\mathbf{u}_m^T & \mathbf{u}_{m'}^T\end{bmatrix}^T$. In the following, we first focus on the case that the precoding vectors are randomly chosen from the null space of $\begin{bmatrix}\mathbf{H}_m & -\mathbf{H}_{m'}\end{bmatrix}$, and the details of the precoding vector selection will be provided in Section IV.

Denote $\mathbf{x}_m$ as one normalized vector from the null space of $\begin{bmatrix}\mathbf{H}_m & -\mathbf{H}_{m'}\end{bmatrix}$. So $\mathbf{u}_m = \sqrt{2N}\tilde{\mathbf{I}}_1\mathbf{x}_m$ and $\mathbf{u}_{m'} = \sqrt{2N}\tilde{\mathbf{I}}_2\mathbf{x}_m$ where $\tilde{\mathbf{I}}_1 = \begin{bmatrix}\mathbf{I}_N & \mathbf{0}_{N,N}\end{bmatrix}$, $\tilde{\mathbf{I}}_2 = \begin{bmatrix}\mathbf{0}_{N,N} & \mathbf{I}_N\end{bmatrix}$, $\mathbf{I}_n$ is a $n \times n$ identity matrix and $\mathbf{0}_{n,m}$ is a $n \times m$ zero matrix. The reason to have the factor $\sqrt{2N}$ in the expression of $\mathbf{u}_m$ is due to the fact that $\mathbf{x}_m$ is normalized and the total transmission power for one pair is constrained by two $\mathbf{u}_m^H\mathbf{u}_m + \mathbf{u}_{m'}^H\mathbf{u}_{m'} = 2N$. Note that the power constraint for each antenna is assumed to be 1.

Denote $\mathbf{w}_m = \mathbf{H}_m\mathbf{u}_m = \mathbf{H}_{m'}\mathbf{u}_{m'}$, and the signal model at the relay can be written as

$$\mathbf{y}_R = \sum_{m=1}^{M} \mathbf{w}_m(s_m + s_{m'}) + \mathbf{n}_R = \mathbf{W}\mathbf{s}_{\oplus} + \mathbf{n}_R, \quad (3)$$

where $\mathbf{W} = \begin{bmatrix}\mathbf{w}_1 & \cdots & \mathbf{w}_M\end{bmatrix}$ and $\mathbf{s}_{\oplus}$ is a $M \times 1$ vectors containing the M mixtures, $(s_m + s_{m'})$.

2) Broadcasting phase: Before the start of the second phase, a $M \times M$ precoding matrix $\mathbf{P}$ is applied to the observations at relay to facilitate the detection at each destination node. So the signals transmitted by the relay during the second stage are[1]

$$(\mathbf{P}\mathbf{y}_R)^* = (\mathbf{P}\mathbf{W}\mathbf{s}_{\oplus} + \mathbf{P}\mathbf{n}_R)^*. \quad (4)$$

The design of this precoding matrix will be provided at the end of this section. The averaged transmission power at the relay $tr\left\{\mathbf{P}\mathbf{y}_R\mathbf{y}_R^H\mathbf{P}^H\right\}$, and a formal analysis for the relay has been provided in [11]. So at Node m, we have the observations as

$$\mathbf{y}_m = \mathbf{H}_m^H\mathbf{P}\mathbf{W}\mathbf{s}_{\oplus} + \mathbf{H}_m^H\mathbf{P}\mathbf{n}_R + \mathbf{n}_m. \quad (5)$$

Denote $\mathbf{v}_m$ as the coefficient vector for detection, and the signal model at Node m can be written as

$$\tilde{y}_m = \mathbf{v}_m^H\mathbf{H}_m^H\mathbf{P}\mathbf{W}\mathbf{s}_{\oplus} + \mathbf{v}_m^H\mathbf{H}_m^H\mathbf{P}\mathbf{n}_R + \mathbf{v}_m^H\mathbf{n}_m. \quad (6)$$

where $\tilde{y}_m = \mathbf{v}_m^H\mathbf{y}_m$. The precoding matrix, $\mathbf{P}$, the detection vector $\mathbf{v}_m$ and the precoding vector $\mathbf{u}_m$ are the ones to be optimized, and the goal of the design of these parameters is

[1] Note that the conjugate operation is used to simplify the signal model at the source nodes, where the channel matrices, $\mathbf{H}_m$, will be in a form of hermitian transpose as shown in (5).

to ensure that each node only observes the messages from its partner. For example, Node m does not want its observation to contain the messages from other pairs, $(s_i+s_{i'})$ for $i \neq m$.

B. Choice of Precoding matrices $\mathbf{P}$ and the vector $\mathbf{v}_m$

To simplify the computational complexity at each node, we make the precoding vector and the detection vector of each node the same, which means

$$\mathbf{v}_m = \mathbf{u}_m.$$

By using such an equality, the signal model at Node m can be simplified as

$$\mathbf{v}_m^H\mathbf{y}_m = \mathbf{w}_m^H\mathbf{P}\mathbf{W}\mathbf{s}_\oplus + \mathbf{w}_m^H\mathbf{P}\mathbf{n}_R + \mathbf{u}_m^H\mathbf{n}_m. \quad (7)$$

To avoid the co-channel interference, it is desirable to only have $(s_m+s_{m'})$ presented at Node m, which means the design of the precoding matrix should satisfy the following

$$\mathbf{P} = \frac{1}{\sqrt{2}}(\mathbf{W}^H)^{-1}\mathbf{W}^{-1},$$

where the factor $\frac{1}{\sqrt{2}}$ is to meet the transmission power constraint. More detailed analysis for the relay transmission power can be found from [11].

Without loss of generality, we only focus on the M nodes at the left side in Fig. 1. Define $\tilde{\mathbf{y}}$ as the vector containing the observations from all nodes at the left side, e.g., $\tilde{\mathbf{y}} = \begin{bmatrix}\tilde{y}_1 & \cdots & \tilde{y}_M\end{bmatrix}^T$. By using the above precoding matrix, we can have

$$\tilde{\mathbf{y}} = \frac{1}{\sqrt{2}}\mathbf{s}_\oplus + \frac{1}{\sqrt{2}}\mathbf{W}^{-1}\mathbf{n}_R + \tilde{\mathbf{n}}_m, \quad (8)$$

where $\tilde{\mathbf{n}}_m = \begin{bmatrix}\mathbf{u}_1^H\mathbf{n}_1^T & \cdots & \mathbf{u}_M^H\mathbf{n}_M^T\end{bmatrix}^T$. Since each node only observes a mixture of two messages and it has the priori information about the mixture, it can first subtract its own message from the mixture and subsequently decode the message from its partner.

III. Performance Analysis for the Proposed Protocol with Randomly Chosen $\mathbf{u}_m$

To obtain explicit expressions for the outage probability and diversity gain, the probability density function of the vector $\mathbf{w}_m$ is crucial. So in this section, we first evaluate the probabilistic property of $\mathbf{w}_m$, and then provide a theorem for the outage probability achievable for the proposed network coding protocol. To facilitate the analysis, we first define the following two matrices

$$\mathbf{A}_m = \begin{bmatrix}\mathbf{H}_m & -\mathbf{H}_{m'}\end{bmatrix} \quad \& \quad \mathbf{B}_m = \begin{bmatrix}\mathbf{H}_m & \mathbf{H}_{m'}\end{bmatrix}.$$

Recall that the precoding vector of each node is $\mathbf{u}_m = \sqrt{2N}\tilde{\mathbf{I}}_1\mathbf{x}_m$, where $\mathbf{x}_m$ is a vector from the null space of $\mathbf{A}_m$. Since $\mathbf{H}_m\mathbf{u}_m = \mathbf{H}_{m'}\mathbf{u}_{m'}$, we can express $\mathbf{w}$ as

$$\mathbf{w}_m = \sqrt{\frac{N}{2}}\mathbf{B}_m\mathbf{x}_m$$

The following conjecture provides the probability distribution for the elements in the vector $\mathbf{w}_m$.

Conjecture 1: Consider the two $M \times N$ matrices, $\mathbf{H}_m$ and $\mathbf{H}_{m'}$, and assume $M \leq (2N-1)$. All elements of the two matrices are independent and identically complex Gaussian distributed. Based on the two random matrices, we construct the following, $\mathbf{A}_m = \begin{bmatrix}\mathbf{H}_m & -\mathbf{H}_{m'}\end{bmatrix}$ and $\mathbf{B}_m = \begin{bmatrix}\mathbf{H}_m & \mathbf{H}_{m'}\end{bmatrix}$. Denote $\mathbf{x}_m$ is a normalized vector randomly chosen from the null space of $\mathbf{A}_m$. The following holds

$$\mathbf{B}_m\mathbf{x}_m \sim CN(0, \mathbf{I}_M). \quad (9)$$

Conjecture 1 implies that all elements of $\mathbf{B}_m\mathbf{x}_m$ are still complex Gaussian distributed with zero mean and variance one. We are yet to find a formal proof of this, although our simulations indicate that it is the case as shown in [11].

By using Conjecture 1, we can obtain the following theorem about the outage probability achieved by the proposed network coding protocol.

Theorem *2:* Consider all source nodes are equipped with N antennas and the number of the relay antennas is M, where $M \leq (2N-1)$. The outage probability of the m-th source node achieved by the proposed network coding protocol with randomly chosen $\mathbf{u}_m$ and $\mathbf{u}_{m'}$ is

$$P(\mathcal{I}_m \leq R) \leq 1 - e^{-\frac{2/N}{\frac{\rho}{2^R-1}-4N}}, \quad (10)$$

and the achievable diversity-multiplexing tradeoff can be shown as

$$d_m(r) = 1 - r, \quad 0 \leq r \leq 1, \quad (11)$$

where R is the targeted data rate for each source, ρ denotes the signal to noise ratio, $d_m(r)$ is the diversity gain and r is the multiplexing gain as defined in [12], [13].

Proof: The details for the proof can be found from [11]. ■

Theorem 2 provides the outage performance for each source, and the following corollary provides the overall system performance.

Corollary *3:* Consider all source nodes are equipped with N antennas and the number of the relay antennas is M, where $M \leq (2N-1)$. The precoder vectors, $\mathbf{u}_m$ and $\mathbf{u}_{m'}$, are randomly chosen. The diversity-multiplexing tradeoff achievable for the source with the worst outage performance is

$$d_m(r) = 1 - r, \quad 0 \leq r \leq 1. \quad (12)$$

Proof: The proof can be completed by first expressing the outage probability for the user with the worst performance as $P(\mathcal{I}_{min} < R) = \sum_{i=1}^{2M} P(\mathcal{O}_i)$, where $\mathcal{O}_i$ denotes the event that there are i sources experiencing outage. It can be easy to evaluate that the event $\mathcal{O}_1$ dominates the others at high SNR. On the other hand, the probability for $\mathcal{O}_1$ is upper bounded as $P(\mathcal{O}_i) \leq \sum_{m=1}^{2M} P\left(SNR_m < \frac{2^R-1}{\rho}\right)$. By using Theorem 2, the corollary can be easily proved. ■

As shown in Theorem 2 and Corollary 3, The diversity-multiplexing tradeoff for each node is exactly the same as the scenario where one source and destination pair communicate with each other. Or in the other words, the use of the proposed network coding protocol can effectively suppress the co-channel interference across the multiple pairs, and each source can communicate to its partner in a interference-free manner. Another interesting observation is that the proposed protocol can only achieve the diversity gain 1, although we have multiple antennas equipped at each node. This is due to the use of a randomly chosen precoding vector $\mathbf{u}_m$, and it is going to show that a carefully chosen precoding vector can increase the achievable diversity gain, as shown in the next section.

IV. The Proposed Network Coding Protocol with Precoding Vector Selection

Previously we have discussed the scenario where the precoding vector of each node $\mathbf{u}_m$ is chosen randomly from the null space of the matrix $\mathbf{A}_m$. In this section, we discuss how to select appropriate precoding vectors and study the impact of such selected precoders on the outage performance.

Recall that the precoding vectors $\mathbf{u}_m$ and $\mathbf{u}_{m'}$ are obtained from the null space of $\mathbf{A}_m$,

$$\left[\mathbf{u}_m^H \quad \mathbf{u}_{m'}^H\right] \mathbf{A}_m^H = \mathbf{0}_{1\times M}. \tag{13}$$

Given the fact that the size of the dimension of the null space is $(2N-M)$, there is a degree of freedom to choose a vector from the null space of $\mathbf{A}_m$. Since the choice of the precoding vectors $\mathbf{u}_m$ will have direct impacts on the choice of $\mathbf{w}_m$ and the vectors $\mathbf{w}_m$ are important to the SNR at each node, a careful selection of the precoding vectors $\mathbf{u}_m$ is potentially useful to increase reception reliability at all nodes. Specifically the task to select $\mathbf{u}_m$ from the null space of $\mathbf{A}_m$ and maximize the SNR of each node can be formulated as the following optimization problem

$$\begin{aligned} \arg\max_{\mathbf{u}_m^H, \mathbf{u}_{m'}^H} \quad & \min\left\{SNR_1, \cdots, SNR_{M'}\right\}. && (14)\\ s.t. \quad & SNR_m = \frac{\frac{1}{2}}{\frac{1}{2}\frac{1}{\mathbf{w}_m^H(\mathbf{I}_M-\mathbf{Q}_m)\mathbf{w}_m} + \mathbf{u}_m^H\mathbf{u}_m} \\ s.t. \quad & \left[\mathbf{u}_m^H \quad \mathbf{u}_{m'}^H\right] \mathbf{A}_m^H = \mathbf{0}_{1\times M}, \\ & \frac{1}{2}\left[\mathbf{u}_m^H \quad \mathbf{u}_{m'}^H\right] \mathbf{B}_m^H = \mathbf{w}_m^H. \end{aligned}$$

Note the worst user's performance is the bottleneck of the whole system, and hence in the above optimization problem we focus on the user which has the least reception reliability.

Solving the maximization problem in (14) is challenging since the addressed optimization problem is not convex. As a result, the optimal solution of this optimization problem cannot be found in an efficient and low complexity way, and the exhaustive search has to be relied on. In Table I, we show one example of such an exhaustive search based algorithm, where during each iteration a vector is randomly chosen from the null space. Although the dimension of the null space of each $\mathbf{A}_m$ is only $(2N-M)$, there are infinite choices to combine the basis of the null space. As a result, we need to provide a criterion for the algorithm indicating when the search should stop. In practice, such a criterion can be based on the quality of service defined in the system. Obviously the more iterations we use, the better performance we can obtain. But more iterations cause more computational complexity and system overhead. So an important question is *how many iterations are sufficient to obtain a good-enough solution for the optimization problem in* (14).

Since the dimension of the null space of each $\mathbf{A}_m$ is $(2N-M)$, intuitively the maximized diversity gain, or the available degree of freedom, is also $(2N-M)$. And in the following, we show that for the simple exhaustive search based algorithm shown in Table I, $(2N-M)$ iterations are sufficient to guarantee a diversity gain $(2N-M)$ is achievable. As the first step, it is important to understand the impact of different vectors from the null space on the SNR. Denote $\mathbf{x}_{m,i}$ as the i-th vector from the null space of $\mathbf{A}_m$ and $\mathbf{w}_{m,i}$ as the corresponding vector, $\mathbf{w}_{m,i} = \sqrt{\frac{N}{2}}\mathbf{B}_m\mathbf{x}_{m,i}$, $i \in \{1, \dots, 2N-M\}$, and the following corollary gives us the statistic relationship between $\mathbf{w}_{m,i}$ and $\mathbf{w}_{m,j}$, $i \neq j$.

Corollary *4:* Provided that $M < 2N-1$ and all elements of $\mathbf{A}_m$ are independent and identically complex Gaussian distributed, the two vectors, $\mathbf{w}_{m,i}$ and $\mathbf{w}_{m,j}$, are independent to each other.

Proof: The details of the proof can be found from [11]. ■

By using Corollary 4, we can easily obtain the diversity gain achieved by the algorithm shown in Table I as stated in the following theorem.

Theorem *5:* Consider all source nodes are equipped with N antennas and the number of the relay antennas is M, where it is assumed $M \leq (2N-1)$. The precoding vectors for the proposed network coding protocol are obtained by the exhaustive searched-based algorithm shown in Tabel I. $(2N-M)$ iterations are sufficient to achieve the following outage probability

$$P\left(\mathcal{I}_m \leq R\right) \quad \leq \quad (2M)^{2N-M}\left(1 - e^{-\frac{2/N}{\frac{\rho}{2^R-1}-4N}}\right)^{2N-M} \tag{15}$$

and the achievable diversity-multiplexing tradeoff

$$d_m(r) = (2N-M)(1-r), \quad 0 \leq r \leq 1, \tag{16}$$

for each source. And the above outage probability and diversity-multiplexing tradeoff is also achievable for the source with the worst performance.

Proof: The details of the proof can be found from [11]. ■

Comparing Theorem 2 to Theorem 5, evidently the use of the selected precoding vectors can increase the reception reliability. While solving the optimization problem shown in (14) can cause high computational complexity and system overhead, Theorem 5 shows that $(2N-M)$ iterations are sufficient to achieve the diversity gain from one to $(2N-M)$. In practice, the proposed network coding protocol offers a flexible tradeoff between the system complexity and quality of service. Provided that quality of service is the top priority, more iterations will be carried out to refine the system performance. For other scenarios, such as power constrained sensor networks, it is more reasonable to just apply the proposed protocol without precoder selection.

V. Numerical Results

We will evaluate the performance of the proposed network coding protocol by using Monte-Carlo simulations in this

TABLE I

AN EXAMPLE OF EXHAUSTIVE SEARCH BASED ALGORITHMS TO FIND THE OPTIMAL PRECODING VECTORS $\mathbf{u}_m$ AND $\mathbf{u}_{m'}$

Initialization	Set $i = 0$, $\mathbf{u}_m = 0$ and $\mathbf{u}_{m'} = 0$ for all $m \in \{1, \cdots, M\}$ Set the smallest SNR among all nodes $SNR_{min} = 0$
Optimization	while the criterion is not met $i = i + 1$: Randomly choose $\mathbf{u}^i_m$ and $\hat{\mathbf{u}}^i_{m'}$ from the null space of $\mathbf{A}_m$ for all $m \in \{1, \cdots, M\}$ Calculate the corresponding SNR values for all nodes, SNR^i_m Find the smallest SNR among all nodes $SNR^i_{min} = \min\{SNR_1, \cdots, SNR_{M'}\}$ If $SNR_{min} \leq SNR^i_{min}$ $\mathbf{u}_m = \mathbf{u}^i_m$, $\mathbf{u}_{m'} = \mathbf{u}^i_{m'}$ $SNR_{min} = SNR^i_{min}$ end end

section. The traditional physical layer network coding scheme using the time sharing approach will be the comparable scheme, where only one pair of sources are served during each time slot and the M pairs of source nodes are served one by one.

The impact of the precoding selection on the total transmission power has been shown in fig.2. Since we only provide the power constraint for each antenna, the desirable total transmission power should be M, where 1 is the transmission power at each antenna. As can be seen from the figure, the use of the precoding selection can be stabilized the transmission power at the relay. Furthermore, the more choices to select the precoders, the less transmission power the proposed protocol requires. As demonstrated in fig.2, when $2N - M \geq 4$, the proposed network coding scheme consumes the transmission power less than MP.

The outage probability achieved by the proposed transmission protocol has been shown as a function of SNR in fig.3 and fig.4. The number of the source pairs is $M = 3$ in fig.3 and $M = 5$ in fig.4. The targeted data rate is $R = 3$bits/Hz/s in fig.3 and $R = 4$bits/Hz/s in fig.4. In order to show the impact of the precoder selection on the outage performance, different choices of the number of the source antennas have been used. Two criteria for performance evaluation have been used for the two network coding schemes. One criteria is based on the user with the worst performance, and another is based on the average performance. We can easily observe from the two figures that the proposed network coding protocol can achieve better outage performance than the comparable scheme. This is due to the fact that the proposed network coding protocol can ensure M pairs of sources to accomplish information exchanging within two time slots while the comparable scheme requires M time slots. As a result of this, for a targeted data rate, compared to the proposed transmission protocol, the traditional physical layer network coding scheme based on the time sharing approach is more likely to make the outage event happen. Note that by increasing the targeted data rata R such a performance gap between the two network coding schemes can be further enlarged. This fact demonstrates that the proposed network coding protocol is particularly suitable for high-data-rata broadband service.

Particularly, when $2N - M = 1$, the dimension of the null space of $\mathbf{A}_m$ is one, so there is no difference between selecting and not selecting precoding vectors $\mathbf{u}_m$ and $\mathbf{u}_{m'}$. When $2N - M \geq 2$, therefore a careful selection of the precoders can increase the diversity gain achieved by the proposed network coding protocol as indicated by Theorem 5. As can be seen from the two figures, the outage performance achieved by the proposed network coding protocol will be improved by increasing the number of the source antennas, and specifically the slope of the curves of the outage probabilities in the figures becomes larger when increasing N, which demonstrates that the diversity gain becomes larger as increasing N. Note that while the reception reliability achieved by the comparable scheme has also been improved with a larger N, the relationship between the two network coding schemes is remaining the same.

The ergodic capacity achieved by the two network coding schemes have been shown as a function of SNR based on two criteria in fig. 5. The number of the source pairs is M=5. Due to the fact that the co-channel interference has been efficiently handled by using the proposed protocol and M pairs of source nodes can communicate with their partners simultaneously, the proposed network coding protocol can achieve larger ergodic capacity than the time sharing scheme, and such a gap can be further enlarged by increasing the signal-to-noise ratio, which can be seen from the figure. This observation is consistent to Theorem 2 and Theorem 5 which state that the multiplexing gain achievable for each source node is one, exactly the same as the scenario where only one pair of source nodes wish to exchange information with each other.

VI. CONCLUSION

In this paper, we have proposed a new network coding transmission protocol for multiple two way relaying channels. We have developed the analytical results of outage probability and diversity-multiplexing tradeoff, which demonstrate that the proposed transmission protocol can achieve a larger multiplexing gain than the time sharing approach. The simulation results of power consumption and ergodic capacity have also been provided to demonstrate the performance of the proposed network coding scheme.

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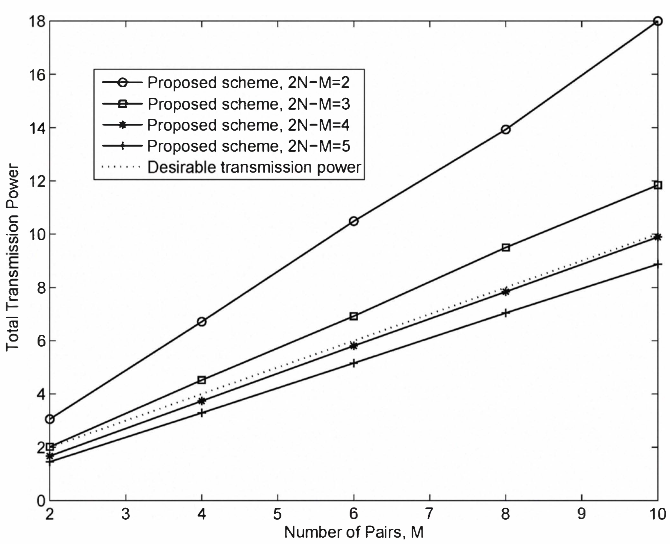

Fig. 2. The impact of the precoder selection on the total transmission power.

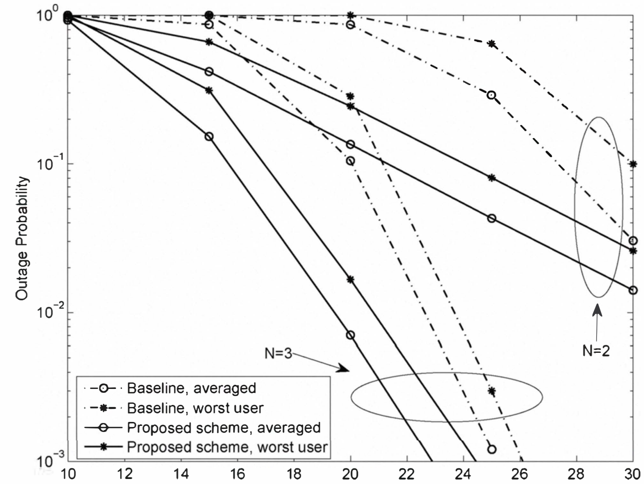

Fig. 3. Outage probability vs SNR. $R = 3$bits/Hz/s. $M = 3$.

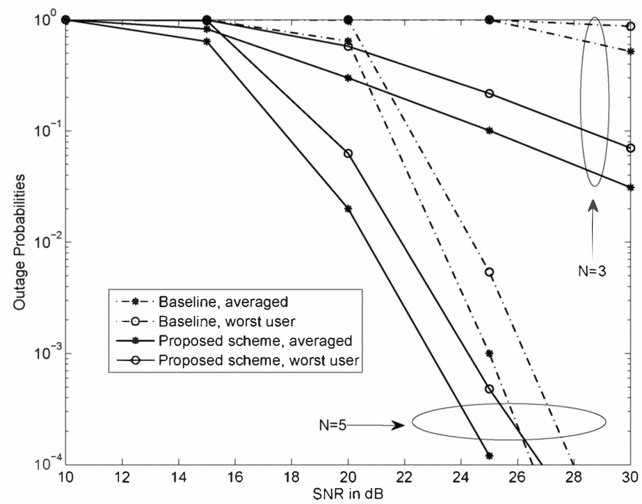

Fig. 4. Outage probability vs SNR. $R = 4$bits/Hz/s. $M = 5$.

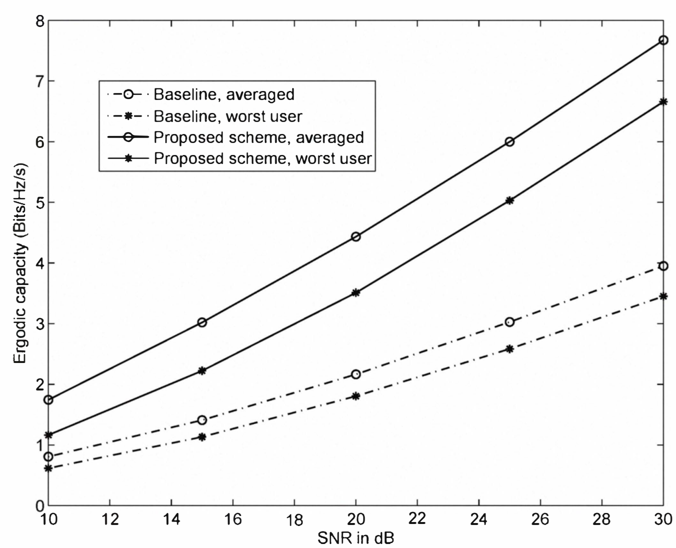

Fig. 5. Ergodic capacity vs SNR. $M = 5$ and $N = 3$.