Adaptive Rate Control over Multiple Spatial Channels in Ad Hoc Networks

Andrew M. Hunter and Jeffrey G. Andrews

Abstract— **In ad hoc networks of nodes equipped with multiple antennas, the tradeoff between spatial multiplexing and diversity gains in each link impacts the overall network capacity. An optimal algorithm is developed for adaptive rate and power control for a communication link over multiple channels in a Poisson field of interferers. The algorithm and its analysis demonstrate that optimum area spectral efficiency is achieved when each communication link in a large distributed wireless network properly balances between diversity and multiplexing techniques. The channel adaptive algorithm is shown to be superior to traditional and static multi-antenna architectures, as well as to certain channel adaptive strategies previously proposed. Lastly, the adaptive rate control algorithm is coupled with an optimum frequency hopping scheme to achieve the maximum area spectral efficiency.**

I. Introduction

While much of the work on wireless network capacity focuses on the limiting capacity as the network grows large or dense [1] and [2], there remain a number of unanswered questions concerning the effect of physical signaling and medium access procedures on the network capacity. A practical approach is to determine which techniques perform best in a given network density or which can support the largest network. This approach is adopted in this paper to analyze spatial multiplexing multiple antenna (MIMO) techniques in ad hoc networks.

In MIMO systems, the need for improved signal quality provided by spatial diversity and the need to increase data rate with spatial multiplexing compete for limited degrees of freedom. In a single link setting, this results in a tradeoff between probability of symbol error and data rate [3]. However, in an interference-limited wireless ad hoc network, a different tradeoff emerges. When outage induced by topology-dependent interference is considered, it was shown by the authors in [4] that diversity techniques that do not enhance interference permit an increase in the density of contending transmitters. The present paper analyzes spatial multiplexing systems, which increase per transmission data rate at the expense of contention density. The tradeoff then between multiplexing and diversity becomes a tradeoff between the spectral efficiency of each link and spatial reuse (or increased density). As an analogy, the tradeoff is between having fewer large pipes or many smaller pipes in the network. Also unlike the tradeoff in point-to-point throughput, from the perspective of network area spectral efficiency, changes in density and changes in link throughput both directly affect network capacity. The goal of this paper is to explore this tradeoff, and develop optimal strategies for using multiple spatial channels in ad hoc networks.

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II. System Model and Mathematical Preliminaries

A. The Transmission Capacity Framework

In a large decentralized wireless network, nodes are often distributed randomly and may interfere with each other in a random fashion depending on the topology of the network and the spatial traffic needs. In a randomly distributed network with mean density λ, the transmission capacity is defined as the density of successful transmissions multiplied by their data rate given some outage constraint. Adopting a technological model, a symbol is successfully decoded if the $\mathrm{SIR} \geq \beta$ for some target β. So with a constraint that attempted transmissions fail with probability no more than ϵ, a contention density is feasible if attempted transmissions experience

$$\mathbf{P}(\mathrm{SIR} \geq \beta) \geq 1 - \epsilon \tag{1}$$

for each transmission. The optimal contention density is then

$$\lambda_\epsilon = \max\{\lambda | \mathbf{P}(\mathrm{SIR} \geq \beta) \geq 1 - \epsilon\} \tag{2}$$

since the SIR statistics are a function of the density of interfering transmitters λ. The transmission capacity then $(1-\epsilon)\lambda_\epsilon$, the density of successful transmissions under the performance constraints, and for a given data rate b in b/s/Hz achievable with SIR β, the area spectral efficiency (ASE) is given by $\mathrm{ASE} = b(1-\epsilon)\lambda_\epsilon$.

The definition of transmission capacity is straightforward to apply when only a single symbol is sent at a time, or even if multiple symbols are sent as part of one datastream but each symbol experiences the same effective channel statistics as in the case of orthogonal designs of space-time block codes. But if multiple datastreams are sent, different SIR statistics may be seen on the different datastreams and so a broader definition is needed. If $\mathrm{SIR}_i \geq \beta_i$ is chosen to be the target SIR to achieve rate b_i and ϵ_i is the outage constraint for the ith datastream, the optimal contention density becomes

$$\lambda_\epsilon = \max\{\lambda | \mathbf{P}(\mathrm{SIR}_i \geq \beta_i) \geq 1 - \epsilon_i,\ \forall i\}. \tag{3}$$

In general, if target SIRs and outage constraints are specified for each subchannel, then one subchannel will become the limiting factor in the optimal contention density, and hence the optimal transmission capacity (or area spectral efficiency).

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B. Analysis in a Poisson Field of Interferers

We consider a receiver-transmitter pair in the midst of a Poisson distributed set of interferers on the plane with intensity λ; the point process of interferers is denoted by Φ. To model propagation through a single wireless channel, let signals be subject to path loss attenuation model $|d|^{-\alpha}$ for a distance d with exponent $\alpha > 2$ as well as small scale Rayleigh fading with unit mean. We will assume the transmitter and receiver have full channel state information on all available channels.

For such a channel, the typical receiver obtains desired signal power $\rho S_0 R^{-\alpha}$ for some fixed transmitter-receiver separation distance R, and with a fading power factor S_0 on the signal from its intended transmitter, labeled 0. The interfering nodes, numbered $1, 2, 3, ...$ constitute the marked process $\Phi = \{(X_i, S_i)\}$, with X_i denoting the location of the ith transmitting node, and with marks S_i that denote fading factors on the power transmitted from the ith node and then received by the typical receiver. Thus the receiver receives interference power $\rho S_i |X_i|^{-\alpha}$ from the ith interfering node. For single-antenna narrowband systems in Rayleigh fading channels, for example, the power factors S_0 and S_i are distributed exponentially with unit mean so that the mean interfering power is governed by transmit power and path loss.

Given a target SINR β, we say a symbol is successfully decoded if on its particular spatial mode

$$\frac{\rho S_0 R^{-\alpha}}{\rho I_\Phi + N_o} \geq \beta \tag{4}$$

is satisfied for S_0 the channel fading factor for the received signal for this particular subchannel which is a squared singular value, R the transmitter-receiver separation, aggregate co-channel interference ρI_Φ on its particular spatial mode and N_o the background thermal noise power. The aggregate interference is a Poisson shot noise process (scaled by ρ), which is a sum over the marked point process:

$$I_\Phi = \sum_{X_i \in \Phi} S_i |X_i|^{-\alpha}. \tag{5}$$

From here on, it will be assumed that the network is interference limited, with $\rho I_\Phi \gg N_o$ so that thermal noise is negligible, and normalized by setting $\rho = 1$. Following [5] and [4], the probability of successful transmission for a typical receiver in a uniformly random network is:

$$\begin{aligned} \mathbf{P}(\mathrm{SIR} \geq \beta) &= \mathbf{P}\left(\frac{\rho S_0 R^{-\alpha}}{\rho I_\Phi} \geq \beta\right) \\ &= \int_0^\infty F^c_{S_0}(s\beta R^\alpha) \mathrm{dPr}(I_\Phi \leq s) \end{aligned} \tag{6}$$

where the third step is reached by conditioning on s and $F^c(\cdot)$ denotes a CCDF. In the SISO case, the received signal power is exponentially distributed with $F^c_{S_0}(s\beta R^\alpha) = e^{-s\beta R^\alpha}$ so that

$$\mathbf{P}(\mathrm{SIR} \geq \beta) = \int_0^\infty e^{-s\beta R^\alpha}\ f_{I_\Phi}(s)\mathrm{d}s\ . \tag{7}$$

This is now a Laplace transform of the PDF of I_Φ which gives:

$$\mathbf{P}(\mathrm{SIR} \geq \beta) = \mathcal{L}_{I_\Phi}(\beta R^\alpha). \tag{8}$$

The Laplace transform for a general Poisson shot noise process in $\mathbb{R}^2$ with i.i.d. marks S_i is given by [6]

$$\mathcal{L}_{I_\Phi}(\zeta) = \exp\left\{-\lambda \int_{\mathbb{R}^2} 1 - E\left[e^{-\zeta S_i |x|^{-\alpha}}\right] \mathrm{d}x\right\} \tag{9}$$

where the expectation, denoted by $E[\cdot]$, is over S_i. To generalize (6), we have the following Lemma derived in [4]:

Lemma 1: Let the interfering transmitters form a Poisson process of intensity λ around a typical receiver with the outage probability being $\mathbf{P}(\mathrm{SIR} \leq \beta) = \mathbf{P}(\frac{\rho S_0 R^{-\alpha}}{\rho I_\Phi} \leq \beta)$ with fixed ρ, β, R, and α. Suppose $F^c_{S_0}$ takes the form

$$F^c_{S_0}(x) = \sum_n e^{-nx} \sum_k a_{n,k} x^k \tag{10}$$

for $n, k \in \mathbb{N}$, and suppose S_0 is independent of I_Φ, then

$$\mathbf{P}(\mathrm{SIR} \geq \beta) = \sum_n \sum_k \left[a_{n,k}\left(-\frac{\zeta}{n}\right)^k \frac{d^k}{d\zeta^k}\mathcal{L}_{I_\Phi}(\zeta)\right]_{\zeta = n\beta R^\alpha}. \tag{11}$$

Furthermore, for a small outage constraint ϵ, the optimal contention density is given by the first order Taylor expansion of (11) around $\lambda \beta^{\frac{2}{\alpha}} R^2 C_\alpha = 0$:

$$\lambda_\epsilon = \frac{K_\alpha}{C_\alpha} \frac{\epsilon}{R^2 \beta^{\frac{2}{\alpha}}} + o(\epsilon)\cdot \tag{12}$$

for

$$K_\alpha = \left[\sum_n \sum_k a_{n,k} n^{\frac{2}{\alpha} - k} \prod_{l=0}^{k} (l - 2/\alpha)\right]^{-1} \tag{13}$$

and $C_\alpha(\beta R^\alpha)^{\frac{2}{\alpha}} = \int_{\mathbb{R}^2} 1 - E[e^{-\zeta S_i |x|^{-\alpha}}]\mathrm{d}x$.

C. Decomposing the MIMO Channel

We consider a network in which each transmitter and receiver has N antennas and each transmitter performs spatial multiplexing sending $M = N$ data streams, each with a separate packet[1]. In Rayleigh fading, the channel between the receiver of interest and its intended transmitter is $R^{-\frac{\alpha}{2}} \mathbf{H}_{00}$ which is an $N \times N$ matrix of i.i.d. complex Gaussian entries with unit variance. This channel can be decomposed into spatial modes by means of its SVD:

$$\mathbf{H}_{00} = \mathbf{U}_{00} \mathbf{\Sigma}_{00} \mathbf{V}^H_{00}\ , \tag{14}$$

where $(\cdot)^H$ denotes a conjugate transpose, $\mathbf{V}_{00}$ and $\mathbf{U}_{00}$ are the unitary matrices of input and output singular vectors respectively and $\mathbf{\Sigma}_{00}$ is the diagonal matrix of singular values, which are the square roots of the eigenvalues of $\mathbf{H}_{00}\mathbf{H}^H_{00}$. The ith interferer has the Rayleigh channel

$$R^{-\frac{\alpha}{2}} \mathbf{H}_{ii} = R^{-\frac{\alpha}{2}} \mathbf{U}_{ii} \mathbf{\Sigma}_{ii} \mathbf{V}^H_{ii} \tag{15}$$

between itself and its intended receiver with $|X_i|$ denoting the distance between it and the receiver of interest; and $|X_i|^{-\frac{\alpha}{2}} \mathbf{H}_{0i}$ is the Rayleigh channel between the ith interferer and the receiver of interest. The M spatial modes can be accessed by precoding

[1]This assumption, while not standard, and not completely necessary, makes the definition of outage and application of Lemma 1 clear.

the vector of symbols $\mathbf{s}_0$ by $\mathbf{V}_{00}$ and postcoding by $\mathbf{U}_{00}^H$ and the receiver of interest sees interference according to:

$$\begin{aligned}\mathbf{y} &= \sqrt{\frac{\rho}{N}}R^{-\frac{\alpha}{2}}\mathbf{H}_{00}\mathbf{V}_{00}\mathbf{s}_0 + \sqrt{\frac{\rho}{N}}\sum_i |X_i|^{-\frac{\alpha}{2}}\mathbf{H}_{0i}\mathbf{V}_{ii}\mathbf{s}_i + \mathbf{n}\\ \mathbf{z} &= \mathbf{U}_{00}^H\mathbf{y}\\ &= \sqrt{\frac{\rho}{N}}R^{-\frac{\alpha}{2}}\mathbf{U}_{00}^H\mathbf{H}_{00}\mathbf{V}_{00}\mathbf{s}_0\\ &\quad+\sqrt{\frac{\rho}{N}}\sum_i |X_i|^{-\frac{\alpha}{2}}\mathbf{U}_{00}^H\mathbf{H}_{0i}\mathbf{V}_i\mathbf{s}_i + \mathbf{U}_{00}^H\mathbf{n}\\ &= \sqrt{\frac{\rho}{N}}R^{-\frac{\alpha}{2}}\mathbf{\Sigma}_0\mathbf{s}_0 + \sqrt{\frac{\rho}{N}}\sum_i |X_i|^{-\frac{\alpha}{2}}\mathbf{H}_{\text{eff},i}\mathbf{s}_i + \tilde{\mathbf{n}} \qquad (16)\end{aligned}$$

where the SINR is calculated on the statistic $\mathbf{z}$. In a Rayleigh fading environment, $\mathbf{H}_{\text{eff},i}$ is a standard random Gaussian matrix. In this analysis, it is assumed that Gaussian signaling is used over each datastream so that an SINR of β guarantees that $\log_2(1+\beta)$ spectral efficiency is achievable on that stream. Furthermore, the total power of each node will be fixed at ρ though nontrivial power allocation strategies may be employed by each user over its various available channels (e.g., spatial modes). As shown in [7], without knowledge of interfering users' channels (both at the receiver of interest and at the interferers' intended receivers), power control can be harmful to network throughput. It is assumed in the above that $\mathbf{s}_0$ and $\mathbf{s}_i$ are unit norm with power allocated among their respective entries.

D. Analysis of Spatial Multiplexing Systems

The interference is modeled as a sum over the marked point process defined by (5). When multiplexing across M modes, the interfering power mark has the form:

$$S_i = \sum_{k=1}^{M}\left|\mathbf{u}^H\mathbf{H}_{0i}\mathbf{v}_i^{(k)}s_i^{(k)}\right|^2. \qquad (17)$$

Here $\mathbf{H}_{0i}$ is the Rayleigh fading channel between the receiver and the ith interferer, $\mathbf{u}^H$ is the combining vector applied at the receiver for the packet of interest, $\mathbf{v}_i^{(k)}$ is the kth column of the precoding matrix applied by the interferer to transmit the symbol $s_i^{(k)}$. All $\mathbf{u}^H$, $\mathbf{v}_i^{(k)}$, $\mathbf{H}_{0i}$, and $s_i^{(k)}$ are independent of one another. The interfering power is the sum of the interference from the M independent data symbols transmitted by the interferer. Each factor $\left|\mathbf{u}^H\mathbf{H}_{0i}\mathbf{v}_{ik}\mathbf{s}'\right|^2$ is an exponential random variable and all M factors are independent. Thus the factor S_i for spatial multiplexing systems is a Gamma distributed variate with scale parameter one and shape parameter M. The mean of S_i is M and its MGF is

$$E\left[e^{-\zeta S_i|x|^{-\alpha}}\right] = \frac{1}{(1+\zeta|x|^{-\alpha})^M}. \qquad (18)$$

Carrying out the integral in (9)

$$\mathcal{L}_{I_\Phi}(\zeta) = e^{-\lambda\zeta^{\frac{2}{\alpha}}C_{\alpha,M}} \qquad (19)$$

with

$$C_{\alpha,M} = \frac{2\pi}{\alpha}\sum_{k=0}^{M-1}\binom{M}{k}B\left(\frac{2}{\alpha}+k; M-\left(\frac{2}{\alpha}+k\right)\right) \qquad (20)$$

and $B(a;b) = \Gamma(a)\Gamma(b)/\Gamma(a+b)$ being the beta function, and $\zeta = \beta R^\alpha$. Note that while the terms and factors in (17) were all independent, the interference powers seen by different simultaneous datastreams across the link of interest are not independent at all. The reason is that they depend heavily on $|X_i|^{-\alpha}$, the distance-dependent attenuation of the signal from the ith interferer. When an interferer happens to be nearby, it will likely cause high interference to all datastreams at the same time, while if no interferers are nearby, it is likely that all datastreams will experience light interference. This is the motivation for an ergodic analysis, i.e., of determining the statistics of a random symbol (or packet) through the reference link.

For a typical (i.e., randomly selected) spatial mode, the channel gain of the received signal has the distribution of an unordered eigenvalue of $\mathbf{H}_{00}\mathbf{H}_{00}^H$ from among the spatial modes used. If all are used, the marginal pdf is: $f_\phi(\phi) = \frac{1}{M}\sum_{k=0}^{M-1}[L_k(\phi)]^2e^{-\phi}$ where the Laguerre polynomial of order k is $L_k(x) = \sum_{i=0}^{k}\binom{k}{k-i}\frac{(-x)^i}{i!}$. Writing $F_{S_0}^c$ in terms of $f_\phi(\phi)$ and integrating term by term,

$$F_{S_0}^c(x) = 1 - \int_0^x f_\phi(\phi)d\phi = e^{-x}\sum_{j=0}^{2(M-1)}a_M^{(j)}x^j \qquad (21)$$

where the coefficients $a_M^{(j)}$ are given by the formula

$$a_M^{(j)} = \frac{1}{j!M}\sum_{m=0}^{M-1}\frac{1}{2^{2m}}\sum_{k=0}^{m}\frac{(2k)!}{(k!)^2}\binom{2m-2k}{m-k}\sum_{i=j}^{2k}\frac{(-2)^i}{i!}\binom{2k}{2k-i} \qquad (22)$$

If instead the weakest spatial mode is not used and the power is split among the other modes, then the CCDF of an unordered eigenvalue from among the top $M-1$ is

$$F_{S_0}^c(x) = e^{-x}\sum_{j=0}^{2(M-1)}\frac{M-1}{M}a_M^{(j)}x^j - \frac{1}{M}e^{-Mx}. \qquad (23)$$

Via Lemma 1, this permits the transmission capacity of these cases to be computed: For small outage constraints ϵ, the optimal contention density is given by

$$\lambda_\epsilon = \frac{K_{\alpha,M}^{sm}\epsilon}{\beta^{2/\alpha}R^2C_{\alpha,M}}. \qquad (24)$$

When all spatial modes are used with an equal power allocation, the factor $K_{\alpha,M}^{sm} \geq 1$ is given by

$$K_{\alpha,M}^{sm} = \left[1+\sum_{k=1}^{2(M-1)}a_M^{(k)}\prod_{l=0}^{k}(l-2/\alpha)\right]^{-1}. \qquad (25)$$

When the smallest spatial mode is abandoned, the factor becomes

$$K_{\alpha,M}^{sm'} = \left[1+\sum_{k=1}^{2(M-1)}\frac{M-1}{M}a_M^{(k)}\prod_{l=0}^{k}(l-2/\alpha) - \frac{2}{\alpha}M^{\frac{2}{\alpha}-1}\right]^{-1}. \qquad (26)$$

In [8], the CCDF of the square of the maximum singular value of the desired channel (which is the largest eigenvalue of a complex Wishart matrix), has been reported (originally given by [9]):

$$F_{\phi_{\max}^2}^c(x) = 1 - \frac{|\Psi(x)|}{\Pi_{k=1}^{q}\Gamma(q-k+1)\Gamma(s-k+1)}$$

where $|\cdot|$ denotes a determinant, $q = \min\{N_t, N_r\}$, $s = \max\{N_t, N_r\}$, and the entries of the $q \times q$ matrix $\Psi(x)$ are

$$\{\Psi(x)\}_{i,j} = \gamma(s-q+i+j-1, x)\ ,\ i,j = 1,\ \ldots,q$$

where $\gamma(\cdot,\cdot)$ is the lower incomplete gamma function. Recall $\gamma(n,x) = (n-1)!\left(1 - e^{-x}\sum_{k=0}^{n-1}\frac{x^k}{k!}\right)$. For the $2 \times N$ channel when only the top spatial mode is used,

$$F^c_{S_0} = \frac{\gamma(N-1,x)\gamma(N+1,x) - \gamma^2(N,x)}{\Pi_{k=1}^2\Gamma(N-k+1)} \tag{27}$$

and when both are used,

$$F^c_{S_0} = 1 - \int_0^x \frac{1}{2}\sum_{k=1}^{2}[L_k^{N-2}(t)]^2 t^2 e^{-t}\mathrm{d}t. \tag{28}$$

While the full sum of exponentials and polynomials expressions are somewhat complicated and yield little intuition (and are hence omitted), they lend themselves to numerical evaluation and to the application of Lemma 1.

III. The Optimal Static, Network-wide Strategy

A. Power Allocation for Fixed Rate Multiplexing

Let ρ_i be the fraction of power allocated to the ith channel with the constraint $\sum_i \rho_i \leq 1$. Given a set of channels for a reference transmitter-receiver pair to select from whose signal distributions meet the requirements of Lemma 1, then for each channel

$$\lambda = \frac{K_i \rho_i^{\frac{2}{\alpha}} \epsilon_i}{C_{\alpha,M} R^2 \beta^{\frac{2}{\alpha}}}. \tag{29}$$

To maintain a total outage constraint on the typical packet which may be sent on any of M channels, we must have that $\frac{1}{M}\sum_i \epsilon_i \leq \epsilon$. Solving for ϵ_i in terms of the rest of the parameters above,

$$\begin{aligned} M\epsilon &= \sum_{i=1}^{M} \frac{\lambda C_{\alpha,M} R^2 \beta^{\frac{2}{\alpha}}}{K_i \rho_i^{\frac{2}{\alpha}}} \\ &= \lambda C_{\alpha,M} R^2 \beta^{\frac{2}{\alpha}} \sum_{i=1}^{M} \frac{1}{K_i \rho_i^{\frac{2}{\alpha}}} \end{aligned} \tag{30}$$

For fixed ρ, β, R, M, K_i, $C_{\alpha,M}$, and ϵ, the transmission capacity is maximized when $\sum_{i=1}^{M} K_i^{-1}\rho_i^{-\frac{2}{\alpha}}$ is minimized since λ is then maximized. To solve for the optimal power allocation ρ_i, we have the Lagrangian:

$$\mathcal{L}(\rho_i;\mu) = \sum_{i=1}^{M} K_i^{-1}\rho_i^{-\frac{2}{\alpha}} + \mu(\sum_i \rho_i - 1) \tag{31}$$

for which the solution can easily be found to be

$$\rho_i = \frac{K_i^{-1/\delta}}{\sum_j K_j^{-1/\delta}} \tag{32}$$

for $\delta = \frac{2}{\alpha} + 1$. Note that the solution for the optimal power allocation is dependent only on the relative channel gains and the path loss exponent. The optimal contention density can then be found by solving for λ in (30):

$$\lambda_\epsilon = \frac{K^{'}\epsilon}{C_{\alpha,M} R^2 \beta^{\frac{2}{\alpha}}} \tag{33}$$

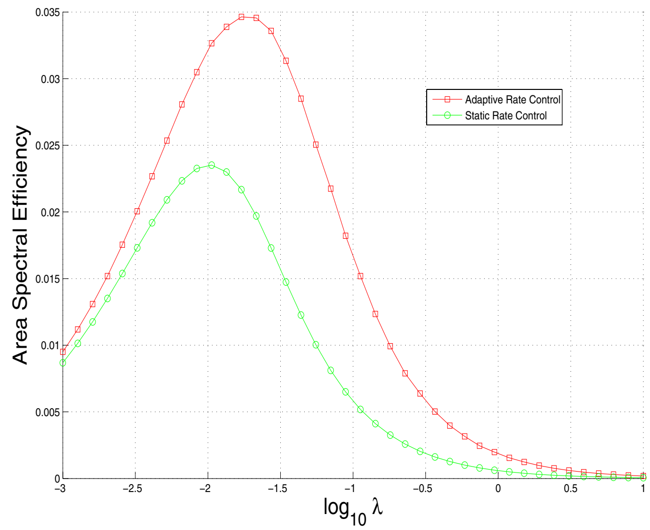

Fig. 1. Adaptive vs. static rate control for Rayleigh channels in a Poisson field of interferers.

for

$$K^{'} = M\left(\sum_{i=1}^{M} K_i^{-\delta}\right)^{-\delta}. \tag{34}$$

Since an outage constraint is applied, more power is allocated to the weaker modes to make them more robust to interference, thus allowing a higher density of interferers.

B. The Tradeoff between Multiplexing and Diversity

As an initial, straightforward example of determining the optimal tradeoff between multiplexing and diversity, we can determine the largest number of nodes per unit area which can transmit at fixed power and over a fixed distance so that a given target β is achieved with outage ϵ: $\mathbf{P}(\frac{\rho S_0 R^{-\alpha}}{\rho I_\Phi} \geq \beta) \geq 1-\epsilon$. From (17), regardless of the choice of weights at the transmitter and receiver when the power is fixed, the statistics of the interference are unchanged from the single antenna case when only one datastream is sent. Clearly if S_0 corresponds to the square of the largest singular value of the channel, this stochastically dominates any other effective channel gain [10] and so results in the largest possible contention density. In addition, it is obvious that devoting any power to lesser channels reduces λ_ϵ so that there is a clear tradeoff the number of transmitters and the number of independent symbols sent by each transmitter.

However, one can also ask how many independent datastreams can be transmitted per unit area each of which achieves the target β with total outage ϵ. This is equivalent to the ASE $= mb(1-\epsilon)\lambda_{\epsilon,m}$ and depends on the number m of independent datastreams sent by each node. The optimal tradeoff maximizes the area spectral efficiency by selecting the number of spatial modes m^* and the corresponding λ_{ϵ,m^*}:

$$m^* = \mathrm{argmax}_m\left\{m\lambda_{\epsilon,m},\ m = 1,\ldots,M\right\} \tag{35}$$

for M the number of non-zero singular values in the MIMO channel. For ϕ_i the ith largest singular value of the channel, since

$\phi_1 \geq_{\text{st}} \phi_2 \geq_{\text{st}} ... \geq_{\text{st}} \phi_M$, then $\lambda_{\epsilon,1} \geq \lambda_{\epsilon,2} \geq ... \geq \lambda_{\epsilon,M}$, but this does not guarantee that $j\lambda_{\epsilon,j} \geq k\lambda_{\epsilon,k}$ for any particular j, k. Since the distributions of eigenvalues of complex Wishart matrices fall under the criteria of Lemma 1, the K factors can be readily computed.

Remark: The best case increase in ASE would come from a set of (statistically) equally good channels for which the gain would be $M^{1-\frac{2}{\alpha}}$ over any single channel. Since $K_i = K\ \forall i$, then $K^{'} = M^{-\frac{2}{\alpha}}K$ indicating that multiplexing reduces the optimal contention density, while it linearly increases the per transmission data rate. This provides an upperbound on the potential improvement in transmission capacity of spatial multiplexing, even with the optimal power allocation, since the eigenmodes of the channel have a strict ordering (i.e., they are not equal): Using M modes must achieve less than an $M^{1-\frac{2}{\alpha}}$ gain over using only a single eigenmode.

IV. Channel Aware Adaptive Rate Control in a Poisson Field

While the above develops an intuitive relation for the tradeoffs in terms of per user throughput, contention density, and ASE, the derivations thus far are for static, network-wide rate control. The network can perform better, not surprisingly, if each communicating pair is able to adapt to its particular channel conditions.

A. Adaptive Rate Control over a Single Channel

Consider first a transmitter-receiver pair with full CSI and a single channel over which to communicate with current power fade level $\|\mathbf{h}\|^2$, yet in the midst of an *a priori* unknown Poisson field of interferers[1]. The following relates the probability of outage to the current propagation and interference environment:

$$\lambda = \frac{\|\mathbf{h}\|^{\frac{4}{\alpha}}\epsilon}{C_\alpha^{'}\beta^{\frac{2}{\alpha}}R^2} \tag{36}$$

where $C_\alpha^{'} = \frac{2\pi}{\alpha}\Gamma(\frac{2}{\alpha})$. Hence, given information on the current channel, the optimum target SINR should be

$$\beta_{\text{opt}} = \frac{\|\mathbf{h}\|^2}{R^\alpha}\left(\frac{\epsilon}{\lambda C_\alpha^{'}}\right)^{\frac{\alpha}{2}} \tag{37}$$

with corresponding spectral efficiency $\log_2(1+\beta_{\text{opt}})$. Note that $\frac{\|h\|^2}{R^\alpha}$ is the actual path loss, and hence is easily measured at the receiver. The link at all times retains a $(1-\epsilon)$ probability of success and so achieves throughput:

$$(1-\epsilon)E[\log_2(1+\beta)] = (1-\epsilon)\int_0^\infty \log_2(1+\beta)f_\beta(\beta)\mathrm{d}\beta \tag{38}$$

For a Rayleigh fading link, letting $\gamma = \left(\lambda C_\alpha^{'} R^2\epsilon^{-1}\right)^{\frac{\alpha}{2}}$,

$$E[\log_2(1+\beta)] = \int_0^\infty \log_2(1+\beta)\gamma e^{-\gamma\beta}\mathrm{d}\beta = \frac{1}{\ln(2)}e^{\gamma}\mathrm{Ei}(\gamma) \tag{39}$$

[1]Note that [11] treated adaptive modulation in a Poisson field numerically for specific SISO digital modulation schemes without outage constraints.

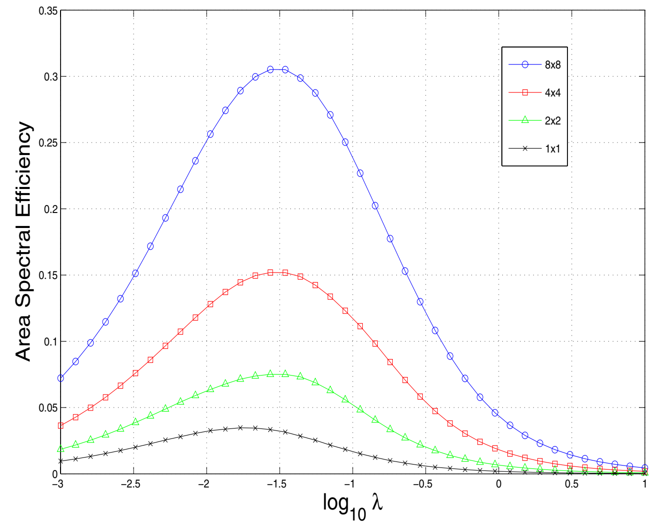

Fig. 2. Area spectral efficiency versus contention density for one, two, four, and eight antennas with optimal adaptive rate control.

for $\mathrm{Ei}(x) = \int_x^\infty \frac{e^{-t}}{t}\mathrm{d}t$, a modification of the exponential integral. This gives the mean data rate over each link, but for a network of such receivers, the ASE is maximized by:

$$\lambda^* = \arg\max_\lambda\ \lambda \exp\left\{(\lambda C_\alpha^{'}R^2\epsilon^{-1})^{\frac{\alpha}{2}}\right\}\mathrm{Ei}\left((\lambda C_\alpha^{'}R^2\epsilon^{-1})^{\frac{\alpha}{2}}\right) \tag{40}$$

This expression has a nontrivial solution, but can be readily found numerically.

B. Adaptive Rate Control over Multiple Channels

Now consider a transmitter-receiver pair with full CSI among a Poisson field of interferers for which:

$$\lambda = \frac{K_i\rho_i^{\frac{2}{\alpha}}\epsilon}{C_\alpha\beta_i^{\frac{2}{\alpha}}R^2} \tag{41}$$

for the ith channel available to the communicating pair where ρ_i is the fraction of the total power assigned to channel i. Letting

$$\gamma_i = \left(\frac{\lambda R^2 C_\alpha}{K_i\epsilon}\right)^{\frac{\alpha}{2}} \tag{42}$$

to determine the optimum rate and power for each channel the algorithm maximizes the expression $\sum_i \log_2(1+\beta_i)$ subject to the constraints $\rho_i = \gamma_i\beta_i$ and $\sum_i \rho_i = 1$. Applying Lagrange multipliers, the optimum target SINR for the ith channel is:

$$\beta_i = \frac{1}{N}\frac{1+\sum_i \gamma_i}{\gamma_i} - 1 \tag{43}$$

If a β_i is found to be negative, the channel is abandoned and the algorithm is rerun with a smaller set of channels. This is reminiscent of the waterfilling solution in which better channels are assigned more power and carry greater information content. Finally, the ASE of a network composed of many such communicating pairs of density λ employing this algorithm would be:

$$\mathrm{ASE} = (1-\epsilon)\lambda \cdot E\left[\sum_i \log_2(1+\beta_i(\lambda))\right] \tag{44}$$

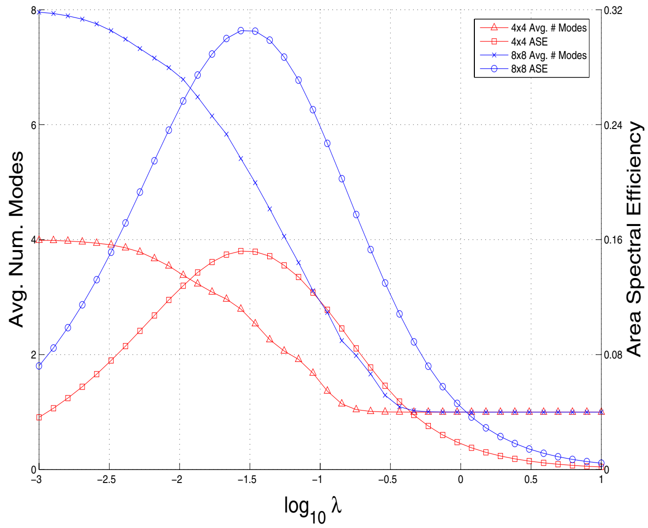

Fig. 3. Area spectral efficiency and mean number of spatial modes used with adaptive rate control. As density increases, the optimal rate control abandons spatial modes in favor of increased power (diversity) on the remaining modes.

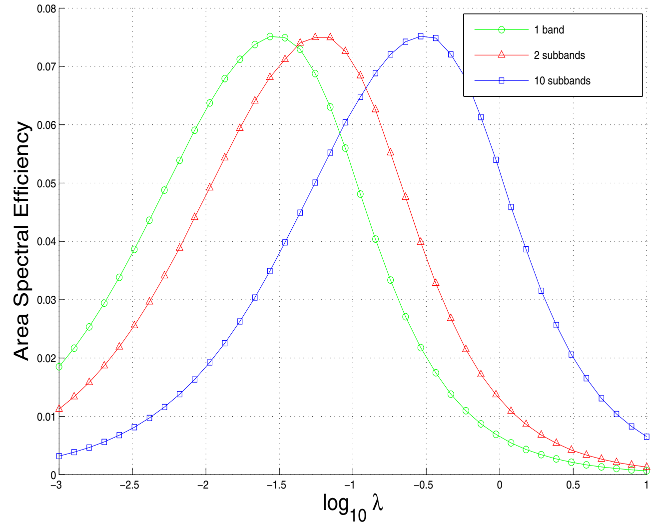

Fig. 4. Area spectral efficiency versus contention density for one, two, and ten subbands with adaptive rate control over 2x2 MIMO Rayleigh channels. The effect of changing the number of subbands is simply a shift of the curve.

where the expectation is over all realizations of the β_i which for each realization are the above static function of λ.

C. Optimal Bandwidth Allocation

Equipped with an algorithm for optimal rate and power control across multiple channels with outage constraints, the optimum number of channels which should be made available to nodes in the network becomes a tunable parameter, as in an OFDMA system. As discussed in [12], an optimal bandwidth partition exists (for frequency flat, single antenna systems) that balances between increasing point-to-point spectral efficiency by reducing spatial reuse. In general, employing a frequency (slow) hopping scheme over N slots for which each node selects only a single frequency slot, the contention density is thinned by a factor $1/N$. This modifies the area spectral efficiency to be:

$$\mathrm{ASE} = (1-\epsilon)(\lambda/N) \cdot E\left[\sum_i \log_2(1+\beta_i(\lambda/N))\right]. \quad (45)$$

As shown in Fig. 4, in which the area spectral efficiency is plotted versus contention density, for a fixed network density, employing a frequency hopping scheme merely shifts the network operating point on the curve. Clearly an optimal partition of the available bandwidth moves the peak of the curve as near as possible to the current density of transmitters in the network.

V. Conclusion

This paper established a tradeoff between multiplexing and diversity in the operation of multiple antenna wireless ad hoc networks. It was shown that spatial diversity increases the tolerable density of contending nodes in a network while spatial multiplexing increases point-to-point throughput at the expense of contention density. In terms of area spectral efficiency, an optimal tradeoff for random wireless networks was found. It was found that good diversity techniques such as maximal ratio transmission/combining tend to maximize $network$ area spectral efficiency for smaller numbers of antennas and for lower path loss, while increased multiplexing is preferred for larger arrays, larger path loss, and lower density.

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