Towards Efficient and Fair Resource Allocation in Wireless Networks

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***Abstract*—This paper addresses the problem of incorporating soft and hard QoS support into the traditional utility-based power control problem. We present some novel problem formulations, prove relevant properties of optimal solutions and propose decentralized recursive algorithms with global convergence. Finally, the convergence behavior and the throughput performance are verified numerically.**

I. Introduction

The main objective of classical QoS-based power control is to allocate transmit powers to the users such that each user meets its QoS (Quality of Service) requirement expressed in terms of the signal-to-interference ratio (SIR) (see, for instance, [1] and references therein). In contrast to that, utility-based power control aims at optimizing the overall network performance with respect to some aggregate utility function (see [2] and references therein). This approach has attracted a great deal of attention recently, mainly because of a more efficient utilization of wireless resources. In view of many important applications, however, the main drawback is that, in general, no QoS can be guaranteed, even if given QoS requirements are feasible.

One possible solution is to enforce the QoS requirements by projecting the transmit powers on a set of so-called *valid* transmit powers that provide the necessary QoS values (hard QoS support). Here one of the main problems is to perform the projection operation in a distributed environment. For instance, a simple cyclic projection algorithm [3] in which the users successively perform projections onto the individual sets may require a lot of coordination in a network. In addition to the issue of implementing the projection operation, another key disadvantage of this approach is that there may be no valid power allocation due to the fading effects, in which case no solution exists to the problem as it is impossible to meet the QoS requirements. In distributed wireless networks, an efficient admission control to cope with the infeasibility problem poses a significant challenge as the communication overhead for such a control can explode. For this reason, we actually argue in favor of soft QoS support in which case a solution to the power control problem, which always exists, attempts to approach the QoS requirements closely, provided that the utility functions are chosen appropriately.

In this paper, we reformulate the conventional utility-based power control problem (Eq. (4)) so as to take into account the QoS requirements of the users. Section III is devoted to the utility-based power control with hard QoS support. We consider two different approaches to the problem. In Section IV, we address the problem of utility-based power control with soft QoS support. First we show that a widely-studied max-min SIR balancing solution can be arbitrarily closely approximated by a solution to a slightly modified utility-based power control problem, with a class of utility functions considered, for instance, in [4]. In Section IV-B, we combine this approach with the conventional utility-based power control problem. The main result (Proposition 4) states that under a solution to this problem,

(a) the QoS requirements are met provided that some parameter is sufficiently large and the requirements are feasible,
(b) "extra" resources[1] are allocated to interested users so as to maximize some aggregate utility function.

Section IV-D shows that the problem can be solved in a distributed manner. In Section V, we present some simulation results. The following section introduces the system model and some definitions. We point out that most of the proofs are omitted in this paper. They will be published elsewhere.

II. System Model, assumptions and definitions

We consider a wireless network with an established network topology, in which all links share a common wireless spectrum. Let $K \geq 2$ users compete for access to the wireless links and let $\mathcal{K} = \{1, \dots, K\}$ denote the set of all users. We assume that transmission of all users occur concurrently (no scheduling in the time domain).[2] The transmit powers $p_k, k \in \mathcal{K}$, of the users are collected in the vector $\mathbf{p} = (p_1, \dots, p_K) \geq 0$. For brevity, we assume individual power constraints so that $\mathbf{p} \in \mathrm{P} := \{\mathbf{x} \in \mathbb{R}^K_+ : \forall_{k \in \mathcal{K}} x_k \leq P_k\}$ for some given $P_k > 0$. The main figure of merit is the SIR at the output of each receiver given by

(A.1) $\mathrm{SIR}_k(\mathbf{p}) := p_k / I_k(\mathbf{p}), k \in \mathcal{K}$, where the interference function I_k is $I_k(\mathbf{p}) := (\mathbf{V}\mathbf{p} + \mathbf{z})_k = \sum_{l=1}^{K} v_{k,l} p_l + z_k$.

[1] By "extra" resources, we mean transmit powers that can be allocated after all QoS requirements are satisfied.

[2] However, note that our power control algorithms can be combined with any scheduling policy.

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Here and hereafter, $\mathbf{V} := (v_{k,l}) \in \mathbb{R}_+^{K\times K}$ with

$$v_{k,l} = \begin{cases} V_{k,l}/V_{k,k} & k \neq l \\ 0 & k = l \end{cases}$$

is the gain matrix where $V_{k,l} \geq 0, V_{k,k} > 0$, is the attenuation of the power from transmitter l to receiver k. The kth entry of $\mathbf{z} := (z_1, \ldots, z_K)$ is $z_k = \sigma_k^2/V_{k,k}$, where $\sigma_k^2 > 0$ is the noise variance at the receiver output. It is often reasonable to assume that

(A.2) $\mathbf{V} \geq 0$ with $\mathrm{trace}(\mathbf{V}) = 0$ is *irreducible* [5], [6].

In this case, a network is entirely coupled by interference, meaning that no subnetwork is interference-isolated in the sense that its links perceive no interference from links outside this subnetwork. Note, however, that many results presented in this paper hold for any nonnegative (not necessarily irreducible) matrix.

Let $\omega_k \in \mathrm{Q} \subseteq \mathbb{R}$ be a QoS requirement of user k. Throughout the paper, it is assumed that

(A.3) $\gamma : \mathrm{Q} \to \mathbb{R}_{++}$ is a twice continuously differentiable and strictly *decreasing* function such that $\gamma_k := \gamma(\omega_k)$ is the minimum SIR which is necessary to provide ω_k to user k.

We refer to $\gamma_1, \ldots, \gamma_K \geq 0$ as the *SIR targets*. If $\gamma_k = 0$ for some $k \in \mathcal{K}$, then user k has no QoS requirements, and thus is called a *best-effort user*. Otherwise, we call it a *QoS user*. Now the QoS vector $\boldsymbol{\omega} := (\omega_1, \ldots, \omega_K) \in \mathrm{Q}^K$ is said to be *feasible* if there is a power vector $\mathbf{p} \in \mathrm{P}$ such that $\forall_{k\in\mathcal{K}} \mathrm{SIR}_k(\mathbf{p}) \geq \gamma_k$. Note that since best-effort users have no QoS requirements, they could be denied access to the channel under some power control strategies such as those of [1]. In this paper, however, we consider power control strategies under which each user is allocated a positive transmit power. As a result, we can focus on positive transmit powers and consider all the problems in the *logarithmic* power domain:

$$\mathrm{S} := \{\mathbf{s} \in \mathbb{R}^K : \forall_k e^{s_k} \leq P_k\} = \cap_{k\in\mathcal{K}} \mathrm{S}_k \neq \emptyset$$

where $\mathrm{S}_k := \{\mathbf{s} \in \mathbb{R}^K : e^{s_k} \leq P_k\}$. Throughout the paper, $\mathbf{s} := \log(\mathbf{p}), \mathbf{p} > 0$, (component-wise) is used to denote the (logarithmic) power vector.[3]

Definition 1: Given $\boldsymbol{\omega} \in \mathrm{Q}^K$, we say that $\mathbf{s} \in \mathbb{R}^K$ is *valid* if $\mathrm{SIR}_k(e^{\mathbf{s}}) \geq \gamma_k, k \in \mathcal{K}$. So, the closed set $\mathrm{S}(\boldsymbol{\omega}) := \cap_{k\in\mathcal{K}} \mathrm{S}_k(\boldsymbol{\omega})$ with

$$\mathrm{S}_k(\boldsymbol{\omega}) := \big\{\mathbf{s} \in \mathbb{R}^K : \mathrm{SIR}_k(e^{\mathbf{s}}) = e^{s_k}/I_k(e^{\mathbf{s}}) \geq \gamma_k\big\} \qquad (1)$$

is the set of all valid power vectors. The set

$$\bar{\mathrm{S}}(\boldsymbol{\omega}) := \mathrm{S}(\boldsymbol{\omega}) \cap \mathrm{S} = \cap_{k\in\mathcal{K}} \big(\mathrm{S}_k(\boldsymbol{\omega}) \cap \mathrm{S}_k\big) \qquad (2)$$

is the set of all *feasible* power vectors, which is closed as well.

Definition 2: Let $\boldsymbol{\omega} \in \mathrm{Q}^K$ be given (but not necessarily feasible). Then, $\underline{\mathbf{s}}(\boldsymbol{\omega}) \in \mathrm{S}$ (if exists) is said to be max-min SIR-balanced if and only if (iff)[4]

$$C\big(\boldsymbol{\omega}, \underline{\mathbf{s}}(\boldsymbol{\omega})\big) = C(\boldsymbol{\omega}) := \inf_{\mathbf{s}\in\mathrm{S}} C(\boldsymbol{\omega}, \mathbf{s}) \qquad (3)$$

where $C(\boldsymbol{\omega}, \mathbf{s}) := \max_{k\in\mathcal{K}}(\gamma_k/\mathrm{SIR}_k(e^{\mathbf{s}}))$.

By the definitions, we have the following observations.

Observation 1: $\boldsymbol{\omega} \in \mathrm{Q}^K$ is feasible if $\bar{\mathrm{S}}(\boldsymbol{\omega}) \neq \emptyset$. The converse holds if $\gamma_k > 0, k \in \mathcal{K}$, in which case the infimum in (3) is attained on S and $\underline{\mathbf{s}}(\boldsymbol{\omega}) \in \mathrm{S}$ exists.

Observation 2: For any $\boldsymbol{\omega} \in \mathrm{Q}^K$, $\mathrm{S}_k(\boldsymbol{\omega})$, $\mathrm{S}(\boldsymbol{\omega})$ and $\bar{\mathrm{S}}(\boldsymbol{\omega})$ are convex sets.

Proof: This follows from log-convexity of I_k [2] and the fact that empty sets are convex sets. ■

Observation 3: $\boldsymbol{\omega}$ is feasible iff $C(\boldsymbol{\omega}) \leq 1$.

A. Utility-based power control without QoS support

The traditional utility-based power control problem is[5]

$$\mathbf{s}^* := \arg\min_{\mathbf{s}\in\mathrm{S}} F_e(\mathbf{s}) \qquad (4)$$

where the minimum is assumed to exist and $F_e : \mathbb{R}^K \to \mathbb{R}$ is the aggregate utility function defined to be

$$F_e(\mathbf{s}) := \sum_{k\in\mathcal{K}} w_k \psi(\mathrm{SIR}_k(e^{\mathbf{s}}))\,. \qquad (5)$$

Here and hereafter, $\mathbf{w} = (w_1, \ldots, w_K) > 0$ is a given weight vector and $\psi : \mathbb{R}_{++} \to \mathrm{Q}$ is the inverse function of γ so that $\gamma(\psi(x)) = x, x > 0$. Therefore, $\psi(\mathrm{SIR}_k(\mathbf{p}))$ is the QoS level of user $k \in \mathcal{K}$ under $\mathbf{p}$ or, in other words, $-\psi(\mathrm{SIR}_k(\mathbf{p}))$ represents the degree of user satisfaction with the QoS.

Utility-based power control schemes have been considered in [7], [8], [9]. These papers balance the throughput and fairness performance against power consumption. In [7], the utility function is a decreasing function of power and a concave increasing function of SIR. Reference [9] considers a sigmoid-like utility of SIR and a linear decreasing function of power. The utility function considered in [8] is related to the notion of power efficiency defined as the ratio of data rate and transmit power. All the papers model the power control problem as a noncooperative game where users maximize their utilities. A game-theoretic approach is also taken in [10]. For a class of increasing and strictly concave functions of SIR, the authors propose power control strategies that converge to a global maximum. This function class constitutes a *proper* subset of utility measures defined in this paper. In [11], the problem of joint power control and end-to-end congestion control is addressed. The power control part of [11] assumes $\psi(x) = -\log(x), x > 0$.

In this paper (as in [2]), it is assumed that ψ satisfies the following two conditions (in addition to Condition (A.3))

(A.4) $\lim_{x\to 0} \psi(x) = +\infty \Rightarrow \lim_{x\to 0} \psi'(x) = -\infty$.

(A.5) $\psi_e(x) := \psi(e^x)$ is convex on $\mathbb{R}$.

Prominent examples of functions that satisfy (A.3)–(A.5) are the (negative) logarithmic function $\psi(x) = -\log(x), x > 0$ and $\psi(x) = 1/x^n, x > 0, n \geq 1$. The main reason for considering this class of functions is the following [12], [2].

[3] $\mathbb{R}_{++} \to \mathbb{R} : x \to \log(x)$ denotes the natural logarithm.

[4] Typically, it is reasonable to assume $\gamma_k > 0, k \in \mathcal{K}$.

[5] We formulate the utility-based power control problem as a minimization problem to be conform with standard results from the optimization theory.

Observation 4: F_e is convex on $\mathbb{R}^K$, and thus, by convexity of S, the problem (4) is convex if (A.1) and (A.3)–(A.5) hold. Moreover, if (A.2) is true, then F_e is strictly convex.

This result evolved from the work on the geometry of the so-called feasible QoS region [13], [14], [15]. In [13], [14], we showed that this region is a convex set if the SIR is a log-convex function of a QoS parameter of interest. As a consequence, the problem of optimizing the aggregate QoS of a network over the set of all feasible QoS levels is a convex problem. In [12], it was shown that if the SIR is log-convex in the QoS, then the Karush-Kuhn-Tucker conditions for the corresponding power control problem are necessary and sufficient to characterize an optimal power allocation. Furthermore, if the log-convexity property holds, the power control problem can be converted into a convex optimization problem by the logarithmic transformation of the power vector.

Distributed power control algorithms for the problem (4) that do not resort to the use of classical flooding protocols can be found in [16], [17]. The key ingredient in distributed implementation of these power control schemes is the use of an adjoint network to efficiently distribute some locally measurable quantities to other (logical) transmitters. More precisely, instead of each transmitter sending its message separately as in case of classical flooding protocols, some information is transmitted simultaneously over the adjoint network such that each transmitter can estimate its gradient component from the power of the received signal [16], [2].

The algorithms are recursive in nature and mostly require an appropriate choice of step sizes. Since (adaptive) step size control is difficult to implement in distributed wireless networks, the step size sequence $\{\delta(n)\}$ is usually a non-increasing real-valued sequence satisfying one of the following.

(A.6) $\delta(n)=\delta, n\in\mathbb{N}_0$, for some *sufficiently small* $\delta>0$.

(A.7) $\delta(n)>0, \sum_{n=0}^{\infty}\delta(n)=\infty$ and $\lim_{n\to\infty}\delta(n)=0$.

Note that the second condition might be more reasonable if an algorithm is based on some noisy measurements. An appropriate choice of the step size is a key ingredient to the effectiveness of the algorithm [18]. Throughout the paper, $\{\delta(n)\}$ is an appropriate step size sequence assumed to fulfill either (A.6) or (A.7).

III. Hard QoS Support

Power control with hard QoS support ensures that each link satisfies its QoS requirement, provided that the QoS vector $\boldsymbol{\omega}$ is feasible. So, the problem takes now the following form

$$\mathbf{s}^*(\boldsymbol{\omega}) := \arg\min_{\mathbf{s}\in\bar{\mathrm{S}}(\boldsymbol{\omega})} F_e(\mathbf{s}) \tag{6}$$

where $\bar{\mathrm{S}}(\boldsymbol{\omega})\subset\mathbb{R}^K$ is given by (2) and $F_e:\mathbb{R}^K\to\mathbb{R}$ is defined by (5) with (A.3)–(A.5). Note that $\gamma_k=\gamma(\omega_k)\geq 0$ is not necessarily positive. By Observations 2 and 4, the problem is convex if (A.1) and (A.3)–(A.5) hold. Moreover, (6) has a solution (minimum) if $\bar{\mathrm{S}}(\boldsymbol{\omega})\neq\emptyset$. In this section, we have

(A.8) $\mathrm{int}(\bar{\mathrm{S}}(\boldsymbol{\omega}))\neq\emptyset$. Here and hereafter, $\mathrm{int}(\mathrm{A})$ is used to denote the interior of a set $\mathrm{A}\subset\mathbb{R}^K$ relative to $\mathbb{R}^K$.

This with Observation 2 implies that the Slater constraint qualification for the problem (6) is satisfied [19, p. 371].

Comparing (6) with (4) reveals that the only difference to the traditional utility-based power control is that the projection must be performed on the set $\bar{\mathrm{S}}(\boldsymbol{\omega})=\mathrm{S}(\boldsymbol{\omega})\cap\mathrm{S}\subset\mathrm{S}$, instead of S. As will be seen below, this projection operation may be difficult to realize without a central network controller. In what follows, we present three approaches to (6), two of which solve the problem iteratively. In contrast, the third one only approximates a solution to (6).

A. Gradient projection algorithm

A straightforward approach is to apply gradient-projection methods to the problem (6). In this case, the iteration takes the form

$$\mathbf{s}(n+1)=\Pi_{\bar{\mathrm{S}}(\boldsymbol{\omega})}\Big[\mathbf{s}(n)-\delta(n)\nabla F_e(\mathbf{s}(n))\Big] \tag{7}$$

where $\Pi_{\bar{\mathrm{S}}(\boldsymbol{\omega})}(\mathbf{x})$ denotes the projection (with respect to the Euclidean norm) of $\mathbf{x}\in\mathbb{R}^K$ onto $\bar{\mathrm{S}}(\boldsymbol{\omega})$, which is well-defined [19, pp. 88–90], and the kth entry of the gradient vector $\nabla_k F_e(\mathbf{s})$ yields [2]

$$\nabla_k F_e(\mathbf{s}) = e^{s_k}\Big(g_k(\mathbf{s})-\sum_{l\neq k} v_{l,k}\mathrm{SIR}_l(e^{\mathbf{s}})g_l(\mathbf{s})\Big) \tag{8}$$

with

$$g_k(\mathbf{s}) = w_k\psi'\big(\mathrm{SIR}_k(e^{\mathbf{s}})\big)/I_k(e^{\mathbf{s}})\,. \tag{9}$$

The gradient vector can be computed in a distributed manner using the adjoint network [2], [16]. So, the main problem is that now the projection must be performed on the closed set $\bar{\mathrm{S}}(\boldsymbol{\omega})$. Since $\bar{\mathrm{S}}(\boldsymbol{\omega})$ is the intersection of convex sets $\mathrm{S}_k(\boldsymbol{\omega})\cap\mathrm{S}_k$ (see (2) and Observation 2), the projection operation can be accomplished by means of a so-called cyclic projection algorithm [3], in which case the users successively perform their own projections, with user k projecting on $\mathrm{S}_k(\boldsymbol{\omega})\cap\mathrm{S}_k$. Writing $\mathrm{S}_k(\boldsymbol{\omega})$ as

$$\mathrm{S}_k(\boldsymbol{\omega}) := \big\{\mathbf{s}\in\mathrm{S}:\mathbf{a}_k^T e^{\mathbf{s}}=\mathbf{a}_k^T\mathbf{p}\leq-\gamma_k z_k\big\} \tag{10}$$

with $\mathbf{a}_k=\big(\gamma_k v_{k,1},\dots,\gamma_k v_{k,k-1},-1,\gamma_k v_{k,k+1},\dots,\gamma_k v_{k,K}\big)$ shows that the projection on valid power region $\mathrm{S}_k(\boldsymbol{\omega})$ of user $k\in\mathcal{K}$ is ($\mathbf{p}=e^{\mathbf{s}}$)

$$\Pi_{\mathrm{S}_k(\boldsymbol{\omega})}(\mathbf{s})=\log\Big(\mathbf{p}-\frac{\max\{0,\mathbf{a}_k^T\mathbf{p}+\gamma_k z_k\}}{\|\mathbf{a}_k\|_2^2}\mathbf{a}_k\Big) \tag{11}$$

which, if needed, must be corrected appropriately to satisfy the power constraint. However, we already see from (11) that the cyclic projection algorithm is in general not amenable to distributed implementation as the operation may require a lot of coordination between the nodes.

B. A distributed primal-dual algorithm

In order to simplify the projection problem, we now consider a primal-dual algorithm and show that it can be implemented in a distributed manner.

To this end, let $L : \mathrm{S} \times \mathbb{R}_+^{|\mathcal{A}|} \to \mathbb{R}$ be an associated Lagrangian function defined to be[6]

$$L(\mathbf{s}, \boldsymbol{\lambda}) := F_e(\mathbf{s}) + \sum\nolimits_{k\in\mathcal{A}} \lambda_k\, f_k(\mathbf{s}) \tag{12}$$

where $\mathbf{s} \in \mathrm{S}$, $\mathcal{A} = \{k \in \mathcal{K} : \gamma_k > 0\}$, $\boldsymbol{\lambda} = (\lambda_1, \dots, \lambda_{|\mathcal{A}|}) \geq 0$ are dual variables and $f_k : \mathbb{R}^K \to \mathbb{R}, k \in \mathcal{A}$, are defined to be

$$f_k(\mathbf{s}) := I_k(e^{\mathbf{s}})/e^{s_k} - 1/\gamma_k,\ k \in \mathcal{A}\,. \tag{13}$$

Observation 5: The function $f_k : \mathbb{R}^K \to \mathbb{R}$ given by (13) are convex for each $k \in \mathcal{A}$. Hence, by Observation 4, the Lagrangian function is a *convex-concave function* [20].

Proof: This immediately follows since $I_k(e^{\mathbf{s}})/e^{s_k}$ is log-convex on $\mathbb{R}^K$ and hence also convex [2]. ■

Since $f_k(\mathbf{s}) \leq 0, k \in \mathcal{A}$, for any fixed $\mathbf{s} \in \mathrm{S}(\boldsymbol{\omega}) \neq \emptyset$, $L(\mathbf{s}, \boldsymbol{\lambda}) \leq L(\mathbf{s}, \mathbf{0})$ for all $\mathbf{s} \in \mathrm{S}(\boldsymbol{\omega})$ and all $\boldsymbol{\lambda} \geq 0$. Hence, for any given $\mathbf{s} \in \bar{\mathrm{S}}(\boldsymbol{\omega}) \neq \emptyset$, $L(\mathbf{s}, \boldsymbol{\lambda})$ attains its maximum (which exists) at $\boldsymbol{\lambda} = \mathbf{0}$, From this and unboundedness of $L(\mathbf{s}, \cdot)$ on $\mathbb{R}_+^{|\mathcal{A}|}$ for any $\mathbf{s} \notin \mathrm{S}(\boldsymbol{\omega})$, it follows that $\min_{\mathbf{s}\in\bar{\mathrm{S}}(\boldsymbol{\omega})} F_e(\mathbf{s}) = \min_{\mathbf{s}\in\mathrm{S}} \max_{\boldsymbol{\lambda}\geq\mathbf{0}} L(\mathbf{s}, \boldsymbol{\lambda})$, and therefore

$$\mathbf{s}^*(\boldsymbol{\omega}) = \arg\min_{\mathbf{s}\in\mathrm{S}} \max_{\boldsymbol{\lambda}\geq 0} L(\mathbf{s}, \boldsymbol{\lambda})\,. \tag{14}$$

The corresponding dual problem to (6) is defined to be

$$\boldsymbol{\lambda}^*(\boldsymbol{\omega}) := \arg\max_{\boldsymbol{\lambda}\geq 0} \min_{\mathbf{s}\in\mathrm{S}} L(\mathbf{s}, \boldsymbol{\lambda}) \tag{15}$$

where the minimum (for any $\boldsymbol{\lambda} \geq 0$) and the maximum can be shown to exist. Now an application of standard results from convex optimization theory [19, pp. 355–371] and [20] together with (A.8) and Observation 5 proves the following.

Observation 6: Strong duality holds and $(\mathbf{s}^*(\boldsymbol{\omega}), \boldsymbol{\lambda}^*(\boldsymbol{\omega})) \in \bar{\mathrm{S}}(\boldsymbol{\omega}) \times \mathbb{R}_+^{|\mathcal{A}|}$ is a saddle-point of the Lagrangian L. Moreover, the complementary slackness conditions are satisfied.

All these observations imply that the Karush-Kuhn-Tucker (KKT) conditions provide necessary and sufficient conditions for optimality. In other words, the pair $(\mathbf{s}^*(\boldsymbol{\omega}), \boldsymbol{\lambda}^*(\boldsymbol{\omega}))$ provides optimal solutions to the primal and dual problems iff it satisfies the KKT conditions [21]. As a consequence, we can solve the problem (6) by solving the KKT conditions. Since the complementary slackness conditions are satisfied and the problem is convex, this is equivalent to finding a stationary point of the Lagrangian function (12), which is the saddle point $\mathbf{x}^*(\boldsymbol{\omega}) := (\mathbf{s}^*(\boldsymbol{\omega}), \boldsymbol{\lambda}^*(\boldsymbol{\omega})) \in \bar{\mathrm{S}}(\boldsymbol{\omega}) \times \mathbb{R}_+^{|\mathcal{A}|}$.

In order to find $\mathbf{x}^*(\boldsymbol{\omega})$, we apply a primal-dual algorithm of the following form (with $\mathbf{x} = (\mathbf{s}, \boldsymbol{\lambda}) \in \mathbb{R}^K \times \mathbb{R}_+^{|\mathcal{A}|}$):

$$\mathbf{x}(n+1) = \Pi_{\mathrm{X}}\Big[\mathbf{x}(n) - \delta(n)\Big(\begin{smallmatrix}\mathbf{I}_K & \mathbf{0}\\ \mathbf{0} & -\mathbf{I}_{|\mathcal{A}|}\end{smallmatrix}\Big)\nabla L(\mathbf{x}(n))\Big] \tag{16}$$

where $\mathbf{I}_m$ is the identity matrix of dimension m, $\mathbf{0}$ is a suitable zero matrix, $\nabla L(\mathbf{x})$ is the gradient vector with respect to $\mathbf{x} = (\mathbf{s}, \boldsymbol{\lambda})$, and

$$\begin{aligned}\Pi_{\mathrm{X}}(\mathbf{x}) = \big(&\min\{s_1, \log(P_1)\}, \dots, \min\{s_K, \log(P_K)\},\\ &\max\{0, \lambda_1\}, \cdots, \max\{0, \lambda_{|\mathcal{A}|}\}\big)\,.\end{aligned}$$

[6]In the analysis, it is often useful to define L over $\mathbb{R}^K \times \mathbb{R}^{|\mathcal{A}|}$ with $L(\mathbf{s}, \boldsymbol{\lambda}) = +\infty$ if $\mathbf{s} \notin \mathrm{S}, \boldsymbol{\lambda} \in \mathbb{R}_+^{|\mathcal{A}|}$ and $L(\mathbf{s}, \boldsymbol{\lambda}) = -\infty$ if $\boldsymbol{\lambda} \notin \mathbb{R}_+^{|\mathcal{A}|}$.

Computing the partial derivatives of L shows that the algorithm (16) takes the form

$$\begin{cases} s_k(n{+}1) = \min\Big\{s_k(n) - \delta(n)\Big[g_k(\mathbf{s}(n))e^{s_k(n)} \\ \qquad - \dfrac{\mu_k(n) I_k(e^{\mathbf{s}(n)})}{e^{s_k(n)}} \\ \qquad + e^{s_k(n)}\Sigma_k\big(\mathbf{s}(n), \boldsymbol{\mu}(n)\big)\Big], \log(P_k)\Big\}, k \in \mathcal{K} \\ \lambda_k(n{+}1) = \max\big\{0, \lambda_k(n) + \delta(n) f_k(\mathbf{s}(n))\big\}, k \in \mathcal{A} \\ \mu_k(n{+}1) = \lambda_k(n+1), k \in \mathcal{A} \\ \mu_k(n{+}1) = 0, k \in \mathcal{K}\setminus\mathcal{A} \end{cases} \tag{17}$$

where the iterations are performed simultaneously and $\Sigma_k : \mathbb{R}^K \times \mathbb{R}_+^K \to \mathbb{R}$ is given by (note that $v_{k,k} = 0$)

$$\begin{aligned}\Sigma_k(\mathbf{s}, \boldsymbol{\mu}) &= \sum\nolimits_l v_{l,k}\Big(\frac{\mu_l}{e^{s_l}} - \mathrm{SIR}_l(e^{\mathbf{s}}) g_l(\mathbf{s})\Big)\\ &= \sum\nolimits_l v_{l,k}\Big(\frac{\mu_l}{e^{s_l}} + \big|\mathrm{SIR}_l(e^{\mathbf{s}}) g_l(\mathbf{s})\big|\Big)\\ &= \sum\nolimits_l v_{l,k} m_l(\mathbf{s}, \mu_l),\ \boldsymbol{\mu} = (\mu_1, \dots, \mu_K)\,.\end{aligned} \tag{18}$$

The last step follows from the fact that $g_l(\mathbf{s})$ defined by (9) is negative on $\mathbb{R}^K$ since ψ is strictly decreasing.

Now considering [20, Theorem 10] (and the discussion after the theorem) as well as Observations 4, 5 and 6, we can show the following.

Proposition 1: If (A.2) hold, then the sequence $(\mathbf{s}(n), \boldsymbol{\lambda}(n))$ generated by (16) converges to a saddle point $(\mathbf{s}^*(\boldsymbol{\omega}), \boldsymbol{\lambda}^*(\boldsymbol{\omega})) \in \bar{\mathrm{S}}(\boldsymbol{\omega}) \times \mathbb{R}_+^{|\mathcal{A}|}$ given by (14) and (15). Moreover, $\mathbf{s}^*(\boldsymbol{\omega})$ minimizes $F_e(\mathbf{s})$ over $\bar{\mathrm{S}}(\boldsymbol{\omega})$.

The algorithm can be implemented in distributed wireless networks using the scheme based on the adjoint network [2], [16]. Except for $\Sigma_k(\mathbf{s}(n), \boldsymbol{\mu}(n)) = \sum_{l\neq k} v_{l,k} m_l(\mathbf{s}(n), \mu_l(n))$ given by (18), all the other quantities (such as the weights w_k) are either known locally or can be computed from local measurements (such as the SIR) and, if necessary, conveyed to the corresponding transmitter/receiver by means of a low-rate control channel. In contrast, $\Sigma_k(\mathbf{s}(n), \boldsymbol{\mu}(n))$ can be estimated from the received signal power in the adjoint network with the exchanged roles of transmitters/receivers and a channel inversion on each link as described in [16]. The only difference to the scheme of [16] is that each receiver, say receiver $l \in \mathcal{K}$, transmits in the nth iteration a sequence of independent zero-mean random symbols with the variance being equal to $m_l(\mathbf{s}(n), \mu_l(n))$ defined in (18). The main steps of the distributed power control scheme is summarized below.

The algorithm assumes that the weight w_k is known at both the transmitter side and the receiver side of link $k \in \mathcal{K}$. Also, all transmitters and receivers must know the function ψ and its derivative. If different functions $\psi_k, k \in \mathcal{K}$, are associated with distinct links, then it is sufficient that each transmitter-receiver pair has only a local knowledge of the function associated with the respective link.

The main disadvantages of the algorithm are, in general, the lack of monotonicity and a relatively low convergence rate. Monotonicity of an algorithm means that the performance (with respect to the aggregate utility function) is improved

Algorithm 1 Distributed primal-dual algorithm

Require: $\mathbf{w} > 0, \epsilon > 0, n = 0, \mathbf{s}(0) \in \mathrm{S}$, $\boldsymbol{\omega}$ with $\bar{\mathrm{S}}(\boldsymbol{\omega}) \neq \emptyset$, non-increasing step size sequence $\{\delta(n)\}_{n\in\mathbb{N}_0}$.
Ensure: $\mathbf{s} \in \mathrm{S}$
1: **repeat**
2: Concurrent transmission at transmit powers $e^{s_k(n)}, k \in \mathcal{K}$, with receiver-side estimation of $I_k(e^{\mathbf{s}(n)})$ and $\mathrm{SIR}_k(e^{\mathbf{s}(n)})$.
3: Each transmitter-receiver pair exchanges on a per-link basis some of the estimates and variables including $I_k(e^{\mathbf{s}(n)}), s_k(n), k \in \mathcal{K}$ and $\lambda_k(n), k \in \mathcal{A}$.
4: Concurrent transmission in the adjoint network with transmitter-side estimation of the received power [16], [2]. The variance of the zero-mean input symbols is $m_k(\mathbf{s}(n), \mu_k(n))$ given by (18).
5: Transmitter-side computation of $s_k(n+1)$ and $\lambda_k(n+1), k \in \mathcal{K}$, according to (17).
6: $n = n + 1$
7: **until** $|F_e(\mathbf{s}(n)) - F_e(\mathbf{s}(n-1))| < \epsilon$

in each iteration step, which may be important in practice where only a relatively small number of iterations is carried out. Note that this property is provided by the gradient-projection algorithm discussed before. The convergence rate in turn can be improved by considering a modified (non-linear) Lagrangian function, allowing us to avoid the projection operation completely as discussed in [22], where the problem of power and interference control was addressed. Reference [17] presents a distributed Newton-like algorithm that provides quadratic convergence rate but with global convergence guaranteed for a smaller class of utility functions.

C. A simple barrier algorithm

Another possibility for ensuring the QoS requirements is to exploit barrier properties of the function ψ. Indeed, $\boldsymbol{\omega}$ is feasible if there is $\mathbf{s} \in \mathrm{S}$ such that $\mathrm{SIR}_k(e^{\mathbf{s}}) - \gamma_k > 0$. So, since ψ fulfills (A.4) and (A.8) holds, this suggests using ψ as a barrier function to ensure QoS requirements. More precisely, the idea is to minimize

$$F_\Theta(\mathbf{s}) := \sum_{k\in\mathcal{K}} w_k \psi\big(\Theta_k(\mathbf{s})\big), \mathbf{s} \in \mathrm{int}(\mathrm{S}(\boldsymbol{\omega})) \tag{19}$$

over $\mathrm{int}(\bar{\mathrm{S}}(\boldsymbol{\omega})) \neq \emptyset$ where $\Theta_k : \mathbb{R}^K \to \mathbb{R}$ is defined to be

$$\Theta_k(\mathbf{s}) := \mathrm{SIR}_k(e^{\mathbf{s}}) - \gamma_k,\ k \in \mathcal{K}\,. \tag{20}$$

Formally, the power control problem under consideration is formulated as follows:

$$\bar{\mathbf{s}}^*(\boldsymbol{\omega}) := \underset{\mathbf{s}\in\mathrm{int}(\bar{\mathrm{S}}(\boldsymbol{\omega}))}{\arg\min}\ F_\Theta(\mathbf{s}) \tag{21}$$

where (A.8) is assumed to hold.

Lemma 1: If $\psi : \mathbb{R}_{++} \to \mathrm{Q}$ satisfies (A.3)–(A.5), then $F_\Theta(\mathbf{s})$ defined by (19) is a convex function of $\mathbf{s} \in \mathrm{int}(\mathrm{S}(\boldsymbol{\omega}))$.

Proof: The lemma follows from the fact that $\Theta_k(\mathbf{s}(\mu))$ is a log-concave function of $\mu \in (0, 1)$. Thus, due to (A.5), proceeding as in [16] shows that F_Θ is convex. ∎

As an immediate consequence of Observation 2 and the above lemma, we have the following proposition.

Proposition 2: If (A.1) and (A.3)–(A.5) holds, then the problem (21) is a convex optimization problem.

The problem (21), can be solved using, for instance, a gradient projection algorithm similar to that used for solving the problem (4) and presented in [16], [2]. In fact, the only differences are:

(a) A start point $\mathbf{s}(0)$ must be a valid power allocation so that $\mathbf{s}(0) \in \mathrm{int}(\mathrm{S}(\boldsymbol{\omega})) \neq \emptyset$.
(b) The entries of the gradient vector ∇F_Θ are given by the right-hand side of (8) with $g_k(e^{\mathbf{s}})$ given by

$$g_k(\mathbf{s}) = w_k \psi'(\Theta_k(\mathbf{s}))/I_k(e^{\mathbf{s}}), k \in \mathcal{K}\,. \tag{22}$$

Again the algorithm can be implemented in a distributed manner using the handshake protocol from [16], [2]. All the differences to the scheme presented in [16], [2] result from using $\Theta_k(\mathbf{s})$ as the basic performance measure of link k, instead of the SIR. However, note that $\Theta_k(\mathbf{s})$ and with it also $g_k(\mathbf{s})$ can be easily computed at the node where link k originates if the corresponding SIR is known. The variance of random symbols transmitted over the adjoint network should be equal to $|\mathrm{SIR}_l(e^{\mathbf{s}}) g_l(\mathbf{s})|$ with g_l defined by (22).

IV. Soft QoS support

A. Approximation of Max-Min SIR balancing

One possibility for incorporating QoS into the utility-based power control is to approximate a max-min SIR-balanced power vector $\underline{\mathbf{s}}(\boldsymbol{\omega})$, which, by Observation 1, exists if

(A.9) $\gamma_k > 0$ for each $k \in \mathcal{K}$ (assumed in this subsection).

The approximation is motivated by Observation 3, which implies that the SIR targets are met under $\underline{\mathbf{s}}(\boldsymbol{\omega})$ whenever they are feasible. Unfortunately, a max-min SIR balancing problem (3) is (for general power constraints) notoriously difficult to solve in a distributed manner (see, for instance, the discussion in [4] about achieving max-min fairness in wired networks). On the positive side, however, we show that a max-min SIR-balancing solution can be approximated by

$$\bar{\mathbf{s}}(\alpha) := \underset{\mathbf{s}\in\mathrm{S}}{\arg\min} \bar{F}_\alpha(\mathbf{s})\,. \tag{23}$$

Here and hereafter, for some given $\gamma_k > 0, k \in \mathcal{K}$,

$$\bar{F}_\alpha(\mathbf{s}) := \sum_{k\in\mathcal{K}} w_k \psi_\alpha\big(\mathrm{SIR}_k(e^{\mathbf{s}})/\gamma_k\big) \tag{24}$$

where $\psi_\alpha : \mathbb{R}_{++} \to \mathrm{Q}$ is given by

$$\psi_\alpha(x) := \begin{cases} \frac{x^{1-\alpha}}{\alpha-1} & \alpha \geq 2 \\ -\log x & \alpha = 1 \end{cases} \quad x > 0\,. \tag{25}$$

Now $\bar{\mathbf{s}}(\alpha)$ can be shown to converge to some $\underline{\mathbf{s}}(\boldsymbol{\omega}) \in \mathrm{M}(\boldsymbol{\omega})$ as $\alpha \to \infty$ where $\mathrm{M}(\boldsymbol{\omega})$ denotes the set of all max-min SIR-balanced power vectors.[7]

[7] Note that there may be more than one solution to the max-min SIR-balancing problem.

Lemma 2: If (A.1) and (A.9) hold, then there is $\mathbf{s} \in \mathrm{M}(\boldsymbol{\omega})$ with $\lim_{\alpha\to\infty} \|\bar{\mathbf{s}}(\alpha) - \mathbf{s}\| = 0$ for any norm $\|\cdot\|$ on $\mathbb{R}^K$.

Now using this lemma yields the following result.

Proposition 3: Assume (A.1) and (A.9). Then, $\boldsymbol{\omega} \in \mathrm{Q}^K$ is feasible iff, for any $\epsilon > 0$, there exists $\alpha(\epsilon) \geq 1$ such that

$$\forall_{\alpha\geq\alpha(\epsilon)}\, \forall_{k\in\mathcal{K}}\, \mathrm{SIR}_k\big(e^{\bar{\mathbf{s}}(\alpha)}\big)/\gamma_k \geq 1-\epsilon\,. \tag{26}$$

By the results, we can arbitrarily closely approximate a solution to the max-min SIR-balancing problem by choosing the parameter α in (23)–(25) sufficiently large. The approximation becomes more accurate as α increases. In fact, the parameter α can be used to achieve different tradeoffs between fairness and efficiency in terms of total throughput: If $\mathbf{V}$ is irreducible, the total throughput $\sum_k \log(1+\mathrm{SIR}_k(e^{\bar{\mathbf{s}}(\alpha)}))$ and the ratio $\frac{\max_k(\mathrm{SIR}_k(\bar{\mathbf{s}}(\alpha))/\gamma_k)}{\min_k(\mathrm{SIR}_k(\bar{\mathbf{s}}(\alpha))/\gamma_k)}$ decreases as α increases. Thus, in some sense, the fairness performance becomes better at the expense of throughput performance (see also Figure 1).

Practically, the proposition implies that if $\boldsymbol{\omega}$ is feasible, then each user meets its QoS requirement under power control (23) if $\alpha \geq 2$ is large enough. The choice of α, however, is influenced by $\mathbf{V}$, and hence also by the fading channel. So, if α is fixed, the QoS requirements may be violated for some realizations of the channel, *even if they are feasible*. Thus, power control (23) provides only *soft* QoS support.

Finally, note that as ψ_α satisfies (A.3)–(A.5) and the ratio $\mathrm{SIR}_k(e^{\mathbf{s}})/\gamma_k$ is used instead of $\mathrm{SIR}_k(e^{\mathbf{s}})$, all the results presented in [16], [2] hold. In particular, the problem is convex and can be solved in a distributed manner using a gradient projection scheme based on the concept of the adjoint network, if the kth receiver knows $\gamma_k > 0$. An additional problem is that the function $\bar{F}_\alpha$ can be difficult to minimize for large values of α since then the gradient vector varies rapidly for relatively small SIRs, resulting in low convergence rates due to small step sizes. This problem may be mitigated by starting the algorithm for a small value of α, and then gradually increasing this parameter after a number of steps.

B. Incorporation of Best Effort Traffic

One drawback of the power control strategy (23) is certainly the inability of choosing $\gamma_k = 0$ for some $k \in \mathcal{K}$. This raises the question of how to incorporate best-effort users. Also, note that, once all the SIR targets γ_k are met for sufficiently large α, users with relatively high SIR requirements may be preferred when allocating "extra" resources. As a result, the throughput-performance of (23) may be much worse than that of the strategies with hard QoS support (and $\psi(x) = -\log(x)$).

As a possible solution, we consider a strategy that combines the power control strategy (23) with the traditional utility-based approach (4). To this end, we define two user subsets $\mathcal{A} \subseteq \mathcal{K}$ and $\mathcal{B} \subseteq \mathcal{K}$ such that $\mathcal{A}\cup\mathcal{B} = \mathcal{K}$. The set $\mathcal{A}$ is assumed to include indices of those users which have some (positive) SIR targets $\gamma_k > 0, k \in \mathcal{A}$, while $\mathcal{B}$ contains all users without any QoS requirements (best effort links) and may also contain all other users. Hence, these sets are not necessarily disjoint. Without loss of generality, we can assume that the users are ordered so that $\mathcal{A} = \{1,\dots,m_a\}, \mathcal{B} = \{m_b+1,\dots,K\}, 0 \leq m_b \leq m_a \leq K$. Now consider the following power control problem

$$\tilde{\mathbf{s}}(\alpha) := \tilde{\mathbf{s}}(\alpha,\boldsymbol{\omega}) = \arg\min_{\mathbf{s}\in\mathrm{S}}\ \tilde{F}_\alpha(\mathbf{s}) \tag{27}$$

where

$$\tilde{F}_\alpha(\mathbf{s}) = \sum_{k\in\mathcal{A}} a_k \psi_\alpha\Big(\frac{\mathrm{SIR}_k(e^{\mathbf{s}})}{\gamma_k}\Big) + \sum_{k\in\mathcal{B}} b_k \psi\big(\mathrm{SIR}_k(e^{\mathbf{s}})\big) \tag{28}$$

and where

(A.10) $\mathbf{a} > 0$ and $\mathbf{b} > 0$ are given weight vectors. Without loss of generality, it is assumed that $\|\mathbf{a}\|_1 = \|\mathbf{b}\|_1 = 1$.

(A.11) $\psi_\alpha : \mathbb{R}_{++} \to \mathrm{Q}, \alpha \geq 2$, is given by (25).

(A.12) $\psi : \mathbb{R}_{++} \to \mathrm{Q}$ is any function that satisfies (A.3)–(A.5).

Observation 7: Each of the following is true: (i) The minimum in (27) exists. (ii) The problem is convex. (iii) $\nabla\tilde{F}_\alpha(\mathbf{s})$ is Lipschitz continuous on every bounded subset of $\mathbb{R}^K$.

Proof: The observation follows from [2] and some standard results from convex optimization theory. ■

A reasonable choice for ψ is $\psi(x) = -\log(x), x > 0$. If α is sufficiently large, the choice of the weight vector $\mathbf{a} \in \Pi_K^+$ has negligible impact on the optimal power vector $\tilde{\mathbf{s}}(\alpha)$ so that one can often assume $\mathbf{a} = \mathbf{1}/|\mathcal{A}|$. Notice that (27) with $\mathcal{A} = \emptyset, \mathcal{B} = \mathcal{K}$, is the utility-based power control problem (4), and if we choose $\mathcal{A} = \mathcal{K}, \mathcal{B} = \emptyset$, the problem reduces to (23).

Lemma 3: Consider (27). If $\bar{\mathrm{S}}(\boldsymbol{\omega}) \neq \emptyset$, then, for any $\mathbf{s} \in \bar{\mathrm{S}}(\boldsymbol{\omega})$, there are constants $c_1 > 0$ and $c_2 = c_2(\mathbf{s}) < +\infty$ such that $c_1 \leq \tilde{F}_\alpha\big(\tilde{\mathbf{s}}(\alpha)\big) \leq \tilde{F}_\alpha(\mathbf{s}) \leq c_2$ for all $\alpha \geq 2$. So, if $\bar{\mathrm{S}}(\boldsymbol{\omega}) \neq \emptyset$, there are $0 < c_3 \leq c_4 < \infty$ such that

$$\forall_{\alpha\geq2}\, c_3 \leq \min_{k\in\mathcal{K}} \mathrm{SIR}_k(e^{\tilde{\mathbf{s}}(\alpha)}) \leq \max_{k\in\mathcal{K}} \mathrm{SIR}_k(e^{\tilde{\mathbf{s}}(\alpha)}) \leq c_4 \tag{29}$$

and hence the entries of $\tilde{\mathbf{s}}(\alpha) \in \mathrm{S}$ are bounded for all $\alpha \geq 2$.

Proof: The lower bounds follow from Observation 7(i) and the fact that $\psi_\alpha(x) > 0$ for all $x > 0$. The upper bounds hold as $\tilde{F}_\alpha(\mathbf{s}(\alpha)) \leq \tilde{F}_\alpha(\mathbf{s})$ for all $\mathbf{s} \in \mathrm{S}$ and $\tilde{F}_\alpha(\mathbf{s})$ is bounded above for any $\mathbf{s} \in \bar{\mathrm{S}}(\boldsymbol{\omega})$. ■

This simple lemma is used to show the following.

Proposition 4: Suppose that $\mathcal{A} \neq \emptyset, \mathcal{B} \neq \emptyset$ and $\mathcal{A} \setminus \mathcal{B} \neq \emptyset$. Then, for any $\epsilon > 0$ and an irreducible matrix $\mathbf{V} \geq 0$ (Condition (A.2)), there exists $\alpha(\epsilon, \mathbf{V}) \geq 1$ such that

$$\max_{k\in\mathcal{A}\setminus\mathcal{B}} \mathrm{SIR}_k(e^{\tilde{\mathbf{s}}(\alpha)})/\gamma_k \leq 1+\epsilon \tag{30}$$

for all $\alpha \geq \alpha(\epsilon, \mathbf{V}) \geq 1$. Moreover, if $\bar{\mathrm{S}}(\boldsymbol{\omega}) \neq \emptyset$, then, for any nonnegative but not necessarily irreducible matrix $\mathbf{V}$,

$$1-\epsilon \leq \min_{k\in\mathcal{A}} \mathrm{SIR}_k(e^{\tilde{\mathbf{s}}(\alpha)})/\gamma_k\,. \tag{31}$$

Notice that irreducibility of $\mathbf{V}$ (no isolated subnetworks) is a key ingredient in the proof of (30).

C. Approximation of $\mathbf{s}^*(\boldsymbol{\omega})$

In a special case when $\mathcal{B} = \mathcal{K}$ and $\mathbf{b} = \mathbf{w} > 0$, the power vector $\tilde{\mathbf{s}}(\alpha)$ given by (27) for any $\mathbf{a} > 0$ tends to $\mathbf{s}^*(\boldsymbol{\omega})$ defined by (6) as $\alpha \to \infty$. This can be shown by proceeding in a similar fashion as in [21, pp.564–568]. Intuitively, this can be explained if we write the problem (6) as

$$\mathbf{s}^*(\boldsymbol{\omega}) = \arg\min_{\mathbf{s}\in \mathrm{S}} \big(F_e(\mathbf{s}) + D(\mathbf{s})\big)$$

where $D : \mathbb{R}^K \to \{0, \infty\}$ is a penalty (or indicator) function defined to be

$$D(\mathbf{s}) = \begin{cases} \infty & \exists_{k\in\mathcal{A}} \mathrm{SIR}_k(e^{\mathbf{s}}) < \gamma_k \\ 0 & \text{otherwise}\,. \end{cases}$$

Now considering (28) with $\mathcal{B} = \mathcal{K}$ and $\mathbf{b} = \mathbf{w}$ shows that $|F_e(\mathbf{s})+D(\mathbf{s})-\tilde{F}_\alpha(\mathbf{s})| = |D(\mathbf{s})-\sum_{k\in\mathcal{A}} a_k\psi_\alpha(\mathrm{SIR}_k(e^{\mathbf{s}})/\gamma_k)|$ for any $\mathbf{s} \in \mathbb{R}^K$. Thus, $\tilde{\mathbf{s}}(\alpha)$ is close to $\mathbf{s}^*(\boldsymbol{\omega})$ if $\sum_{k\in\mathcal{A}} a_k\psi_\alpha(\mathrm{SIR}_k(e^{\mathbf{s}})/\gamma_k)$ is a good approximation of $D(\mathbf{s})$. Now since $D(\mathbf{s}) = \infty$ iff $\exists_{k\in\mathcal{A}}\mathrm{SIR}_k(e^{\mathbf{s}})/\gamma_k < 1$, it follows from the properties of the functions ψ_α that the approximation becomes better as α increases.

D. A Distributed Algorithm

The gradient-projection algorithm for the problem (27) is

$$s_k(n+1) = \min\Big\{s_k(n) - \delta_k(n)\nabla_k\tilde{F}_\alpha(\mathbf{s}(n)), \log(P_k)\Big\} \quad (32)$$

for each $k \in \mathcal{K}$ with $\mathbf{s}(0) \in \mathrm{S}$. The kth entry of the gradient vector $\nabla_k\tilde{F}_\alpha(\mathbf{s})$ is given by

$$\nabla_k\tilde{F}_\alpha(\mathbf{s}) = e^{s_k}\Big(\phi_k(\mathbf{s}) - \sum\nolimits_{l\neq k} v_{l,k}\mathrm{SIR}_l(e^{\mathbf{s}})\phi_l(\mathbf{s})\Big) \quad (33)$$

where

$$\phi_k(\mathbf{s}) = \begin{cases} u_k(\mathbf{s}) & k \in \mathcal{A}\setminus\mathcal{B} \\ u_k(\mathbf{s}) + v_k(\mathbf{s}) & k \in \mathcal{A}\cap\mathcal{B} \\ v_k(\mathbf{s}) & k \in \mathcal{B}\setminus\mathcal{A} \end{cases} \quad (34)$$

and where

$$u_k(\mathbf{s}) = a_k\psi'_\alpha\big(\mathrm{SIR}_k(\mathbf{s})/\gamma_k\big)/(\gamma_k I_k(e^{\mathbf{s}}))$$
$$v_k(\mathbf{s}) = b_k\psi'(\mathrm{SIR}_k(e^{\mathbf{s}}))/I_k(e^{\mathbf{s}})\,.$$

Now standard results from convex optimization theory together with Observation 7 lead to the following observation.

Observation 8: A sequence $\{\mathbf{s}(n)\}$ generated by (32) converges to $\tilde{\mathbf{s}}(\alpha) \in \mathrm{S}$ given by (27).

A distributed implementation is similar to that discussed in Section III-B. Again, except for $\Sigma_k(\mathbf{s}) = \sum_{l\neq k} v_{l,k}\mathrm{SIR}_l(e^{\mathbf{s}})\phi_l(\mathbf{s}), k \in \mathcal{K}$ in (33) with $\phi_l, l \in \mathcal{K}$, given by (34), all the other quantities (such as the weights) are either known locally or can be computed from local measurements (such as the SIR) and, if necessary, conveyed to the corresponding transmitter/receiver by means of a low-rate control channel. In contrast, $\Sigma_k(\mathbf{s})$ can be estimated using a scheme based on the concept of the adjoint network as described in Section III-B. The only difference is that each receiver, say receiver $l \in \mathcal{K}$, transmits in the nth iteration a sequence of independent zero-mean random symbols with the variance equal to $\phi_l(\mathbf{s}(n))$ given by (33).

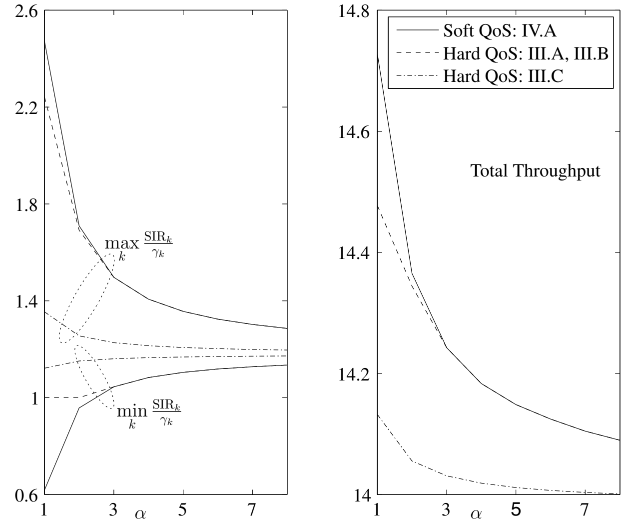

Fig. 1. Minimum and maximum of SIR_k/γ_k and total throughput over α.

V. Numerical Results

This section presents some simulations for a network with $K = 8$ users (transmitter-receiver pairs) and a randomly chosen irreducible gain matrix $\mathbf{V} \geq 0$. The weight vector and the SIR targets are chosen to be $\mathbf{w} = \mathbf{1}$ and $\gamma_k = 6, k \in \mathcal{K}$, respectively. Each user operates at $\mathrm{SNR} = 40$dB.

Figure 1 shows the impact of the parameter $\alpha \geq 1$ on the achievable SIRs and the total throughput for different power control strategies. The simulations confirm Proposition 3, which says that we can arbitrarily closely approximate a max-min SIR-balancing solution as $\alpha \to \infty$. So, if the SIR targets are feasible, then they can be achieved provided that the parameter α is sufficiently large. In contrast, in the case of hard QoS support, $\alpha = 1$ seems to be the most reasonable choice because the SIR targets are achieved by means of projecting the power vectors onto $\bar{\mathrm{S}}(\boldsymbol{\omega})$. Depending on the choice of $\mathbf{V}$, the link rate discrepancy may be significant. The barrier algorithm of Section III-C has, as expected, a better fairness performance than the other algorithms (III-A, III-B).

Figure 2 depicts exemplarily the convergence behavior for two algorithms: The gradient projection algorithm and the primal-dual algorithm. The step sizes are chosen to achieve a fast convergence rate. Simulations suggest that the gradient projection algorithm converges faster than the primal-dual one. Moreover, the gradient-projection algorithm exhibits monotonicity, which is not guaranteed by the primal-dual algorithm.

Figure 3 illustrates Proposition 4. As α increases, then (a) the SIR of the QoS user in $\mathcal{A}$ approaches the SIR target according to (30) and (31), (b) the QoS user in $\mathcal{A}\cap\mathcal{B}$ is allocated "extra" resources in addition to the SIR target and (c) the both best-effort users benefit since the second summand in (28) becomes dominant.

VI. Conclusions

The paper has addressed the problem of incorporating QoS requirements of the users (expressed in terms of some

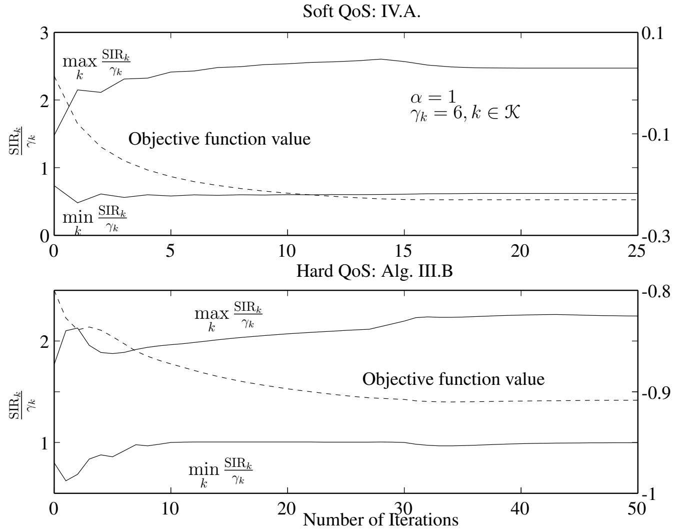

Fig. 2. Convergence examples.

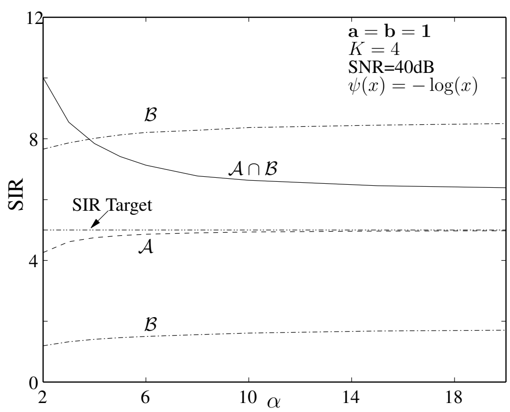

Fig. 3. Given some $\mathbf{V}$, the figure shows the SIR performance of the users as a function of α.

SIR targets) into a traditional utility-based power control. A straightforward approach is to maximize the aggregate utility function over the set of feasible power vectors (hard QoS support). Here, an additional challenge (in comparison with the traditional approach) is to perform the projection operation, which may require a lot of coordination in a distributed environment. Therefore, interesting alternatives are primal-dual algorithms to find stationary points of associated Lagrangian functions. In this paper, we have shown that a primal-dual algorithm based on the standard Lagrangian function can be efficiently implemented in a distributed manner.

Soft QoS support is of interest since the QoS requirements might be infeasible for some channel states, in which case the above problem with hard QoS support has no solution. This paper shows that the QoS requirements can be achieved by means of the traditional utility-based power control, provided that they are feasible and the utility functions are chosen suitably. Moreover, one can arbitrarily closely approximate a solution to the utility-based power control problem with hard QoS support.

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