Optimal user pairing in downlink MU-MIMO with transmit precoding

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Abstract**—*In this paper consider a downlink multiuser-MIMO transmission system in which at most two users can transmit simultaneously on different spatial channels or beams. We show how to find the optimal user subsets for K users in a system where inter-user interference is mitigated via transmit precoding and sum capacity is maximized using a linear programming algorithm that defines the optimal user subsets under a particular fairness criterion.*

Index terms**—** *Capacity, Scheduling, beamforming, MIMO, assignment problem.*

I. Introduction

Transmit precoding has been studied extensively in recent years in multiuser MIMO or MISO applications in broadcast channels, see e.g. [4], [6], [13], [12]. In a conventional broadcast channel, or in cellular downlink, the receivers do not cooperate and the inter-user interference must be mitigated with appropriate precoders at the transmitter. In effect, transmit precoding enables spatial division multiple access in downlink even if the receivers have only one receive antenna. For example, with zero-forcing precoding using n_t transmit antennas we can simultaneously serve n_t users on the same time/frequency/code (TFC) slot.

Typically, the number of users exceeds n_t and multiplexing over several TFC slots is needed. The users need to be divided/scheduled into subsets, with at most n_t users per subset and per slot. While the subsets can obviously be determined randomly, such a solution is unable to benefit from multiuser diversity. Indeed, since the user channels are generally different, it is possible to improve the overall system capacity (compared to any *a priori* defined subsets) by using channel state information when determining the subsets [1], also in a multicell network [8]. There are different ways of determining the subsets and each result in different notions of *fairness* and strikes a different balance between optimality (total capacity), computational computational complexity. Despite the conceptual simplicity of determining/scheduling optimal user subsets, e.g. via exhaustive search, it is evident that a brute solution is notoriously complex [14], [2], [13] and impractical even for moderate sized user populations.

In this paper we will focus on a scheduling scheme based on user pairing in conjunction with transmit beamforming and assume as objective function the total instantaneous mutual information between the source and the destination nodes when matched filter receivers are considered. After presenting the signal model, we pose the combinatorial optimization problem, which yields an optimal subsets for each slot. It is shown that this problem can be solved efficiently by posing the problem as an assignment problem [9], [10], [11].

II. Signal model

A downlink MU-MIMO system model comprises a transmitter with n_t transmit antennas and K receivers each occupying at most n_r receive antennas. The signal received by user/receiver k is given by

$$\mathbf{y}^{(k)} = \mathbf{H}^{(k)} \sum_{\ell=1}^{K} \mathbf{U}^{(\ell)} \mathbf{x}^{(\ell)} + \mathbf{z}^{(k)} \tag{1}$$

where $\mathbf{H}^{(k)} \in \mathbb{C}^{n_r \times n_t}$ is the *channel coefficient matrix* to user k, $\mathbf{U}^{(\ell)} \in \mathbb{C}^{n_t \times N}$ is the channel precoding/beamforming matrix to user ℓ, $\mathbf{x}^{(\ell)} \in \mathbb{C}^{N}$ represent the transmitted vectors to users $\ell = 1, \ldots, K$ containing N information symbols, and $\mathbf{z}^{(k)} \in \mathbb{C}^{n_r}$ is white Gaussian noise vector distributed as $\mathcal{N}(0, \mathbf{I}_{n_r})$. Let P be the total transmitted power by the base station and $\mathsf{SNR} = P$. We rewrite (1) in equivalent matrix form

$$\mathbf{y}^{(k)} = \mathbf{H}^{(k)} \left[\mathbf{U}^{(1)} | \cdots | \mathbf{U}^{(K)} \right] \begin{bmatrix} \mathbf{x}^{(1)} \\ \vdots \\ \mathbf{x}^{(K)} \end{bmatrix} + \mathbf{z}^{(k)} \tag{2}$$

In a downlink MU-MISO system (where in each receiver $n_r = 1$) only one stream is transmitted per user ($N = 1$). The corresponding signal model is accordingly simplified to

$$\mathbf{y} = \mathbf{H}\mathbf{U}\mathbf{x} + \mathbf{z}, \tag{3}$$

where $\mathbf{U} \in \mathbb{C}^{n_t \times K}$ is the precoding matrix, the kth row of $\mathbf{H}$ corresponds to $\mathbf{H}^{(k)}$ in equation (2), and the kth element of $\mathbf{x}$ designates the symbol transmitted to the kth user.

The precoding matrices can be determined e.g. using a matched-filter (MF) or zero-forcing (ZF) criterion. With MF $\mathbf{U} = \mathbf{H}^{\dagger}$ and only useful signal combines coherently when passing through the channel of user k. Interference power is reduced due to incoherent combining. The ZF precoder is defined by the pseudo-inverse or by a generalized-inverse of

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the channel matrix $\mathbf{H}$ [15]. For the purposes of this paper these two precoders suffice although any other linear precoder could naturally be used.

The effect of the linear precoder is apparent in the signal-to-interference-and-noise ratio (SINR) of receiver k. Assuming uncorrelated source signals, where the transit power of each source is equal under a given total power constraint, the SINRs are given by

$$\gamma_k(K) = \frac{[\mathbf{HU}]^2_{kk}}{\sum_{j\neq k}[\mathbf{HU}]^2_{kj}+1}, k=1,...,K \tag{4}$$

With zero-forcing precoding $[\mathbf{HU}]_{kj}=0$ whenever $k\neq j$ and subsequently only the signal of interest penetrates the receiver. With ZF precoder, the effective downlink channel decouples into non-interfering subchannels.

III. Subset selection and user pairing

The signal model in previous section corresponds to one arbitrary channel use or slot. In this given slot up to n_t users transmit simultaneously using transmit precoding. In this section we restrict the number of users in any given slot to be at most two, i.e. $K=1$ or $K=2$, and the total number of users is $\widetilde{K}>2$. We jointly optimize subset selection, or user pairing, and map the $\widetilde{K}$ users to the available slots, while ensuring that the energy allocated to each user (over all assigned slots) is identical. This constitutes our fairness criterion.

We next discuss the scheduling scheme and then show how this scheduling problem can be posed as a bipartite weighted matching problem (or linear assignment problem) and solved efficiently via specialized algorithms. We also exemplify the structure of typical scheduling patterns. A similar scheduling problem has been considered for uplink MU-MIMO systems in [14], in the absence of transmit precoding.

A. Scheduling algorithm

In order to keep the scheduling algorithm linear, we assume that the total number of available transmission slots depends only on the number of scheduled users $\widetilde{K}$. This can be accomplished if the paired users always access two slots whereas unpaired or single users access only one slot. In particular, we set the number of slots to be $N=2N_{pair}+N_{sing}=\widetilde{K}$. Under this constraint, the scheduling algorithm has to determine which users are paired and which are left to transmit alone in the channel. The N_{sing} unpaired users, that only use one slot, are allowed to double their transmit power, in order to maintain (energy) fairness. In addition to fairness, this constraint will also lead to comparable out-of-cell interfering power for each transmission slot. Naturally, by transmitting with double power, unpaired users can employ a higher order modulation in order to increase their spectral efficiency and compensate for their use of only one slot.

The utility for pairing users k_1 and k_2 is defined as

$$\gamma_{k_1,k_2} = \begin{cases} \gamma_{k_1}(2)+\gamma_{k_2}(2) & \text{if } k_1\neq k_2, \\ \gamma_{k_1}(1) & \text{if } k_1 = k_2 \end{cases} \tag{5}$$

where the notations $\gamma_{k_1}(1), \gamma_{k_1}(2)+\gamma_{k_2}(2)$ are apparent from equation (4) with the interpretation that the model captures only channel information of users k_1 and k_2. The interpretation for $\gamma_{k_1}(1)$ is that only user k_1 transmits and interference term in denominator obviously vanishes.

The scheduling algorithm attempts to find the optimal permutation σ that maximizes the diagonal sum

$$C=\max_{\sigma}\sum_k \gamma_{k,\sigma(k)}. \tag{6}$$

This formulation states that in slot k users k and $\sigma(k)$ are paired. If $k=\sigma(k)$ the user is not paired and is transmitting alone (with double power). Problem (6) is equivalent to solving

$$\max_{x_{k,j}}\sum_k\sum_j \gamma_{k,j}x_{k,j} \tag{7}$$

where $(x_{k,j})$ is a permutation matrix, which again is analogous to the linear assignment problem [3] or to the weighted bipartite matching problem, for which there exist many polynomial-time algorithms.

B. Scheduling patterns

To illustrate the scheduling patterns, let us now show how a pairing configuration, denoted as $\pi=\{\pi_{pair},\pi_{sing}\}$, can be mapped to a permutation σ of K elements of the form

$$\sigma: \quad \begin{pmatrix} 1 & 2 & \cdots & \widetilde{K} \\ \sigma(1) & \sigma(2) & \cdots & \sigma(\widetilde{K}) \end{pmatrix}. \tag{8}$$

Let the pairs $(k_1,k_2)\in\pi_{pair}$ correspond to the two columns of (8) $(k_1, k_2=\sigma(k_1))^T$ and $(k_2,k_1=\sigma(k_2))^T$, while the unpaired users $(k_3)\in\pi_{sing}$ correspond to the fixed elements of the permutation, i.e., columns of (8) of the type $(k_3,k_3)^T$. For example

$$\pi=\{(1,5)(2,4)(3)\} \quad\Rightarrow\quad \sigma: \quad \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 3 & 2 & 1 \end{pmatrix}$$

Clearly, under the assumptions of the previous section this will limit the permutations σ to have at most cycles of length 2 of the type (k_1,k_2), [5].

We can further expand the possible pairing configurations to include any user permutation σ, i.e., we will consider $\widetilde{K}$ pairs of users $(k,\sigma(k))$. For example we can have

$$\pi=\{(1,5)(2,4)(3,3)(4,5)(5,2)\}$$
$$\Rightarrow \quad \sigma: \quad \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 3 & 1 & 2 \end{pmatrix}$$

which is a permutation with a cycle $(1,5,2,4)$ of length 4.

The scheduling patterns that arise from the posed scheduling problem have cyclic structure where a user may be paired with different users in the two assigned slots. To our knowledge, such cyclic user-pairing patterns have not been suggested before.

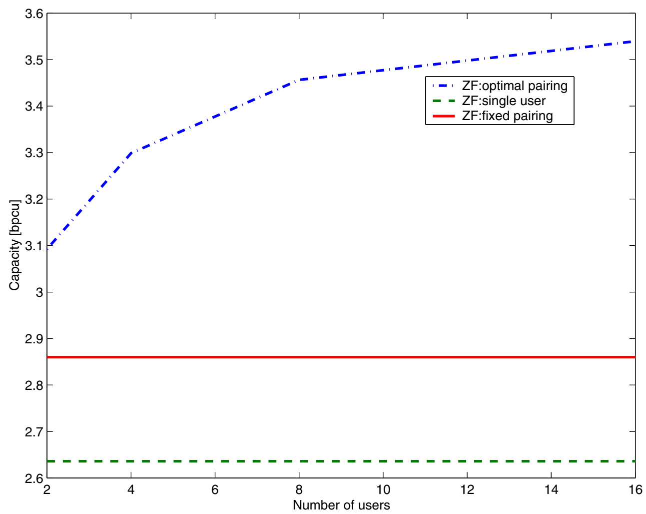

Fig. 1. Capacity per channel use at destination for the proposed scheduler with zero-forcing precoder, for single user transmission (one user per slot), and for fixed pairing (two users per slot).

C. Discussion

Above, we made the assumption that the total number of users is the same as the number of available transmission slots. Without this assumption, the number of required transmission slots could be smaller than $\widetilde{K}$, but the optimization problem becomes more involved, see e.g. [13], [4]. If the number of slots is allowed to depend on the number of paired and unpaired users, we need to search over the number of partitions of a set of $\widetilde{K}$ distinguishable elements into sets of size 1 and 2 or equivalently to the number of $\widetilde{K} \times \widetilde{K}$ symmetric permutation matrices [17] - there are

$$\sum_{k=1}^{\lfloor \widetilde{K}/2 \rfloor} \frac{\widetilde{K}!}{(\widetilde{K}-2k)!\,2^k\,k!}$$

of them. For example, for $\widetilde{K} = 2, 4, 6, 8, 16$ these sets have $2, 10, 76, 764, 46206736$ elements and in the absence of specialized optimization routines the solution becomes easily demanding [14].

IV. Performance

Figure 1 depicts the performance using zero-forcing precoding with different number $\widetilde{K}$ of scheduled users. The results are average over 300 random channel realizations. Each user has an iid Rayleigh fading channel. The number of transmit antennas $n_t = 2$ and the number of receive antennas in each of the $\widetilde{K}$ receivers is $n_r = 1$ with SNR $= 5$ dB. With fixed pairing each slot is occupied by exactly two users and the users are paired randomly. With single user transmission each slot is occupied by exactly one user. With optimal pairing the number of users and their indices are optimized for each slot using the assignment method.

V. Conclusion

In this paper we have proposed a downlink spatial scheduling (user pairing) scheme based on optimal scheduling of user pairs from a population of $\widetilde{K}$ users to $\widetilde{K}$ slots. This proposed scheme provides capacity gains using a polynomial time algorithm and leads often to cyclic scheduling patterns. The method can be used essentially with arbitrary precoding schemes, although in this paper we only focused on zero-forcing and matched filter precoders.

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