Concealing Robots in Environments: Enhancing Navigation and Privacy through Stealth Integration

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Abstract

With the continuous advancement of robotics technology, the integration of robots into diverse human environments has become increasingly prevalent. However, the presence of robots in public spaces can often elicit discomfort or unease among individuals. To address this concern, the concept of concealing robots in various settings has emerged as an innovative approach to improve robot navigation and interaction while minimizing intrusion on human privacy. This paper explores the motivations, challenges, and potential benefits of hiding robots in different environments, particularly within the context of swarm robotics where multiple interconnected robots form a cohesive swarm. Equipped with onboard processing, communication, and sensing capabilities, these robots can autonomously interact with each other and adapt to the environment. The paper investigates the problem of maximizing the number of hidden orthogonal swarm robots, considering scenarios in which robots need to navigate and operate within polygonal environments. Specifically, it presents a 4-approximation algorithm for computing a maximum hidden robot set in such environments. The algorithm provides a practical solution for efficiently arranging robots, minimizing their visibility, and ensuring effective swarm operation. It also presents an $O(n^2)$-time 4-approximation algorithm to compute the maximum hidden robot set in simple polygons. The algorithm utilizes a working space of $O(n)$, where n represents the complexity of the environment provided as input. Concealing robots in diverse environments offers several benefits. Firstly, it alleviates discomfort or unease among individuals, facilitating smoother integration of robots into public spaces. Additionally, concealing robots enhances their navigation capabilities by leveraging stealth techniques, allowing them to move seamlessly and unobtrusively within the environment. This approach also promotes improved human-robot interaction, as the reduced visibility of the robots can alleviate concerns and foster a more natural and comfortable interaction between humans and robots. Furthermore, it presents insights into future directions, including the development of more advanced stealth technologies, ethical frameworks for integrating hidden robots, and the potential impact on urban planning and infrastructure. In conclusion, hiding robots in diverse environments offers a promising approach to enhancing robot navigation, interaction, and privacy.

1. Introduction

In recent years, the field of robotics has witnessed remarkable advancements, establishing itself as a pivotal area of technological innovation [1]. Robots have

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seamlessly integrated into various human environments, revolutionizing industries such as manufacturing, healthcare, retail, and public spaces. The continuous evolution of robotics technology has not only expanded the possibilities of automation but has also significantly transformed the way we live and interact with machines [2, 3]. Algorithms have played a central role in driving the progress of robotics [4]. They have become fundamental in shaping our daily routines, revolutionizing decision-making processes [5, 6], system control [7, 8], artificial intelligence, graph theory, motion planning, robot navigation, and various other applications [9–12]. Acting as the backbone of robotic systems, algorithms empower robots to perform complex tasks, adapt to changing environments, and seamlessly interact with humans. Robot navigation exemplifies how algorithms have become indispensable components of our modern existence. Efficient algorithms enable robots to perceive their surroundings, plan optimal paths, and navigate through dynamic environments with precision and safety [13, 14]. As robots increasingly integrate into human environments, ensuring their smooth navigation and interaction with humans becomes crucial for their successful deployment. However, the integration of robots into public spaces can raise concerns among individuals. The presence of robots may evoke discomfort or unease, primarily due to issues related to privacy, safety, or the potential disruption of social norms. Acknowledging and addressing these concerns is vital for fostering public acceptance and enabling harmonious human-robot coexistence. To overcome these challenges, researchers and engineers have started exploring innovative approaches to enhance robot navigation and interaction while mitigating the intrusion on human privacy. Advancements in artificial intelligence and deep learning [15, 16] have significantly contributed to providing more effective solutions to this issue. The capabilities of AI and deep learning algorithms have opened new possibilities in addressing complex challenges related to robotics, particularly in the context of concealing robots in various environments. One such approach is the concept of hiding robots in various environments. By concealing robots within the environment, they can operate discreetly, blending seamlessly into their surroundings and minimizing the impact on human privacy. This paper aims to delve into the motivations, challenges, and potential benefits of hiding robots in diverse settings. It specifically focuses on the context of swarm robotics, where interconnected robots form cohesive swarms that exhibit collective intelligence and enhanced capabilities [17, 18]. Leveraging the advantages of swarm robotics, hidden robots can seamlessly navigate and interact within their environment while ensuring efficient swarm operation. A key aspect of

hiding robots is to maximize the number of hidden robots within a given environment, striking a balance between effective concealment and operational efficiency. This paper addresses the problem of maximizing the number of hidden orthogonal swarm robots in polygonal environments, presenting an efficient 4-approximation algorithm to compute a maximum hidden robot set. Concealing robots in diverse environments offers several potential benefits. Firstly, it fosters improved acceptance of robots in public spaces by alleviating discomfort and unease among individuals. Secondly, hidden robots can navigate seamlessly and unobtrusively, enhancing their overall navigation capabilities. Furthermore, concealing robots addresses privacy concerns, promoting natural and comfortable interactions between humans and robots. The algorithm presented in this paper is novel, offering an efficient approach to maximizing the number of hidden robots while ensuring effective swarm operation. Moreover, our exploration of hiding robots in diverse settings and the discussion of challenges and potential benefits contribute to the understanding of this emerging field. Throughout this paper, we will explore the motivations behind hiding robots in different environments, discuss the challenges involved, and provide insights into the current state of the field. We will delve into existing research, experimental findings, and real-world deployments to understand the practical implications of hiding robots. Additionally, we will discuss future directions, including advanced stealth technologies, ethical considerations, and the potential impact on urban planning and infrastructure. In conclusion, the concept of hiding robots in diverse environments presents a novel approach to enhancing robot navigation, interaction, and privacy. By addressing concerns surrounding the presence of robots in public spaces, we can pave the way for smoother integration and acceptance of robots in our daily lives. The research presented in this paper contributes to the understanding of hiding robots and opens avenues for further exploration and development in this emerging field.

1.1. Background and Related Work

Extensive research has been conducted in the realm of geometric optimization, focusing on the intriguing challenge of safeguarding polygons [19–21]. The primary focus lies in determining the optimal number of sentinels needed to efficiently cover a polygon [22–24], the paramount objective is to guarantee visibility of every point within the polygon by at least one guard, leaving no area unnoticed or unmonitored. [25] Various techniques and algorithms have been developed to address this problem, aiming to optimize guard placement and minimize the number of guards needed. In our research, we specifically

consider the context of orthogonal polygons, which are polygons with edges aligned with the coordinate axes. Researchers have made significant contributions to this area by establishing bounds and developing efficient algorithms for guard placement in orthogonal polygons [26, 27]. For instance, the Rectilinear Art Gallery Theorem, established by Kahn et al., presents a rigorous upper limit on the minimum number of guards needed to protect orthogonal polygons, providing a valuable insight into the optimization of guarding strategies [26]. Sack and Toussaint expanded on this research by presenting algorithmic proofs and efficient techniques for placing guards in monotone rectilinear polygons. Their contributions offered practical solutions and enhanced our understanding of guard placement optimization in this specific polygonal class [27]. Another related problem is polygon covering, which deals with covering a polygon using various geometric objects. TViewed from a different perspective, the Art Gallery Problem can be regarded as a specific instance of polygon covering. It revolves around achieving comprehensive coverage by utilizing the minimum number of star-shaped subpolygons [28]. The field has witnessed significant contributions from Avis and Toussaint, who introduced algorithms for decomposing polygons into sets of star-shaped subpolygons. Their advancements have significantly aided in solving the Art Gallery Problem by providing efficient methods for partitioning polygons into subpolygons that possess the desired star-shaped property [29]. In the context of our work on hiding robots, we address a variant of the guarding problem known as Hidden Guarding, which adds the constraint that no two guards should have direct visibility of each other. This particular variant can be examined in two distinct forms: the maximum hidden (robot) set and the minimum hidden robot set. In the maximum hidden vertex set variant, guards are placed exclusively on the polygon's vertices. Nonetheless, it has been proven that determining the maximum hidden vertex set is computationally NP-hard in terms of computational complexity [30, 31]. The insights gleaned from exploring the problem of guarding polygons have direct implications and practical applications in the realms of robotics and navigation [32]. Efficiently determining the optimal placement of guards holds immense importance in guaranteeing the safety and security of robotic systems functioning in environments characterized by obstacles and constraints. By leveraging the findings of polygon guarding research, we can enhance the effectiveness and reliability of robotic navigation and operation in complex real-world scenarios. By leveraging the principles and techniques developed in the field of guard placement, we can design algorithms and strategies to enhance robot navigation, path planning, and coverage, while minimizing visibility and

intrusion on human privacy. Our research extends the concepts and methodologies established in the field of geometric optimization to the context of robotics. By investigating the problem of maximizing the number of hidden orthogonal swarm robots in polygonal environments, we aim to develop practical algorithms and approaches that enable robots to navigate and interact unobtrusively within their surroundings while ensuring efficient swarm operation. This research contributes to the broader goal of creating seamless and socially acceptable human-robot interaction in real-world environments. In summary, the background and related work in the field of guarding polygons provide valuable insights and algorithmic techniques that can be leveraged to address the challenges of hiding robots and enhance their navigation capabilities in the context of robotics applications. By bridging the gap between geometric optimization and robotics, we aim to contribute to the development of efficient and privacy-conscious robot navigation algorithms.

1.2. Motivation

In recent years, the integration of robots into human environments has become increasingly prevalent, spanning industries such as manufacturing, healthcare, retail, and public spaces. The advancements in robotics technology have brought about numerous benefits, including increased productivity, improved efficiency, and enhanced task performance. However, the presence of robots in public spaces can sometimes give rise to concerns and challenges that need to be addressed for successful human-robot interaction and acceptance. One of the key concerns associated with the presence of robots in public spaces is the potential discomfort or unease experienced by individuals. People may feel apprehensive or uneasy when confronted with robots, particularly when these machines are highly visible or intrude upon personal space. Such discomfort can hinder the acceptance and adoption of robots, limiting their potential to contribute effectively in various domains. Moreover, the issue of privacy intrusion arises when robots operate in close proximity to individuals. Robots equipped with sensors and cameras may inadvertently capture private information or compromise personal privacy. This privacy concern becomes particularly pronounced in public spaces where individuals expect a certain level of privacy and control over their personal information. To mitigate these concerns and challenges, the concept of hiding robots in different environments has emerged as an innovative approach. The overarching motivation behind hiding robots is to enhance their navigation and interaction capabilities while minimizing their visibility and impact on human privacy. By concealing robots within the environment,

they can seamlessly blend into their surroundings, operate unobtrusively, and ensure a more harmonious coexistence with humans. Hiding robots offers several key benefits. Firstly, it addresses the discomfort and unease that individuals may experience in the presence of robots, fostering improved acceptance and integration of robots in public spaces. By reducing the visibility of robots, individuals may perceive them as less obtrusive and disruptive to their daily activities, enabling more natural interactions. Secondly, hiding robots promotes enhanced navigation capabilities. Robots can leverage stealth techniques to move seamlessly and unobtrusively within the environment, avoiding obstacles and minimizing disruptions to human activities. This enhanced navigation not only improves the efficiency of robot operations but also contributes to a safer and more effective coexistence with humans in shared spaces. Furthermore, concealing robots addresses privacy concerns by reducing the visibility of robots and minimizing the potential intrusion on personal privacy. By ensuring that robots are not overtly capturing personal information or impeding individual privacy, the perceived risk associated with robots in public spaces is diminished. This, in turn, fosters a greater sense of trust and comfort in the presence of robots. The motivation to explore and understand the concept of hiding robots extends beyond the immediate challenges of discomfort and privacy. It aligns with the broader objective of creating seamless and effective human-robot interaction in diverse environments. By enabling robots to operate unobtrusively and respectfully within shared spaces, we can harness the full potential of robotics technology to enhance productivity, improve services, and augment human capabilities. In this paper, we delve into the motivations, challenges, and potential benefits of hiding robots in different settings. Specifically, we focus on the context of swarm robotics, where interconnected robots form cohesive swarms capable of exhibiting collective intelligence. By exploring the motivations and addressing the challenges associated with hiding robots, we aim to pave the way for the successful integration of robots into human environments, fostering improved acceptance and enabling robots to contribute effectively in various domains. Overall, the motivation to explore the concept of hiding robots stems from the desire to create a harmonious and productive coexistence between humans and robots. By minimizing discomfort, addressing privacy concerns, and leveraging the advantages of stealthy navigation, we can unlock the full potential of robotics technology and establish a solid foundation for future developments in the field.

1.3. Contribution of this Paper

Within this research paper, we introduce innovative algorithms designed to tackle the hiding robots problem within various polygonal environments. Firstly, we present a linear-time algorithm that effectively calculates a maximum hidden set within histogram polygons. Leveraging the inherent structure of the polygon, this algorithm achieves an impressive time complexity of $O(n)$, where n denotes the total number of edges. Moreover, we propose a 4-approximation algorithm with a time complexity of $O(n^2)$ for identifying the maximum hidden robot set in simple polygons. This approximation algorithm provides a practical and efficient solution to the problem, delivering satisfactory results within a reasonable time frame. This approximation algorithm strikes a balance between computational efficiency and the quality of the solution, providing a practical approach for real-world applications. By providing these algorithms, we offer concrete solutions for optimizing the placement of hidden robots, maximizing their invisibility while ensuring effective swarm operation. These algorithms contribute to the broader goal of seamless human-robot interaction and privacy preservation in diverse environments. Through the integration of insights from geometric optimization and the unique challenges of robotics, we aim to advance the state of the art in robot navigation algorithms. By leveraging the principles and techniques developed in the field of guarding polygons, we can enhance the capabilities of robots, enabling them to navigate unobtrusively and efficiently in real-world scenarios. In the following sections, we will discuss the details of these algorithms, present experimental results, and analyze their effectiveness in different scenarios. By combining the power of geometric optimization with the needs of robotics applications, we strive to create practical solutions that enhance the navigation and interaction capabilities of robots while respecting human privacy and comfort.

2. Preliminaries

Within this section, we will delve into essential concepts and terminologies that hold significance in our investigation of hiding robots within polygonal environments. By familiarizing ourselves with these key components, we can develop a comprehensive understanding of the research context and lay the foundation for our subsequent analysis and findings. These definitions and descriptions lay the foundation for a better understanding of the subsequent discussions and algorithms proposed in this paper. Graph theory [33–35] plays a crucial role in translating real-world scenarios into mathematical problems. In our research, we will harness various components of graph theory to model and analyze the complexities involved in our study. We

begin by emphasizing that in our research, we simulate the environment using polygons and represent other components using geometric entities. Considering an orthogonal polygon P comprising n edges, the fundamental objective of the Polygon Covering Problem is to determine a set of rectangles R with minimal size that completely covers the polygon P. It is crucial to ensure that the union of these rectangles precisely matches the original polygon P, thereby providing a comprehensive and efficient covering solution. The coverage of P by these rectangles ensures that every point within P is contained in at least one rectangle. Within the realm of Polygon Covering, the concept of *rectilinear visibility* (r-visibility) plays a pivotal role. When considering an orthogonal polygon P, two points p and q are deemed *r-visible* if the axis-aligned rectangle formed by these points intersects solely the interior and boundary of P. This form of visibility, rooted in r-visibility, serves as the fundamental criterion for evaluating visibility and facilitating the attainment of thorough coverage within the Polygon Covering problem. In this paper, the term "visibility" specifically refers to r-visibility, unless explicitly mentioned. The *maximum hidden set* in P refers to the largest possible subset of points within P where every pair of points remains invisible from each other. This subset represents an optimal guard set that achieves maximum coverage while minimizing visibility. Similarly, the *maximum hidden vertex set* consists of the largest subset of vertices within P where every pair of vertices remains hidden from each other. Importantly, every maximum hidden set automatically qualifies as a guard set. The concept of the *visibility region* of a point p, denoted as $V_P(p)$, defines a set comprising all points within the polygon P that are visible from p. It is noteworthy that the boundary segments of $V_P(p)$ that do not align with the boundary of P are known as *windows*. These windows offer valuable insights into the visibility properties within P, providing a deeper understanding of the visibility dynamics at play. Let p, x, and y be three points within P. We say that p is *shadow-visible* to line segment xy if there exists a line segment perpendicular to xy that connects p to a point on xy. The *shadow-visibility area* of xy, denoted as $V_P(xy)$, defines a specific subset of points within the polygon P that exhibit shadow-visibility to xy. It is important to note that, when discussing line segments, visibility generally refers to the notion of shadow-visibility area, unless explicitly specified otherwise. An orthogonal polygon P earns the designation of an *r-star polygon* if there exists a point p inside the polygon from which the entire polygon is visibly observable. The existence of such a point guarantees complete visibility coverage of the polygon, emphasizing its significance within the context of r-star polygons. Additionally, we introduce the following definitions: A *vertically monotone polygon*, known as an *x-monotone polygon*, is an orthogonal simple polygon

where every vertical line intersects its boundary at most twice. Similarly, horizontally monotone or *y-monotone polygons* have corresponding properties. A *histogram* is an orthogonal simple polygon that is either vertically or horizontally monotone, with a base edge denoted as b, where the total length of the other horizontal edges equals b. The *upper chain* of a histogram includes all edges except the base edge. A *staircase* is an orthogonal polygon that is both vertically and horizontally monotone. It shares two vertices with the opposite vertices of its bounding box and divides the boundary into an upper chain and a lower chain. An *orthogonal path* represents a connected series of axis-aligned line segments. In graph theory, an *independent vertex set* refers to a subset of vertices within a graph where no two vertices are connected by an edge. This characteristic ensures that the vertices in the independent vertex set do not share any direct adjacency, highlighting their independent nature within the graph. The size of the largest independent vertex set, also known as the *vertex independence number*, represents the independent number of the graph. If an orthogonal polygon exhibits properties of being a histogram, a staircase, and an r-star polygon, it is referred to as a *stairs-shaped* polygon. By establishing these preliminary definitions, we provide a solid foundation for the subsequent discussions and algorithms proposed in this paper. These concepts enable a comprehensive exploration of hiding robots in polygonal environments and the development of effective strategies for maximizing visibility and enhancing robot navigation in such settings.

3. Maximizing Hidden Robot Placement in Pathways: Algorithm for Histogram Polygons

In the following section, we will delve into the algorithm dedicated to identifying a maximum hidden set within a specific type of pathway known as a histogram polygon. A histogram polygon is a simplified form of pathways that exhibits specific characteristics. Our research unfolds with the introduction of a significant theorem, namely Theorem 1. This theorem establishes a crucial result: within a histogram polygon, it is indeed possible to discover a maximum hidden set of robots. Furthermore, we unveil an algorithm that accomplishes this task efficiently. Remarkably, the algorithm operates with time complexity of $O(n)$, where n represents the total number of vertices encompassing the polygon. This theorem and its associated algorithm contribute to advancing our understanding and practical implementation of optimal hiding strategies within histogram polygons.

Theorem 1: Within the realm of histogram polygons, it is possible to construct an algorithm that can identify a maximum hidden set of robots within the polygon.

Remarkably, this algorithm operates with time complexity of $O(n)$, where n denotes the total number of vertices encompassing the polygon.

Proof: Let us consider a histogram polygon as a simplified type of pathway with distinct properties. It consists of connected line segments, resembling pathways, but with a simpler structure and fewer complexities. We begin by initializing two sets: C and S. The set C will store the locations of the robots in the histogram polygon, and the set S will represent the maximum hidden set of robots that we aim to find. Next, we extend each vertical edge of the histogram polygon from its reflex-angle endpoint until it touches the boundary. This extension is performed to ensure complete coverage of the polygon. We add all the obtained rectangles, resulting from the extension of vertical edges to the set C. Each rectangle represents the coverage area of a robot. To address any overlapping or intersecting regions, we merge the rectangles with the same y-coordinate of their upper horizontal edges. This merging process ensures that the coverage areas do not overlap, resulting in an optimized robot placement within the histogram polygon. As a result of the merging process, we obtain a modified set C that represents the minimum polygon covering of the histogram polygon. It ensures that there are no horizontal edges with the same y-coordinate. We compute the middle point of the leftmost horizontal edge on the upper chain of each rectangle in C. These middle points are added to the set S, representing the maximum hidden set of robots. The inclusion of these points ensures that no two robots within the set S have direct visibility to each other. By following the steps outlined above, we can find the maximum hidden set of robots within a histogram polygon. The algorithm in question exhibits a linear time complexity due to the nature of its operations. Tasks such as extending vertical edges, merging rectangles, and computing middle points can all be executed in $O(n)$ time, where n corresponds to the number of vertices present within the histogram polygon. Figure 1 provides a visual reference to better comprehend the algorithm's implementation. Additionally, it is crucial to emphasize that the correctness of the algorithm is guaranteed by the distinct properties inherent to histogram polygons. These characteristics play a vital role in ensuring the algorithm's accuracy and reliability throughout the process of identifying the maximum hidden set of robots. On the upper chain of the histogram polygon, for each horizontal edge with a unique y-coordinate, there exists a robot that provides coverage. When multiple horizontal edges share the same y-coordinate, the algorithm handles the merging process appropriately to guarantee a correct and optimized hidden set of robots. Thus, by utilizing the algorithm designed for histogram polygons, we can efficiently find a maximum hidden set of robots within pathways. The simplicity and specific characteristics of
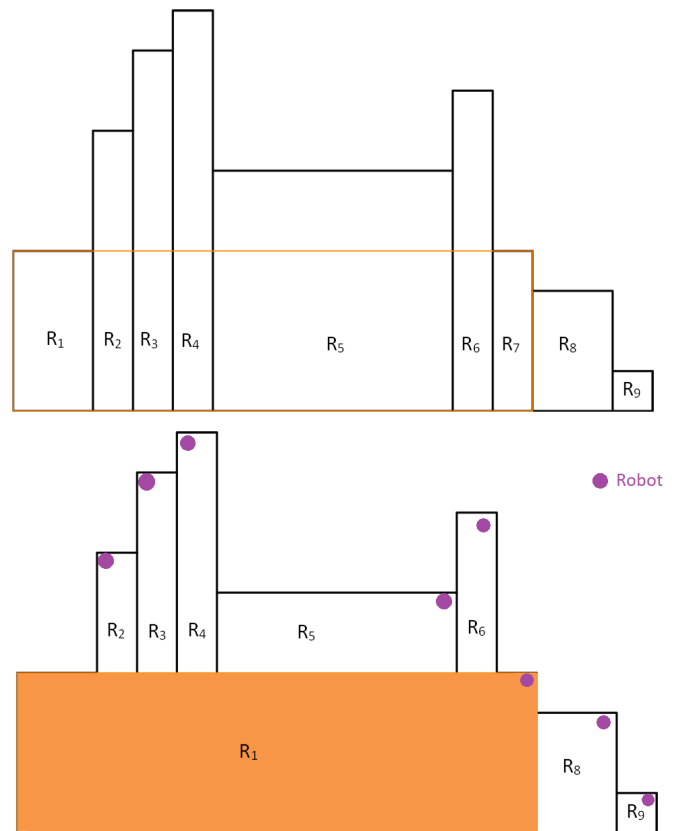

Figure 1. Illustrating the proof of Theorem 1

histogram polygons make them suitable for practical applications where optimizing robot visibility and coverage are crucial.

4. A 4-Approximation Algorithm for Maximizing Robot Hiding in Polygonal Environments

Within this section, our primary objective is to introduce a 4-approximation algorithm specifically tailored to optimize the hiding of robots within polygonal environments. The algorithm's main focus lies in identifying the maximum hidden set of robots within general polygons, which encompass diverse and intricate environments. By capitalizing on the geometric properties and structures inherent in these polygons, we present an efficient algorithm composed of three pivotal steps. The algorithm guarantees a 4-approximation, ensuring a near-optimal solution for maximizing robot hiding within the given polygonal settings.

Step 1: Partitioning into Histograms
We begin by partitioning the given polygon into histograms, which serve as simplified representations of the original environment. This partitioning process

divides the polygon into non-overlapping regions, capturing the essential structure of the environment. Algorithm 1 outlines the procedure for this partitioning, ensuring the creation of distinct histograms that facilitate efficient hiding strategies.

Figure 2. Visualization of an Orthogonal Polygon Partitioned into Histogram Polygons

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Algorithm 1 Orthogonal Polygon Partition into Histograms

Input: An orthogonal polygon

Output: A partition into histogram polygons

1. Compute the visibility region of the lowest horizontal edge.

2. Store this visibility region (*source region*) separately, then remove it from the polygon P and update P.

3. while *All windows are not processed* **do**

- Identify each boundary region of the visibility region that is not part of the boundary of P as a *window*.

- Calculate the visibility region for each window present within the polygon P and add the resulting window(s) to the queue for further processing.

- Save the visibility region of each window, remove it from P, and update P.

end

Algorithm 1 aims to divide an orthogonal polygon into histogram polygons. Firstly, it computes the visibility region of the lowest horizontal edge and stores it separately as the "source region/part" Then, it removes this region from the original polygon and updates it. Next, the algorithm iterates through all windows that are not part of the updated polygon's boundary within the visibility region. For each window, it calculates its visibility region and adds the resulting window(s) to a queue for further processing. The algorithm repeats this process until all windows have been processed. Overall, this algorithm efficiently partitions the orthogonal polygon into histogram polygons.

Step 2: Assigning Numbers to Histograms

Once the partitioning is complete, we assign a number between 1 and 4 to each histogram. Algorithm 2 presents the algorithmic steps to assign numbers based on the adjacency relationships between histograms. This numbering scheme enables the efficient organization and identification of histograms within the polygonal environment.
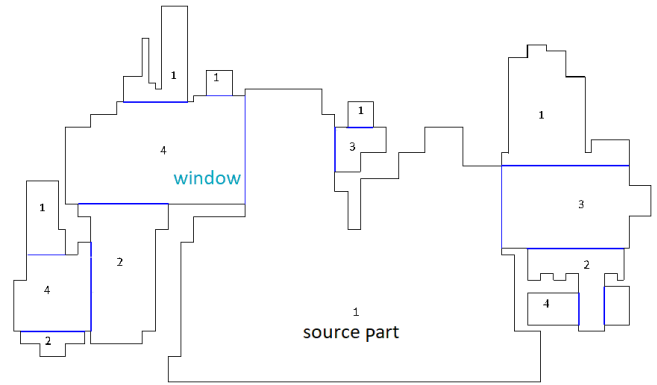

Algorithm 2 Number Assignment Algorithm for Partitioned Histograms

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Input: The output of Algorithm 1, which is a set of histograms after partitioning an orthogonal polygon

Output: A set of labeled histograms

1. Initialize set U containing all parts as unassigned.

2. Create an empty set A for assigned parts.

3. Assign number 1 to the source part.

4. Remove the source part from U and add it to A.

5. **while** $U \neq \{\}$ **do**

   for *each unassigned part* **do**

      if *it is adjacent to a part labeled with 1 or 2 on the right side* **then**

        | Assign it number 4

      else if *it is adjacent to a part labeled with 1 or 2 on the left side* **then**

        | Assign it number 3

      else if *it is adjacent to a part labeled with 3 or 4 on the top side* **then**

        | Assign it number 2

      else if *it is adjacent to a part labeled with 3 or 4 on the bottom side* **then**

        | Assign it number 1

      end

      Remove the part from U and add it to A

   end

end

Algorithm 2 takes as input a set of histograms obtained after partitioning an orthogonal polygon using Algorithm 1. The goal is to assign numbers to the histogram parts based on their adjacency to other labeled parts. The algorithm starts by initializing two sets, U and A, representing unassigned and assigned parts, respectively. It assigns number 1 to the source part, removes it from U, and adds it to A. The algorithm then iterates through all unassigned parts, assigning numbers based on their adjacency to parts labeled

with 1, 2, 3, or 4. Parts adjacent to 1 or 2 on the right side are labeled 4, those adjacent on the left side are labeled 3, those adjacent to 3 or 4 on the top side are labeled 2, and those adjacent on the bottom side are labeled 1. The assigned part is then removed from U and added to A. This process continues until all parts are assigned, resulting in a set of labeled histograms. The algorithm ensures that adjacent parts are given appropriate numbers, contributing to the efficient and accurate organization of the partitioned histogram polygons.

Step 3: Computing Maximum Hidden Sets
In this crucial step, we leverage the results and techniques discussed in Section 3 to compute the maximum hidden set within each histogram. Given the simpler structure of histograms, we can efficiently compute their maximum hidden sets in linear time. This step ensures optimal hiding within each individual region, maximizing the overall hiding capability in the polygonal environment.

In the given scenario, after assigning regions 1, 2, 3, or 4 to the corresponding parts, one of these assigned regions contains the maximum number of hidden robots. We select this region and place hidden robots in it. Although there are three other parts that remain uncovered, even with this selection, we have achieved at least one-fourth of the optimal number of hidden robots. This is due to the efficient allocation of robots in the chosen region and maximizing their concealment.

4.1. Visibility Among Parts Sharing the Same Assigned Number

The inherent invisibility between robots located in different parts of the polygon, sharing the same assigned number (1, 2, 3, or 4), becomes evident upon analysis. Our study demonstrates that when two parts with the same assigned number contain robots not positioned on the base edges, they are unable to perceive each other. This fundamental observation highlights the lack of visibility between robots in separate parts marked with the same number, which plays a crucial role in enhancing the efficiency of our algorithm. By computing the maximum hidden sets for each subset of parts labeled 1, 2, 3, or 4, denoted as R_1, R_2, R_3, and R_4 respectively, our algorithm ensures a 4-approximation for the overall maximum hidden set. Let C_1, C_2, C_3, and C_4 be the total of maximum hidden sets for all parts in R_1, R_2, R_3, and R_4 respectively. Now, let C denote the number of computed maximum hidden sets in the polygon, and let C_{opt} denote the maximum hidden set of the polygon. Then, we have Equation 1:

$$C = \max_{i\in\{1,2,3,4\}}\{C_i\} \geq \frac{1}{4}\cdot\sum_{i\in\{1,2,3,4\}} C_i \qquad (1)$$

$$C_{\text{opt}} \leq \sum_{i\in\{1,2,3,4\}} C_i \qquad (2)$$

$$C_{\text{opt}} \leq 4\cdot C \qquad (3)$$

Equation (1) states that the computed maximum hidden set (C) in the polygon is at least $\frac{1}{4}$ of the sum of the maximum hidden sets in each subset of parts labeled 1, 2, 3, and 4 (C_1, C_2, C_3, and C_4). Equation (2) states that the maximum hidden set of the polygon (C_{opt}) is less than or equal to the sum of the maximum hidden sets in each subset of parts labeled 1, 2, 3, and 4 (C_1, C_2, C_3, and C_4). Equation (3) states that the maximum hidden set of the polygon (C_{opt}) is less than or equal to four times the computed maximum hidden set (C) in the polygon.

4.2. Time and Space Complexity

Algorithm 1 exhibits a running time belonging to $O(n^2)$, while Algorithm 2 operates with a time complexity of $O(n)$. Consequently, the time complexity of the 4-approximation algorithm amounts to $O(n^2)$. The algorithm effectively utilizes $O(n)$ space to store all histogram parts, demonstrating an efficient memory utilization strategy. In summary, we have presented a 4-approximation algorithm that maximizes robot hiding in polygonal environments. By partitioning the polygon into histograms, assigning numbers to each histogram, and computing the maximum hidden sets within them, our algorithm achieves an efficient and effective hiding strategy. With a 4-approximation guarantee, it provides a balance between hiding efficiency and computational feasibility, making it suitable for real-world robotic applications in diverse and complex environments.

5. Conclusion

In conclusion, this paper presents a comprehensive exploration of hiding robots in polygonal environments, with a focus on maximizing the number of hidden robots in pathways. The importance of privacy and unobtrusive navigation in robotics applications is emphasized, highlighting the relevance of studying guarding polygons and geometric optimization. Key concepts and terminologies related to polygon covering, rectilinear visibility, maximum hidden sets, and visibility regions are introduced. Algorithms and approximation techniques are provided for finding maximum hidden robot sets in orthogonal and simple polygonal environments. The algorithms presented in this study deliver a 4-approximation for simple polygons, providing practical solutions with a time complexity of $O(n^2)$. These algorithmic approaches contribute to the realm of efficient and privacy-conscious robot navigation within polygonal environments. By leveraging these algorithms, optimal hiding configurations can be achieved, ensuring both efficiency and privacy

in robotic navigation tasks. By partitioning polygons into histograms and assigning labels to histograms, the algorithms effectively maximize the number of hidden robots while maintaining coverage and visibility constraints.

Future research directions include refining and optimizing the algorithms, extending them to handle complex environments, integrating with path planning and navigation systems, exploring multi-robot systems, and conducting real-world experiments for validation. These efforts aim to advance the development of efficient and privacy-conscious robot navigation algorithms applicable to various robotic applications.

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