On the Impact of Primary User Localization on Rate Performance of CR systems

Nurul H. Mahmood, Geir E. Øien
Department of Electronics & Telecomunications
Norwegian Univ. of Science & Technology
N-7491, Trondheim, Norway
Email: {nurulhud, oien}@iet.ntnu.no

Maïté Brandt-Pearce
Charles L. Brown Department of
Electrical and Computer Engineering
University of Virginia, Charlottesville, Virginia 22904
Email: mb-p@virginia.edu

***Abstract*—This paper investigates how localization knowledge of primary users impacts the sum-rate performance of a secondary system in an underlay cognitive radio setting. Two different probabilistic models for the primary users' location information are assumed, one uniform distribution and the other with Gaussian distribution. The interplay between the achievable rate of the secondary system and the primary users' location information under the AWGN and a fading channel model are presented and analyzed. Investigations are carried out for different propagation environments, localization models and primary-secondary user settings. Results show that the relationship between the secondary system sum-rate and the primary users' location information depends on the number of primary and secondary users, as well as the propagation environment. Empirical expressions for the sum-rate are proposed for different localization probability distributions.**

Key Terms - Underlay Cognitive Radio, CR achievable rate, Localization information of PU.

I. Introduction

Cognitive Radios (CR) can be broadly defined as communication systems that are aware of their surrounding environment and operating conditions, and have the capability to adapt their behavior accordingly. Cognitive radio models are classified into overlay, interweaved and underlay models [1]. The overlay technique allows concurrent transmission of the primary and the secondary user. The secondary user is assumed to have knowledge of the primary's message, which can be used to perform some sort of dirty paper coding to mitigate the interference at the primary receiver (PR) [2]. A primary user (PU) not using its licensed spectrum results in temporary frequency voids known as spectrum holes. Secondary transmitters use these spectrum holes to opportunistically access the spectrum in the interweaved model [3]. In the underlay model, no interference mitigation is assumed, but secondary users (SU) coexist with PUs by controlling their transmission powers to ensure that resulting interference at the PRs is kept below the maximum tolerable level [4], [5].

Implementing CR under the strict constraint that the interference should be below a threshold at each and every possible operating location will result in virtually zero capacity [6]. However, with knowledge of the PR location, the interference constraint can be redefined to be considered specifically at the *PR locations*. Then the goal becomes ensuring that the constraint is not violated only at those locations, resulting in greater freedom for the STs. For this to be possible, knowledge about the PR locations is necessary.

In [6], the capacity of CR networks under receive and spatial spectrum-sharing constraints is studied, and the AWGN channel capacity for the many-user Gaussian multiple access channel (MAC) with Successive Interference Cancellation (SIC), and for the non-degraded Gaussian relay network, is derived. Here locations of PUs are assumed to be perfectly known. Musavian and Aïssa on the other hand, investigate and propose closed form expressions for different capacities (ergodic, outage etc.) with received power constraints at the PR for fading channels in [7]. However, this work only considers a single SU constrained by a single PR, with imperfect feedback of channel state information.

The present work is motivated by the fact that knowledge about PR locations can potentially benefit a coexisting secondary system's sum-rate. The relationship between accuracy in the knowledge about PR locations, and the sum-rate of a secondary system opportunistically accessing that primary user's licensed band, is studied. Different propagation environments and primary-secondary user scenarios are investigated. For the secondary system, an underlay model is considered.

The rest of the paper is arranged as follows. Section II gives a brief introduction to localization techniques that are relevant to this work. The general system model and problem under consideration is presented in Section III. The sum rate for the AWGN channel model is presented in Section IV, while that for the fading channel model is presented in Section V. Finally concluding remarks and future research directions are discussed in Section VI.

II. Introduction to Localization Techniques

There are many techniques for localization, but in general localization techniques consist of i) Distance or range estimation based on TOA/TDOA (Time of Arrival/Time Difference of Arrival), AOA (Angle of arrival), RSS (Received Signal Strength) or Hop count (measuring the hop count to different anchors at known positions); and ii) Position or Location computation from this range estimate using different mathematical techniques [8]. Common to all these techniques is that localization is not exact. There is always some uncertainty in the estimate.

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If there is cooperation between the PUs and the SUs, the PR location estimate can generally be made more accurate. However, it is not common so far to assume primary-secondary cooperation. Thus, for CR applications, localization techniques that are not facilitated by cooperation between primary and secondary users should be used. Localization information is instead obtained from a third party, for example a sensor network, or by using CR-specific localization techniques as proposed in [9]. In this study, we assume that the location information is provided by a sensor network specifically built for that purpose, as in [10], [11].

III. Basic Scenario description

We consider a CR system coexisting with a primary system, both accessing the PU's licensed spectrum simultaneously. The secondary transmitters and primary receivers are arbitrarily located, while the primary and secondary base stations are at fixed and known locations within that cell.

The exact location of the STs are known to the secondary system, but not that of the PRs. However, the secondary system can localize the PRs within a certain degree of uncertainty, using a dedicated sensor network deployed for that purpose. We study the rate performance of the secondary system for different degrees of accuracy in PR localization and for different models of this localization information. Investigations are carried out for different channel models as well as.

Both the primary and secondary systems are assumed to be single-cell cellular systems. Each of these systems have one base station serving a number of users. The primary and secondary users are mobile devices that can be at any locations within the cell. The considered scenario is illustrated in Figure 1. More details about assumptions are given below.

A. General assumptions and channel model

Since this work is a study of the general trend in the relationship between accuracy in PR location information and the sum-rate of the secondary system, assumptions about the system model are kept general enough to cover a wide range of practical systems, and possible scenarios.

It is assumed that the secondary base station (SBS) does not have the ability to cancel interference; i.e. the channels we compute sum rate for are interference channels where all interferences are treated as noise. Furthermore, the primary and secondary terminals are assumed to be isotropic single antenna devices.

The terminals are assumed mobile, but with low-speed mobility. This makes an estimate of the location of a terminal remain valid over a significant period of time.

In this study, at first a free space propagation channel model is assumed. In this model, the power of the transmitted signal decays exponentially with the distance. The exponent of the decay, i.e. the path loss exponent, is denoted β in this paper. At the receiver, this signal is received in the presence of a Gaussian distributed white noise term. Later, the model is extended to include fading. However, the channel is assumed to change slowly, and therefore a log-normal shadowing model is assumed.

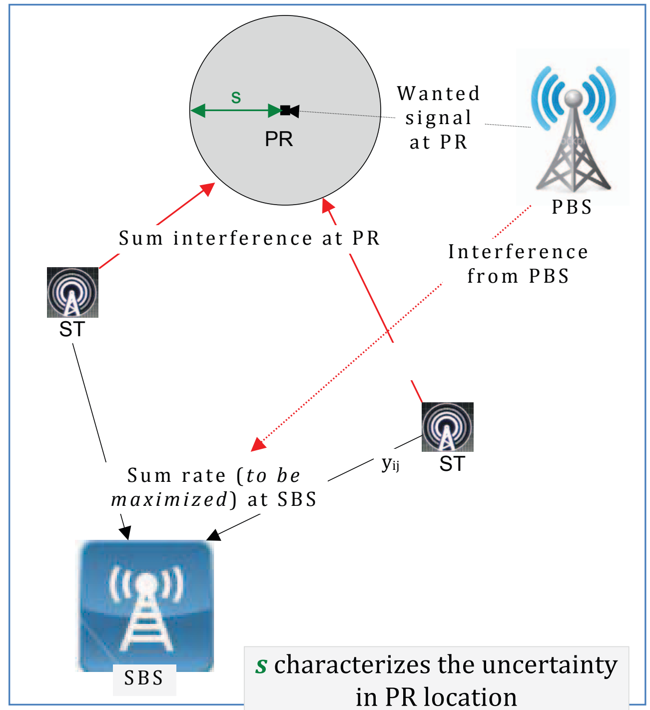

Fig. 1. The basic scenario setup

B. Assumptions about the Primary System

For the primary system, we assume downlink communication of a frequency duplexed (FDD) system. We are only concerned about the interference constraints at the PRs; received interference at the primary base station (PBS) is not a concern since the uplink channel is assumed to reside at a different frequency. Within the downlink channel, each PR is separated by orthogonal time slots (TDMA-like). Hence a PR does not experience any interference from transmissions to other PRs. However, since we assume that the secondary system does not have any mechanism to synchronize with the primary system and identify specifically which PR is receiving at any given time instant, it has to restrict its transmission by considering interference at all PRs. We assume that outage at a PR occurs when the received signal to interference and noise ratio (SINR) at that PR is smaller than a given threshold, γ_I.

C. Assumptions about the Secondary System

For the secondary system, we consider an uplink scenario where all the secondary transmitters access the same channel. When receiving the signal from a given ST, there are no mechanisms to cancel the interference from other STs at the secondary base station. Hence they interfere with each other. The signal from the PBS also adds to this received interference.

With multiple STs, the interference experienced by the PRs is the aggregate interference from all STs that are transmitting. This makes the transmit power that can be accepted at each ST dependent on that of others. To illustrate this, suppose the

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transmit power of one of the STs is such that the resultant interference at any PR equals the interference, γ_I. This means that the remaining STs cannot transmit at all, since any increase in their transmit power will result in the aggregate interference at one or more of the PRs exceeding γ_I.

We assume a centralized secondary system with a master node (which can be the SBS or some other node) assigning the transmit powers of all STs. This node receives multiple estimates of each of the PR's locations from the standalone sensor network. The location of the STs are assumed to be precisely known to the master node. Therefore, it can estimate the distance between any given ST to any other PR. Given these distances, and assuming that the path loss exponent is known, the master node can estimate the path loss between any ST-PR pair.

Given the above described scenario, the transmit powers of the STs are allocated such that some performance metric at the secondary system is maximized. There can be many choices for such a performance metric. This work studies how the secondary system as a whole benefits from accuracy in PR localization. So, the metric considered should reflect the overall performance of the secondary system. The achievable sum rate at the SBS is such a metric, and is considered in this analysis.

Given the received power constraints at the PRs and the estimates of the path loss from each ST to all other PRs, the master node assigns transmit powers to the STs such that the sum rate at the SBS is maximized.

Let us assume that there are M PRs and N STs. Let the channel between PR_i and ST_j be h_{ij}, and that between ST_j and the SBS be g_j. Let the matrix $\mathbf{H}$ be defined as the M $\times$ N matrix with elements h_{ij}. Similarly, let $\mathbf{g}$ be defined as the channel vector with elements g_k. Let the transmit powers of the STs be the vector $\mathbf{P}$. The assumed maximum transmit power constraint at all STs require that $\mathbf{P} \leq P_{max}\mathbf{1}$, where $\mathbf{1} = [1, 1, \cdots, 1]'$.

The signal transmitted by ST_i with power P_i is affected by the channel gain g_i before being received at the SBS in the presence of noise. As mentioned previously, if the signal of interest at the SBS is that from ST_i, then the received signals from other STs are treated as interference. Therefore, the received SINR from i_{th} ST at the SBS, SINR_i, is

$$\text{SINR}_\text{i} = \frac{P_i g_i}{\sigma_z^2 + \sum_{j=1,j\neq i}^{N} P_j g_j + Z} \tag{1}$$

Here, Z is the interference contribution from the PBS.

At the i^{th} PR, let the aggregate interference received from all the STs be I_i. As stated earlier, I_i can not exceed the interference threshold, γ_I. Therefore, we have

$$I_i = \mathbf{h_i}\mathbf{P} \leq \gamma_I \tag{2}$$

Here, $\mathbf{h_i}$ is the i^{th} row of the matrix $\mathbf{H}$. Defining $\mathbf{I}$ as the vector $[I_1, I_2, \cdots, I_M]'$ and γ as the M$\times$1 vector with all its elements equal to γ_I, we can write the received interference constraint at the PRs as

$$\mathbf{HP} \leq \gamma \tag{3}$$

With the above defined parameters, the problem of maximizing the sum-rate at the secondary system given the received power constraints at the PRs can be expressed as

$$\begin{aligned} & \max_{\mathbf{P}} \sum_{i=1}^{N} \log(1 + \text{SINR}_\text{i}) \\ \text{s.t.} \quad & \mathbf{HP} \leq \gamma \\ \text{and} \quad & \mathbf{P} \leq P_{max} \end{aligned} \tag{4}$$

The above optimization problem is non-convex, and hence can not be solved easily [12]. In this work, a simple centralized iterative resource allocation algorithm is proposed and implemented to solve this power assignment problem. Before proceeding further with the analysis, this suboptimal algorithm is briefly explained here.

D. The proposed resource allocation algorithm

If we look at this problem closely, we can see its similarity to traditional resource allocation problems in communication systems [12]. However, the difference here is that there is a received power constraint along with the usual limitation on the transmit power.

For the problem at hand, we would like to maximize the achievable sum rate with received power constraints in addition to the limitation on transmit powers. The goal is now to maximize the sum-rate at the secondary system while not violating the interference constraint at the PRs. Therefore, the transmit powers of the STs has to be allocated such that the achieved sum-rate is maximized for the allowable sum interference at the PRs.

The algorithm proposed is an iterative one. At each iteration, the transmit power of one of the STs is incremented by a small unit, ΔP.

Suppose, after the $(n-1)^{th}$ iteration, $R^{(n-1)}$ is the sum rate of the secondary system, and $P_i^{(n-1)}$ is the transmit power of the i^{th} ST. To choose which ST to allocate resources to at iteration n, the following steps are taken:

For each ST, the gain in the sum rate at the SBS due to the incremental power ΔP is calculated. For the i^{th} ST, this is defined as $\Delta R_i^{(n)}$, where

$$\Delta R_i^{(n)} = \log\left(1 + \frac{(P_{n-1}^{(i)} + \Delta P) g_i}{\sigma_z^2 + \sum_{j=1,j\neq i}^{N} P_{n-1}^{(j)} g_j + Z}\right) - R^{(n-1)} \tag{5}$$

Simultaneously, the maximum of the resultant interference at the PRs, $I_i^{(n)}$, due to this incremental power at ST_i is calculated. Let the powers of the STs be $\mathbf{P^{(n)}} = [P_1^{(n-1)}, P_2^{(n-1)}, \cdots, P_2^{(n-1)} + \Delta P, \cdots, P_N^{(n-1)}]'$. Then $I_i^{(n)}$ is

$$I_i^{(n)} = \max(\mathbf{HP^{(n)}}) \tag{6}$$

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Now, we have N values for $\Delta R_i^{(n)}$ and $I_i^{(n)}$ each. The transmit power is incremented by ΔP for the ST that has the higher incremental *gain to interference ratio* (GIR), where $GIR_i^{(n)} = \frac{\Delta R_i^{(n)}}{I_i^{(n)}}$. Thus, the ST that is allocated an incremental ΔP transmit power at the nth iteration, ST$^{(n)}$ is

$$ST^{(n)} = \arg\max_i \; GIR_i^{(n)} \tag{7}$$

These steps are repeated until the transmit power at none of the STs can be increased any further. This happens when increasing the transmit powers violate either the outage probability constraint, or the maximum transmit power (P_{max}) constraint.

The algorithm is illustrated through the flowchart presented in Figure 2.

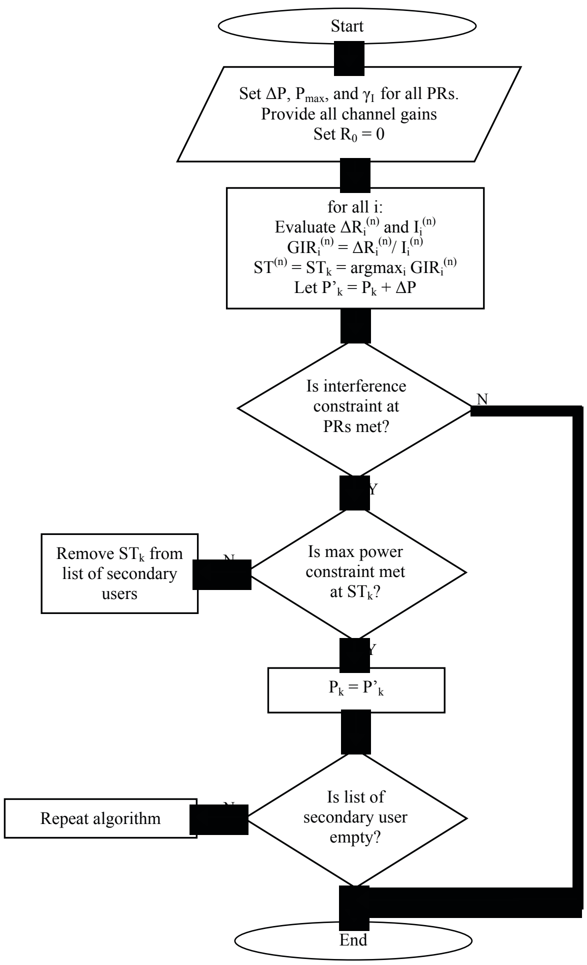

Fig. 2. Flowchart for the proposed resource allocation algorithm

IV. Results for AWGN Channel Model

We begin our analysis by considering the free space propagation channel model with AWGN, and extend it later to the case of fading channel models. Both the primary and the secondary systems are assumed to be within a rectangular area.

For the AWGN channel model, two different probability distributions for the PR location information are considered. The first is a uniform probability distribution with a circular area of radius s as the support; while the second is a Gaussian probability distribution with infinite support.

A. Localization model with uniform probability distribution

With this model, it is assumed that the location information of the PRs can be estimated within a circular region of uncertainty of radius s, as depicted in Figure 1. The distances from each secondary terminal to the centre of each of the circle corresponding to each PR, are calculated. The probability density of the PR being anywhere inside this circle is $\frac{1}{\pi s^2}$, while that of being outside the circle is 0. The PBS, on the other hand, can be exactly localized[1]. For this case, we assume that the transmit powers of the STs are decided such that the probability of outage at the PRs is zero. This implies that the interference caused by the STs at any locations inside the circular regions representing the PR location estimate can not exceed the interference threshold γ_I.

In practice, how large s is will depend, among other factors, on the propagation environment, mobility, and the specific localization technique. With a dedicated sensor network deployed for this purpose, the density and design of the network, and sensing, processing and transmission capabilities of the sensors will also contribute to the precision of PR location information [11], [13].

Simulation results for achievable sum rate of the secondary system for different values of s are presented in Figure 3. Here we have considered different combinations of the number of PRs and STs, and also different radio propagation environments (represented by different values of the path loss exponent β). The presented results are obtained by averaging each point on the sum-rate curve over more than a thousand Monte Carlo simulation runs. At each run, the PRs and STs are randomly located, and the sum-rate is calculated over many different channel realizations.

Discussion

Some key observations can be seen from the above presented results. We observe that the sum rate at the SBS in general depends relatively weakly on the localization knowledge for an AWGN channel model with uniform probability distribution for the location information. However, the dependence varies from scenario to scenario.

With the number of STs higher than the number of PRs, we observe that better localization results in the most significant increase in the sum rate. This means that with better localization, the secondary users can better utilize the available resources. This is because with more secondary transmitters compared to the number of primary receivers, the secondary

[1]The BS is a fixed structure and can easily be localized accurately using existing localization techniques, and sometimes this information is even publicly available.

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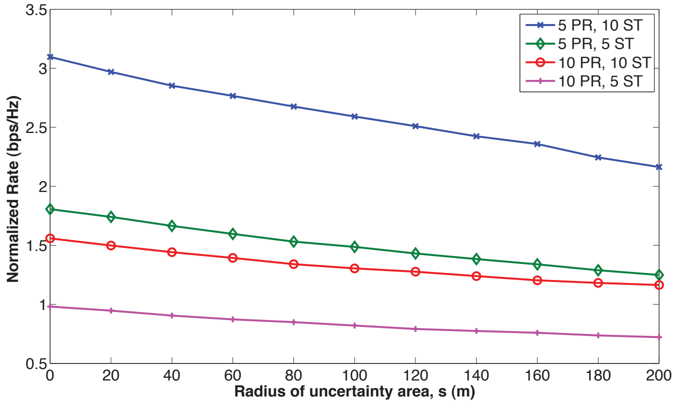

Fig. 3. Sum rate of multiple STs with multiple PRs for deterministic PR localization in AWGN channel

system has more degrees of freedom, and it can utilize these more freely the better the information available about PR locations.

Comparing the different curves, we observe that the relationship between accuracy in localization and sum-rate of secondary system depends on a number of factors. For a fixed number of STs, an increase in the number of PRs result in a fall in the sum rate. This is because increasing the number of PRs mean that there are more locations where the interference has to be limited. This means stricter transmit power constraint at the STs. Furthermore, it is also observed that the sum-rate becomes independent of the accuracy in localization as the number of PRs and STs increase.

On the other hand, for a fixed number of PRs, there is a gain in performance when the number of STs are increased. Increasing the number of STs give the secondary system more options to utilize the available resources, resulting in improved performance.

However, the association of the number of PRs and STs with the secondary system sum-rate becomes more involved when both are increased or decreased simultaneously. As we can observe from the obtained results, as both M and N are increased, initially there is a fall in the rate. But as they are increased further, the rate becomes independent of the localization information, and depends only on N and M.

We can summarize the above observations by the following empirically derived mathematical approximation. For some constants, c_1, c_2 and c_3, the sum rate at the secondary base station as a function of M, N, and s can be approximated by:

$$R_1(M, N, s) \approx c_1 \frac{N^{c_2}}{M} - c_3 \frac{\sqrt{N}}{M} s \tag{8}$$

Here, the constants c_1, c_2 and c_3 depend on the propagation environment as well as the SINR thresholds of the primary receivers. The value of the constant c_2 lies between 0.6 and 0.8. It should be noted that, approximation 8 becomes less accurate as the density of either the primary or the secondary users increase.

B. Localization model with Gaussian probability distribution

So far we modeled the uncertainty in the location information of the PRs by a fixed circular region, and were concerned about not violating the interference threshold anywhere within this region. However, this is not how it is most likely going to be in reality. A more realistic model would be assigning a probability distribution to the location information.

The confidence of a localization technique is expected to be high closer to the mean of the spatial probability distribution, and low for locations further away. Such a varying confidence can be represented by a Gaussian probability model. We therefore, investigate the performance of the secondary system for a Gaussian probability model of the localization information.

The secondary system gets multiple estimates of all primary receivers' locations from the sensor network deployed for localization. The mean and the variance of the Gaussian probability model are then calculated from these measurements or by knowledge of the reliability of a given localization technique based on measurement errors. The accuracy of the measurement is reflected in the variance or in an estimate thereof.

When allocating power and calculating the achievable rate, the ST transmit power is chosen such that for a specific PR, the integral over the all the possible locations of that PR which are affected by the ST transmit power should be less than the given outage probability.

Let the estimated position of the primary transmitter be a random vector $\mathbf{x} = [x, y]^T$, where $\mathbf{x} \sim \mathcal{N}(\mu, \mathbf{C})$. We assume x and y are i.i.d, thus the covariance matrix $\mathbf{C}$ is a diagonal matrix where all non-zero elements equal to σ^2. This gives us the distribution of $\mathbf{x}$ as

$$p(\mathbf{x}) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{1}{2\sigma^2}(\mathbf{x}-\mu)^T(\mathbf{x}-\mu)\right)$$

The mean of the estimated PR location can be expressed in polar coordinates as $\mu = [R\cos\varphi, R\sin\varphi]^T$. Similarly, $\mathbf{x}$ can be written as $[r\cos\theta, r\sin\theta]^T$. With these, the probability distribution of the PR location estimate, $\mathbf{x}$, can be expressed in r and θ as

$$p_{r,\theta}(r,\theta) = \frac{r}{2\pi\sigma^2} \exp\Big(-\frac{1}{2\sigma^2}[r^2 + R^2 - \cdots \\ 2rR(\cos\theta\cos\varphi + \sin\theta\sin\varphi)]\Big) \tag{9}$$

For a given outage probability $P_{r(out)}$, the transmit power of the ST has to be chosen such that the interference threshold is not violated at a distance d from the ST, i.e. $Pd^{-\beta} < \gamma_I$ where d is given by:

$$\int_0^d \int_0^{2\pi} p_{r,\theta}(r,\theta)\, d\theta\, dr = P_{r(out)} \tag{10}$$

The achievable sum rates for different outage probabilities and different values of the standard deviation of the distribution of PR localization are presented in Figure 4 and Figure 5. The secondary system's rate performance for

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different combinations of the number of PRs and STs with 5 % outage probability are shown in Figure 4. Figure 5 illustrates the dependency of sum-rate on the standard deviation for different outage probabilities, with five primary receivers and five secondary transmitters.

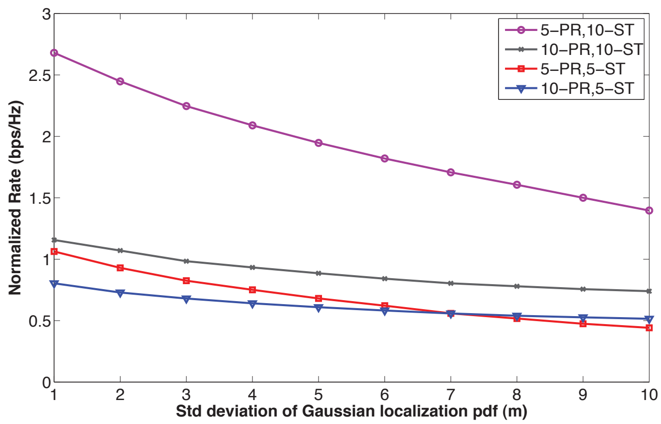

Fig. 4. Achievable rate for many PRs and many STs with Gaussian distributed location information in AWGN channel (5 % outage probability)

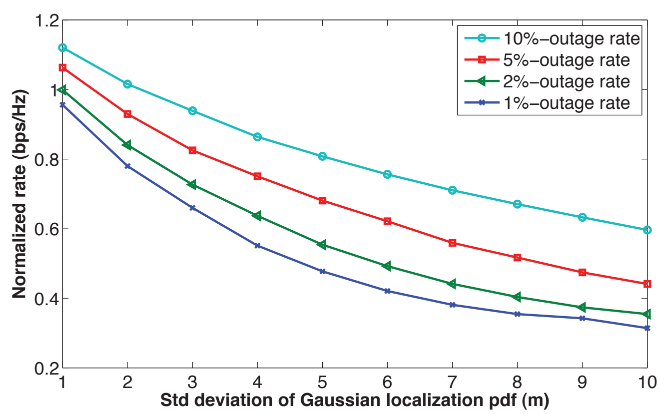

Fig. 5. Achievable rate for different outage probabilities with Gaussian distributed location information in AWGN channel (5 PRs and 5 STs)

Discussion

From the curves presented above, it can be observed that the rate no longer depends linearly on the localization information. We can see that there is also a weak inverse logarithmic relation between the sum rate and the standard deviation of the localization information. This dependence is more evident for low outage probability scenarios, implying that the exponent of the standard deviation weakly depends on the outage probability, ϵ, as well. Such a behavior can most likely be attributed to the Gaussian probability distribution, which is clustered around the mean.

Looking at Figures 4 and 5, we find that the sum-rate increases when the number of STs are increased. On the other hand, increasing the number of PRs result in a fall in the sum rate. Analyzing simulated results for a large number of PR and ST settings, we discover a dependency of the sum-rate on the number of PRs and STs, similar to that for the uniform probability distribution.

The above observations can be summarized in an empirical expression for the sum-rate of a secondary system. For some arbitrary constants k_1, k_2 and k_3, this expression as a function of M, N, σ and ϵ, is approximated by:

$$R_2(M, N, \sigma, \epsilon) \approx k_1 \frac{N}{M} + k_2 \frac{N^{\frac{3}{4}}}{M} s^{-\frac{k_3}{\sqrt[3]{\epsilon^2}}} \quad (11)$$

Here, the constants k_1, k_2 and k_3 depend on the propagation environment as well as the SINR thresholds of the primary receivers. The constant k_3 usually has a value between 0.02 and 0.05. As noted in the previous section, the approximation 11 becomes less accurate as the density of either the primary or the secondary users increase.

V. INVESTIGATION UNDER A FADING CHANNEL MODEL

In the preceding investigation, we have observed some interesting relations between achievable rate of a secondary system and the knowledge about primary receivers. So far, our study has been limited to the AWGN channel model. In this section, we extend it to include fading. Because the signal is subject to fading (both shadowing and multipath propagation), its amplitude can only be expressed in statistical terms. So, we need to express the performance of the secondary system in terms of outage-probability at the primary receiver, which is the probability of unsatisfactory reception (i.e. outage). This outage probability is due to the statistical behavior of the channel gain, and not due to the uncertainty in PR location information.

Outage can occur in two ways. The first is outage due to coverage, which occurs when the magnitude of the signal of interest expressed in dB scale, s_{pr}, falls below the required signal to noise ratio (SNR) threshold, s_0 dB, at the PR. This outage probability (P_{out}^c) is given as the probability that the received signal is below the SNR threshold. Mathematically, this can be expressed as [14]:

$$P_{out}^c = \int_{-\infty}^{s_0} p_r(s_{pr}) ds_{pr} \quad (12)$$

On the other hand, the signal to interference ratio (SIR) criteria implies that s_{pr} must be greater than the interfering signal, s_i, by the SIR margin of γ_{SIR}. Therefore, outage for the primary receiver will also occur when the SIR falls below this threshold, i.e. $s_{pr} - s_i < \gamma_{SIR}$ (in dB scale). Thus, the outage probability due to interference (P_{out}^i) is the probability that ($s_i > (s_{pr} - \gamma_{SIR})$) weighted by the pdf of s_{pr} and integrated over all possible values of s_{pr}. This can be expressed as [14]:

$$\begin{aligned} P_{out}^i &= p_r(s_i > s_{pr} - \gamma_I) \\ &= \int_{-\infty}^{\infty} p_r(s_{pr}) \left[\int_{s_{pr}-\gamma_{SIR}}^{\infty} p_i(s_i) ds_i \right] ds_{pr} \quad (13) \end{aligned}$$

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For satisfactory reception at the primary receiver, these two criteria has to be met simultaneously. Thus, the outage probability is the probability of failing to meet both criteria. This is:

$$P_{out} = 1 - (1 - P^c_{out})(1 - P^i_{out}) \qquad (14)$$

Plugging in Eq 12 and Eq 13 into Eq 14, we get [14]:

$$P_{out} = 1 - \int_{s_0}^{\infty} p_r(s_{pr}) \left[\int_{-\infty}^{s_{pr}-\gamma_I} p_i(s_i) ds_i \right] ds_{pr} \qquad (15)$$

So far, the performance was measured in terms of achievable rate. But with fading channel, the performance is presented in terms of ergodic rate of the secondary system given a certain outage probability at the primary receivers. In rest of this section, we analyze the relation between ergodic sum-rate of secondary system, and the uncertainty in primary users' location for different outage probabilities at the primary receiver. We limit our analysis to the case of shadowing only, since it can be safely assumed that the effects of short term fading can be averaged over.

Let us consider the case of a single primary receiver and a single secondary transmitter, where both signals are subject to shadowing. The SNR and SIR requirement at the primary receiver for satisfactory performance is:

$$S - N \geq \gamma_S (\text{in dB scale}) \qquad (16)$$
$$S - I \geq \gamma_{SIR} (\text{in dB scale}) \qquad (17)$$

Where S and I are the signal of interest and interference respectively, N is the noise and γ_S is the SNR threshold. Both S and I are subject to shadowing. It is well accepted that shadowing can be modeled by a log-normal distribution [14].

S can be written as $S = \bar{S} + e_s$ (in dB scale), where $\bar{S}$ is the mean, and e_s a statistical sample describing the variation and has a Gaussian distribution. e_s has mean, $\mu = 0$ and variance, σ_s^2. Similarly $I = \bar{I} + e_i$, where e_i has mean $\mu = 0$ and variance σ_i^2. Eq 16 and Eq 17 can be rewritten as

$$\bar{S} + e_s - N \geq \gamma_S \qquad (18)$$
$$\bar{S} + e_s - \bar{I} - e_i \geq \gamma_{SIR} \qquad (19)$$

Assuming that the signal and interference are independent of each other, the joint pdf of S and I, $p(S, I)$ with mean at $(\bar{S}, \bar{I})$ is

$$p(S, I) = \frac{1}{2\pi\sigma_s\sigma_i} \exp\left(-\frac{1}{2} \left[\frac{x^2}{\sigma_s^2} + \frac{y^2}{\sigma_i^2} \right] \right) \qquad (20)$$

where $x = S - \bar{S}$ and $y = I - \bar{I}$. From Eq 16 and Eq 17, we have the constraint for successful reception that $S \geq N + \gamma_S = \gamma'_S$ and $I \leq S - \gamma_I$. The outage probability is then $(1 - F(\gamma_I, \gamma'_S))$, where $F(\gamma_I, \gamma'_S)$ is the joint cumulative distribution function (cdf) of S and I. This is given as:

$$F(\gamma_I, \gamma'_S) = \frac{1}{2\pi\sigma_s\sigma_i} \int_{\gamma'_S}^{\infty} \exp\left[-\frac{(S - \bar{S})^2}{2\sigma_s^2} \right] \int_{-\infty}^{S-\gamma_I} \exp\left[-\frac{(I - \bar{I})^2}{2\sigma_i^2} \right] dI dS \qquad (21)$$

A. Results and Findings

There are no closed form solution of (21), and hence it has to be solved numerically. The upper limit of the inner integration can be estimated by $\mu + 3\sigma$, which covers 99.9% of the area under the Gaussian curve. This gives the estimate of the cdf as:

$$F(\gamma_I, \gamma'_S) \approx \frac{1}{4} \left[1 - \operatorname{erf}\left(\frac{\gamma'_S - \bar{S}}{\sqrt{2}\sigma_s} \right) \right] + \frac{1}{2\sqrt{\pi}} \int_{\frac{\gamma'_S - \bar{S}}{\sqrt{2}\sigma_s}}^{\bar{S}+3\sigma_s} \exp(-z^2) \operatorname{erf}\left(z + \frac{\bar{S} - \bar{I} - \gamma_I}{\sqrt{2}\sigma_i} \right) dz \qquad (22)$$

In order to find the achievable rate, Eq 22 is solved numerically to find the mean interference level ($\bar{I}$) that corresponds to a given outage probability. We have assumed that $\gamma_S = \gamma_I = 6.8$ dB, and $\sigma_s = \sigma_i = 8$ dB.

Extending the above analysis to scenario with multiple PRs and multiple STs is quite straight forward. For a given outage-probability, the tolerable mean interference level at each PR is obtained by solving Eq 22. Note that, with multiple interferers σ_i is now the sum of the variances of individual interference contribution.

Given the set of mean interference levels at each PR, transmit power among the STs is allocated using the algorithm described in Subsection III-D. The achievable ergodic rate for different outage probabilities with a log-normal shadowing channel model with three primary receivers and three secondary transmitters is presented in Figure 6.

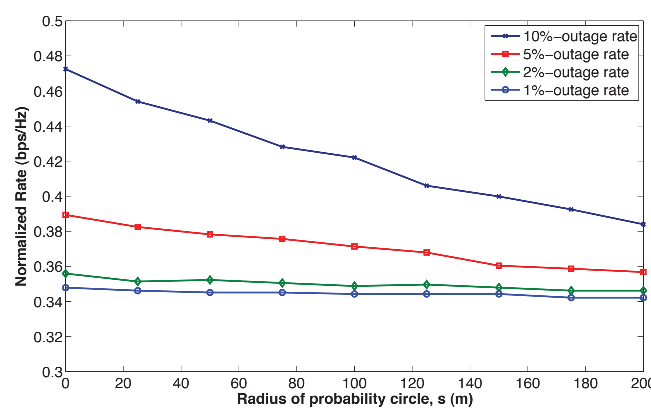

Fig. 6. Achievable rate for three STs and three PRs for different outage probability in shadow-fading environment

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Discussions

From the results presented in Figure 6, we find that for environments with shadow fading, the benefit in knowing the PR location is highly dependent on the outage probability at the PR.

The outage tolerated by systems in practice is in the range of five to ten percents. From Figure 6, we observe that the gain in the sum rate at the secondary system is most significant for these outage probabilities.

VI. Conclusion and Future work

Underlay cognitive radios can coexist with primary systems because spectral opportunities are *geographically localized* phenomena. We have observed that, in general, location information of primary receivers helps to improve the secondary system's rate performance, though the dependency is different for different scenarios and propagation environments. Furthermore, it is found that knowledge of PR location is greatly beneficial when there are more STs than PRs. This is because the relatively larger number of STs provides the secondary system with more options on how the available opportunities can be exploited.

For the AWGN channel model with a uniform probability distribution of the location information, the sum rate at the secondary system is found to depend on the number of PRs, the number of STs and the accuracy in location information. A mathematical approximation of this relationship is proposed by (8).

On the other hand, for AWGN channel model with Gaussian distributed location information, the sum rate at the secondary system is found to depend on the number of PRs, the number of STs, the accuracy in location information, and the probability of outage tolerated at the PR. This relationship is approximated by (11).

In the fading environment, the dependency of the secondary system sum rate on the location information is greatly dependent on the allowed outage probability at the PRs.

This is by no means an exhaustive work; in fact it only opens the door to many possible interesting research directions. Investigating the interplay between PU localization and SU performance for a fading channel model with probabilistic location information; or for more complicated primary and secondary systems, e.g. with directional/multiple antennas - are some examples of possible future work.

We have seen the benefit of PU localization, but have not looked at the cost of obtaining this information (e.g. by having to deploy sensors more densely). Intuitively speaking, this will be higher as the localization gets more accurate, resulting in a tradeoff between performance and cost of information. Finding an appropriate model for the cost, and combining the above results with that model, we can identify an optimum cost-performance operating point for the secondary system for different scenarios.

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