Performance of multiuser MIMO and network coordination in downlink cellular networks

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***Abstract*— We consider a wireless network with multiple cells where base stations with multiple antennas transmit to multiple users, each with multiple antennas. Using single-user (SU) MIMO as a baseline, we evaluate the system throughput performance of multiuser (MU) MIMO with generalized zero-forcing beamforming under a multi-user proportional fair scheduling metric. We consider *network MIMO* extensions of this technique by coordinating transmission among clusters of spatially distributed bases. By significantly mitigating intercell interference, network MIMO provides a tradeoff between improved performance and increased backhaul complexity that depends on the coordination cluster size. We describe a general technique for simulating cellular networks that is applicable to next-generation packet-based cellular standards, and we evaluate the performance of MU and network MIMO in heavily loaded networks. Compared to the SU MIMO baseline, median system throughput can be doubled by coordinating MU MIMO transmission among clusters of 3 adjacent cells.**

I. Introduction

In systems such as downlink cellular networks where a multi-antenna transmitter serves multiple users, information theory indicates that the optimum strategy is to transmit to multiple users simultaneously over multiplexed spatial channels [1]. The class of techniques for transmitting in this manner is known as multiuser (MU) MIMO and has been an active area of research lately [2]. The downlink of a single sector in a cellular network can be modeled as a broadcast channel, and the capacity region can be achieved using beamforming and dirty paper coding (DPC) [1]. Due to its high complexity, simpler techniques based on linear beamforming and conventional single-user coding have been studied [2].

In multicellular networks, a user experiences co-channel interference from neighboring sectors and cells, and the spectral efficiency of the network is limited by this intercell interference. A novel class of techniques known as *network MIMO* coordinates the transmissions among multiple bases for eliminating interference. The transmissions across multiple coordinated bases are beamformed for multiple users simultaneously so that each user ideally receives its desired signal with no interference [3], [4]. Network MIMO is a generalization of MU-MIMO in the sense that multiple users can be served simultaneously by multiple coordinated bases during a given transmission interval. As a result of network MIMO, intercell interference can be eliminated among the coordinating bases, resulting in a significant improvement in system throughput. The tradeoff is that this technique requires user messages and channel state information to be shared among the coordinating bases, resulting in the need for enhanced backhaul capabilities.

In this paper, we evaluate the system spectral efficiency performance of MU and network MIMO techniques in the context of next-generation downlink packet-based cellular networks. Previous work on this topic either impose a per-frame constant rate constraint on the users [3] or consider a simplified model where users have identical average SNRs [4]. These references also assume full coordination over all bases in the network. In contrast, this study uses a multiuser proportional fair scheduling criteria that is well-suited for packet data systems, and it considers network coordination over a limited set of bases. With limited coordination we implicitly reduce the backhaul required for base station cooperation [5], that represents one of the main challenges in future deployments of network MIMO. A study of uplink network MIMO that considers limited coordination is given by [6].

In section II, we describe the system model and a general simulation methodology that is applicable to next-generation packet-based cellular networks. In section III, we describe a general scheduling and transmission strategy based on multiuser proportional fair scheduling [7] which is an extension of the well-known proportional fair scheduling algorithm. We then summarize the capacity expressions for three transmission options: a lower bound using closed-loop single-user spatial multiplexing, a simplified MU-MIMO technique based on zero-

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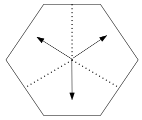

Fig. 1. Hexagonal cell with 3 sectors, where the arrows indicate the orientation of each sector's antennas. If a user lies in the direction of the arrow, then $A(\theta_{k,b}) = 0$

forcing beamforming, and an upperbound using DPC. We present numerical results in section IV and show the potential gains of MU MIMO and network coordination compared to the single-user baseline.

II. System model

We consider the downlink of a cellular network with multiple hexagonal cells. Each cell is partitioned into 3 sectors as shown in Figure 1, and a linear array of M transmit antennas is deployed in each sector. Under network coordination, antennas across multiple sectors transmit in a coordinated fashion. Letting C denote the number of coordinated cells, the baseline case of no coordination ($C = 1/3$) assumes that the sectors transmit independently. For $C = 1$, the 3 sectors of a given cell are coordinated. We also consider $C = 3$ and $C = 7$-cell coordination using non-overlapping, similarly shaped coordination clusters. These cellular architectures are shown in Figure 2. Each cluster of C cells consists of $3MC$ antennas (justifying the use of $C = 1/3$ for the no-coordination case). Users are dropped uniformly in the network, and each is assigned to a cluster. (The user to cluster assignment is described later.) We let $\mathcal{S}_b$ denote the set of users assigned to cluster b, with $b = 0, \ldots, B-1$ where B is the number of clusters in the entire network. From Figure 2, the number of clusters for cluster sizes of $C = 1/3, 1, 3, 7$ are $B = 57, 19, 7, 7$, respectively.

Each user has N receive antennas, and assuming the kth user is assigned to cluster $b = 0$, its received signal is:

$$\mathbf{x}_k = \mathbf{H}_{k,0}\mathbf{s}_0 + \sum_{b=1}^{B-1} \mathbf{H}_{k,b}\mathbf{s}_b + \mathbf{n}_k \tag{1}$$

where $\mathbf{H}_{k,b}$ is the $N \times 3MC$ complex channel matrix between cluster b and the kth user, $\mathbf{s}_b$ is the $3MC$-dimensional transmitted signal from cluster b, and $\mathbf{n}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_N)$ is the additive white Gaussian noise vector. Clusters with indices $1, \ldots, B-1$ correspond to the other clusters in the network that cause interference to this user. We assume linear precoding and that L_k data streams are spatially multiplexed and sent to each user.

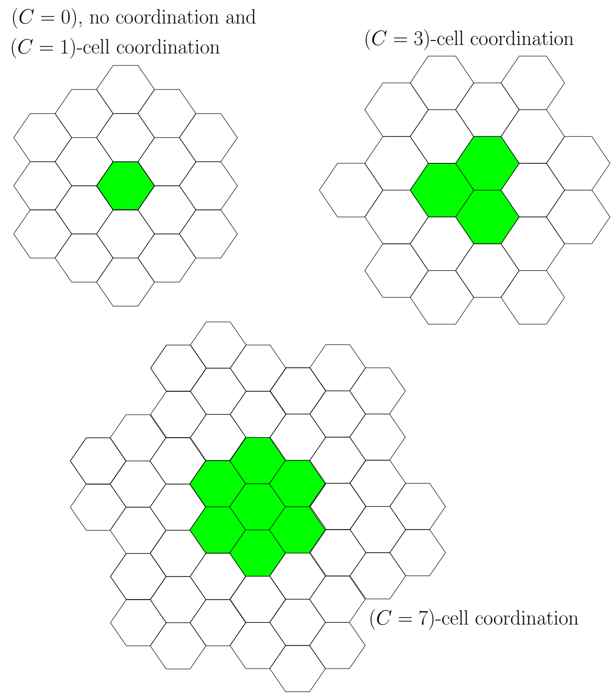

Fig. 2. Network architectures for varying levels of base station coordination

The transmitted signal by base b is given by

$$\mathbf{s}_b = \sum_{k \in \mathcal{S}_b} \mathbf{G}_k \mathbf{u}_k \tag{2}$$

where $\mathbf{G}_k$ is the $3MC \times L_k$ linear precoder matrix for the kth user, and $\mathbf{u}_k$ is the symbol vector for the kth user. We assume a block fading model for the channel so that it is static over the symbol interval. The transmit covariance is given by $\mathbf{Q}_b = \sum_{k \in \mathcal{S}_b} \mathbf{G}_k \mathbb{E}(\mathbf{u}_k \mathbf{u}_k^H) \mathbf{G}_k^H$, and under a sum power constraint P among the $3MC$ cluster antennas, we have that $Tr(\mathbf{Q}_b) \leq P$. We note that as long as cell coordination is performed between spatially separated antenna arrays, it would be necessary to consider a more strict average per-base power constraint instead of the average sum-power constraint. Anyway the CDF of the per-base power allocation, under the relaxed assumption of sum power constraint among all the coordinated antennas, reveals a really steep behaviour. This motivates our simplified approach of sum-power constraint even when coordinating spatially distributed antenna arrays for $C = 3$ and 7.

The channel coefficient between each transmit and receive antenna pair is a function of distance-based pathloss, shadow fading, and Rayleigh fading. We let the (n, m)th element of the kth user's MIMO channel matrix $\mathbf{H}_{k,b}$ from cluster b be given by:

$$\{\mathbf{H}_{k,b}\}^{(n,m)} = \beta_{k,b}^{n,m} \sqrt{A(\theta_{k,b}) \left[d_{k,b}/d_0\right]^{\gamma} \rho_{k,b} \Gamma} \tag{3}$$

where $\beta_{k,b}^{n,m}$ is independent Rayleigh fading, $\beta_{k,b}^{n,m} \sim \mathcal{NC}(0,1)$, $A(\theta_{k,b})$ is the antenna element response as a function of the direction from the bth base to the kth user (all antennas in a sector are assumed to "point" in the same direction), $d_{k,b}$ is the distance between the bth base and the kth user, d_0 is a fixed reference distance, γ is the pathloss coefficient, and $\rho_{k,b}$ is the lognormal shadowing between the bth base and kth user. Since shadowing is caused by large scatterers we assume that antennas of the same cell are close enough to be characterized by the the same shadowing effect. In other words, the log-normal random variable $\rho_{k,b}$ assumes the same value for bases whose antennas belong to co-located sectors. If we are considering coordinated networks, then the shadowing realization will be different across antennas belonging to the same base. This detail is not made explicit in the notation. We assume a time-division duplexed system with stationary users so that channel state information at the transmitter might be considered ideal.

The variable Γ is the reference SNR defined as the SNR measured at the reference distance d_0, assuming a single antenna at the cell center transmits at full power, accounting only for the distance-based pathloss. This parameter conveniently captures the effects of transmit power, cable losses, thermal noise power and other link-related parameters. For example, for a conventional macrocellular system with a 2 km base-to-base distance and with a 30 watt amplifier transmitting in 1 MHz bandwidth, the reference SNR is $\Gamma = 18$ dB [8].

We model the antenna element response as an inverted parabola that is parameterized by the 3 dB beamwidth θ_{3dB} and the sidelobe power A_s measured in dB:

$$A(\theta_{k,b})\mid_{\text{dB}} = -\min\{12(\theta_{k,b}/\Theta_{3dB})^2, A_s\} \tag{4}$$

where $\theta \in [-\pi, \pi]$ is the direction of user k with respect to the broadside direction of the antennas of the bth sector, $\Theta_{3dB} = 70\pi/180$, and $A_s = 20$. These parameters are taken from the 3GPP/3GPP2 spatial channel model for 3-sector cells. The broadside direction is the same for the M antennas in each sector.

In real world wireless systems, data rates are drawn from a discrete set where each rate corresponds to a particular coding rate and modulation type. The efficiency of contemporary codes and modems result in link performance that is within a few dB of the Shannon limit, [9]. In our simulations, with the exception of the capacity-achieving upper bound, we model the link performance using a continuous rate set given by the Shannon limit with a 3dB (factor of 1/2) power penalty. In other words, the achievable rate used for a link with SINR γ_k is given by $R_k^{3dB} = \log_2(1+\gamma_k/2)$. With this approach we are implicitly assuming that there is a rich set of modulation and coding rates but at the same time we provide a practical way to account for link inefficiency.

Intercell interference is modelled with a two-phase methodology as in [9]. In the first phase, the resource allocation and transmit covariance calculations are performed assuming the intercell interference is spatially white and estimating the achievable SINR assuming all bases transmit at full power and accounting for pathloss and shadowing. In the second phase, the actual achievable rates are computed assuming that the transmit covariances are colored according to sample covariances generated from the first phase. The assumption of spatially white noise in the first phase is the worst-case noise and results in a somewhat pessimistic rate. This methodology circumvents the problem of resource allocation when the statistics of the colored spatial noise is not known.

III. Transmission strategies

In order to reflect the operation of a next-generation packet-based cellular network, we assume that the average number of users per sector is much larger than the number of transmit antennas per sector. Due to the limited degrees of freedom, not all users can be served during each transmission interval. Therefore, we employ a scheduler that decides which users, and at what rate, these users are served during each interval. During the nth transmission interval, the scheduler generates a quality of service (QoS) weight $q_k(n)$ for the kth user according to the multiuser proportional fair scheduling (MPFS) algorithm [7] which is an extension of the well-known proportional fair algorithm.

Let $\mathcal{S}(n)$ be the set of users scheduled at slot n (we have dropped the dependence on the cluster index b for convenience) and $R_k(n, \mathcal{S}(n))$ the rate scheduled to user k at slot n. Note that with notation $R_k(n, \mathcal{S}(n))$ we have highlighted the fact that rates achieved by each user are mutually dependent. For MPFS, the average throughput of user k up to slot n is denoted as $T_k(n)$ and is updated as follows:

$$T_k(n+1) = \left(1 - \frac{1}{\tau}\right) T_k(n) + \frac{1}{\tau} R_k(n, \mathcal{S}(n)), \tag{5}$$

where τ is a parameter related to the time over which fairness should be achieved. In [7] it has been shown that proportional fairness, maximizing $\sum_k \log_2(T_k(n))$, is achieved by scheduling users according to the following criterion:

$$\sum_{k \in \mathcal{S}(n)} \log_2 \left(1 + \frac{R_k(n, \mathcal{S}(n))}{(\tau - 1) T_k(n-1)}\right). \tag{6}$$

Using the approximation of $\log_2(1+1/x) \approx 1/x$ for large x, for $\tau \gg 1$ (6) reduces to

$$\sum_{k \in \mathcal{S}(n)} q_k(n) R_k(n, \mathcal{S}(n)) \tag{7}$$

where the QoS weight $q_k(n) = T_k(n-1)^{-1}$ is the reciprocal of the windowed average rate. Therefore on each interval, the scheduler computes $q_k(n)$ for each user, and the resource allocation algorithm determines the users $\mathcal{S}(n)$ and user rates to maximize the weighted sum rate.

We now briefly summarize the weighted sum rate expressions for our three transmission options: a lower bound using closed-loop single-user spatial multiplexing, a simplified MU-MIMO technique based on zero-forcing beamforming, and an upperbound using capacity-achieving dirty paper coding.

The single-user (SU) spatial multiplexing mode represents a near-term cellular network deployment against which we can measure the gains of network coordination. If the channel is known ideally at the transmitter, the achievable rate for the kth user is [10]

$$r_{SU,k}(\mathbf{H}_k, P) = \max_{\mathbf{Q}\succeq \mathbf{0}, \mathbf{tr}\mathbf{Q} \leq \mathbf{P}} \log_2 \left| \mathbf{I}_N + \mathbf{H}_k \mathbf{Q} \mathbf{H}_k^H \right| \tag{8}$$

Because each base transmits to only a single user during a given interval, the set of serviced users $\mathcal{S}(n)$ is simply the single user with the largest weighted rate:

$$\mathcal{S}(n) = \{\tilde{k}\} = \arg\max_k q_k(n) r_{SU,k}(\mathbf{H}_k(n), P) \tag{9}$$

The actual transmitted rate during interval n is

$$R_{SU}(n) = r_{SU,\tilde{k}}(\mathbf{H}_k(n), P) \tag{10}$$

Under zero-forcing beamforming, multiple users are spatially multiplexed so that they do not receive any interbeam interference. If the users have multiple antennas, each could potentially receive spatially multiplexed streams using block diagonalization. However, we restrict each user to receive at most a single stream transmitted on its dominant eigenmode [11]. This restriction is justified by asymptotic analysis of the sum rate capacity [12] for a large number of users and by empirical results showing that the performance penalty is minimal for a moderate number of users [11]. Given this restriction, one could serve up to $3MC$ users per cluster during a transmission interval. For a given set of active users $\mathcal{S}$, the rate achievable by user $k \in \mathcal{S}$ as a function of the power w_k assigned to this user:

$$r_k(w_k) = \log_2(1 + w_k v_k^2(\bar{\mathbf{H}}(n), \mathcal{S})) \tag{11}$$

where $1/v_k^2(\bar{\mathbf{H}}(n), \mathcal{S}))$ is the effective noise power as a result of the zero-forcing beamforming. This power is a function of the users' MIMO channels in the set $\mathcal{S}$ and its derivation is given in [11]. We use $\bar{\mathbf{H}} = \{\mathbf{H}_1, \ldots, \mathbf{H}_K\}$ to denote the collective set of user MIMO channels, with K representing the number of users in the system. The optimal achievable rate vector and active user set for maximizing the weighted sum rate is found by first finding the optimum power vector $\mathbf{w}$ for a given set $\mathcal{S}$ and then maximizing over all possible sets $\mathcal{S}$, subject to constraints on the power:

$$\left\{\mathbf{r}_{ZF}(n), \tilde{\mathcal{S}}\right\} = \arg_{\mathbf{r},\mathcal{S}} \max_{\mathcal{S}} \max_{\mathbf{w}} \sum_{k \in \mathcal{S}} q_k(n) r_k(w_k) \tag{12}$$

$$\text{subject to} \begin{cases} w_k \geq 0, \quad k = 1, \ldots, K \\ F(\mathbf{w}) \leq P, \end{cases} \tag{13}$$

where $r_k(w_k)$ is given by (11) and $F(\mathbf{w})$ is the total transmit power as a function of the individual transmit powers for the users in set $\mathcal{S}$. The optimization with respect to $\mathbf{w}$ is calculated using waterfilling. The outer optimization with respect to $\mathcal{S}$ requires a brute force search over all possible sets. Greedy allocation algorithms have shown to provide near-optimum performance with significantly lower complexity, especially when the number of users K is large, [13]. Given the optimum rate vector and user set, the sum rate is simply

$$R_{ZF}(n) = \sum_{k \in \tilde{\mathcal{S}}} r_{ZF,k}(\mathbf{q}(n), \bar{\mathbf{H}}(n), P). \tag{14}$$

For the capacity-achieving dirty paper coding (DPC) technique, the resource allocator determines the point on the boundary of the capacity region which maximizes the weighted sum rate:

$$\mathbf{r}_{DPC}(\mathbf{q}(n), \bar{\mathbf{H}}(n), P) = \arg\max_{\mathbf{r}(n) \in \mathcal{C}(\bar{\mathbf{H}}(n), P)} \sum_{k=1}^{K} q_k(n) r_k(n). \tag{15}$$

The capacity region $\mathcal{C}$ is defined in [1], and the rates $r_{DPC,1}, \ldots, r_{DPC,K}$ that maximize the metric can be computed numerically [14]. The sum rate during this interval is given by the element sum of the rate vector $\mathbf{r}_{DPC}$:

$$R_{DPC}(n) = \sum_{k=1}^{K} r_{DPC,k}(\mathbf{q}(n), \bar{\mathbf{H}}(n), P). \tag{16}$$

IV. Numerical results

To model near-term antenna architectures, we assume that each sector uses $M = 4$ antennas and that users have $N = 1$ or 2 antennas. Users are dropped uniformly over the entire network, and the average number of users per hexagonal cell is 60. Users are assigned to the base with the highest received SNR, accounting for distance-based pathloss and shadow fading. In the case of base coordination, the user is assigned to the cluster with the highest aggregate received SNR. The pathloss exponent is $\gamma = 3.5$ the shadowing standard deviation is $\sigma_\rho = 8$ dB, the radius of each cell is $R_c = 1$ km and the reference SNR is $\Gamma = 20$ dB. To provide fairness in the network we adopt the proportional fair scheduler with fairness factor $\tau = 10$ time slots.

Cell wraparound is employed in the simulations in order to make interference statistics uniform over the entire network. We emphasize that even if the size of the network changes depending on C, comparisons are consistent because at the transmit powers used, 2 rings of interfering cells provide the most significant part of intercell interference and adding further cells to the network would contribute negligibly to the SINR statistics of the users.

For each drop of users, the complex Gaussian spatial channel is generated assuming i.i.d. spatial channel which models multiple-wavelength spacing between elements or cross-polarized antennas. The channel is modeled as flat to represent a single subchannel of an OFDM system. The channel is also assumed to be static for the duration of the drop (several hundred frames) in order to model stationary users. As a consequence, it is reasonable to assume perfect CSI available at the transmitter if the system is TDD. Perfect CSI is also assumed at the receiver.

For a given drop of users, the scheduler and resource allocation algorithm maximize the weighted sum rate. The sum-rate is given by equations (10), (14), and (16) for the three transmission modes considered. The sum-rate per cell can be obtained by dividing the per-cluster sum rate by the number of cells per cluster C. Realizations of the per-cell sum rate are collected over multiple drops.

Figure 3 shows the CDF of sum-rate per cell for the case of a single receive antenna per user ($N = 1$). Starting from the left, the baseline single-user transmission with no coordination ($C = 1/3$) has a median throughput of about 8bps/Hz per cell. Because there are 4 antennas per sector and because only a single user is served at a time, the spatial dimensions are underutilized. MU-MIMO using zero-forcing allows the possibility of transmitting to multiple users simultaneously. With single-sector transmission ($C = 1/3$), the median throughput improves over the baseline to about 11bps/Hz per cell without any additional hardware. By coordinating transmission among multiple sectors of a single cell ($C = 1$), intersector interference is mitigated, and the median throughput increases to 15bps/Hz per cell. Because the coordinated sectors are co-located, the marginal infrastructure required for sharing data and control information among them is minimal. We also note that the upper tail of the distribution is improved significantly because the intersector interference due to co-located antennas is mitigated, contributing to an improvement of SINR at the upper tail. With $C = 1$ coordination, the median throughput is nearly double that of the single-user baseline.

As the level of coordination is increased to $C = 3$ and $C = 7$ cells, the throughput increases but with diminishing returns. This indicates that the majority of interference is mitigated using coordination among a few cells. For larger coordination clusters, the intercell interference is weaker, and its power is comparable to that of the additive Gaussian noise. Therefore the performance becomes noise-limited as the cluster size increases. By increasing the transmit power, the system becomes more interference limited, and the gains of coordination are increased [6].

Note that we have assumed for both SU and ZF transmission a 3dB margin per link with respect to Shannon capacity. As an upperbound, we consider DPC without this margin. For no coordination ($C = 1/3$), the median DPC throughput is about twice that of ZF, indicating the potential improvement achievable with improved modulation and coding combined with more complex dirty paper coding.

Figure 4 shows the CDF of sum-rate per cell for the case of two receive antennas per user ($N = 2$). For a given transmission mode, the performance gains achieved using an additional antenna is about 20%. Therefore the relative performance of the various options for $N = 2$ is similar to the $N = 1$ case. With multiple receive antennas, each user could potentially receive up to $N = 2$ spatially multiplexed streams. However, we restricted each user to receive at most a single stream. Indeed as the number of users per cell is relatively large, exploiting multiuser diversity there is negligible performance penalty compared to the unrestricted case [11].

If we consider SU transmission as a baseline, then ZF with $C = 7$-cell coordination results in a 2.5-fold improvement in median cell throughput for both $N = 1$ and 2. DPC with $C = 3$-cell coordination results in a factor of 5 improvement.

V. Conclusions

We have studied the throughput performance of multi-user MIMO and network MIMO in a packet data and downlink cellular network context. Network MIMO mitigates interference by treating all base station antennas

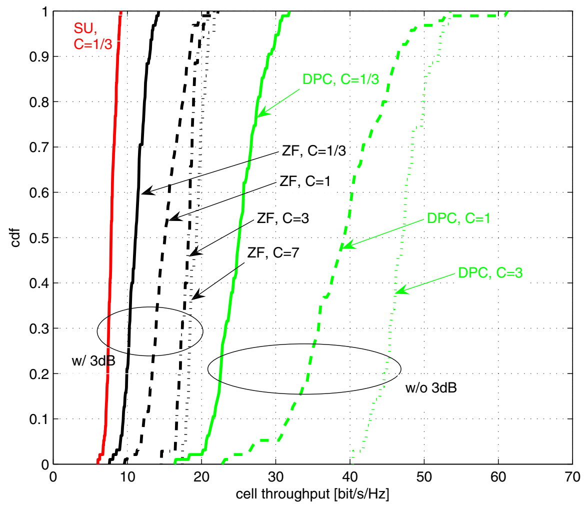

Fig. 3. CDF of throughput per cell (bps/Hz), $N = 1$ antenna per user, 12 antennas per cell, $C = 1/3, 1, 3, 7$ cell cluster coordination. The SU and ZF performance includes a 3dB power penalty per stream.

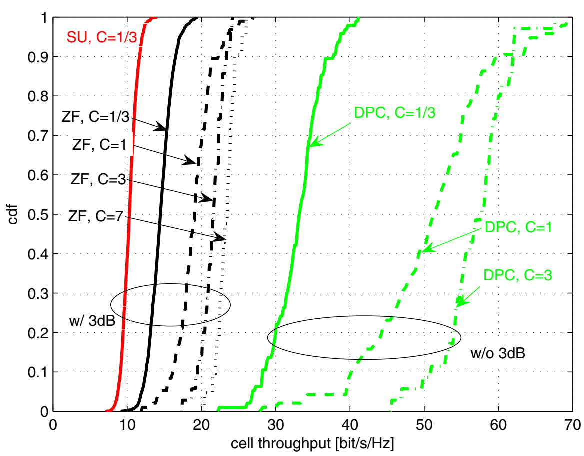

Fig. 4. CDF of throughput per cell (bps/Hz), $N = 2$ antennas per user, 12 antennas per cell, $C = 1/3, 1, 3, 7$ cell cluster coordination. The SU and ZF performance includes a 3dB power penalty per stream.

that are within radio range of a given user in a coordinated manner so as to minimize interference, providing a several fold increase in cell throughput compared to a baseline with single-user MIMO transmission. Network MIMO requires more complex signal processing for computing beamforming coefficients across multiple bases and an enhanced backhaul for exchanging user data and control information among coordinating bases. However, the complexity seems justified in light of the potential gains, and the cost will diminish in time with Moore's Law and the proliferation of high-speed wired networks.

Future work will address practical issues such as the impact of time and frequency synchronization and channel estimation errors. Moreover the possibility of exploiting the additional degrees of freedom provided by frequency selective channels will be studied, extending the proposed techniques to multiband OFDM systems.

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