Location Optimization for Decode-and-forward Opportunistic Cooperative Networks

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***Abstract*— In this paper, we investigate the location optimization for decode-and-forward (DF) opportunistic cooperative networks, in which the impact of the availability of the direct link between the source and the destination has been studied. Under Rayleigh fading channels, the closed-form solution for the optimal location is derived. It is interesting to find that the optimal relay location is related to the power configuration of nodes and the pathloss exponent. When the direct link exists, the optimal location is partly dependent on the total number of the relays. Numerical results show that the approximation analysis matches the theory values very well, and with location optimization, the minimal outage probability can be obtained.**

I. INTRODUCTION

Signal fading due to multipath propagation is a major concern in wireless networks, and an effective way to mitigate fading is to implement diversity. By transmitting redundant signals over essentially independent channel realizations in conjunction with suitable receiver combining, diversity can be achieved to average the channel effects. Recently, cooperative diversity which was first proposed in [1] and [2], has been recognized as a low-cost and efficient technique to combat fading.

The basic idea of cooperative diversity is to exploit the potential of spatially dispersed user antennas to improve communications reliability. Later on, various relay protocols are brought up [3] and they can be categorized as decode-and-forward (DF) and amplify-and-forward (AF). In the former, relays decode the received messages, re-code and then transmit them to the destination, while for the latter, relays simply amplify and forward the source's messages. In [4], Laneman et al. demonstrate the superior performance of Distributed Space-Time Block Coded (DSTBC) transmission over orthogonal transmissions, but it requires symbol and carrier synchronization between distant cooperating nodes.

Opportunistic cooperation, also known as selection cooperation, firstly proposed in [5] - [7], has been shown to be an attractive alternative to traditional fixed cooperative strategies. By selecting only one cooperative partner, selection cooperation provides the same diversity order as DSTC but avoids the known problem of bandwidth expansion of a pure maximum ratio combining (MRC) system, as well as the synchronization and power-splitting problems of distributed space-time codes.

The available research on opportunistic cooperation has largely focused on resource allocation, protocol designing and performance analysis [5] - [8] and little attention has been paid to relay location. In fact, relay location is a very important issue in practical systems, especially when designing and deploying the infrastructures. In [9], the authors analyzed the effect relay location for a three-terminal cooperative system in Rayleigh fading environment, and demonstrated that for AF protocol, the optimal relay location is half-way between the source and the destination in most cases with the exception when the power is too low. For the DF mode, the optimal relay location is around but not extremely close to the source. In [10], Wang Li-Chun et al. investigated the optimal relay location aiming to maximize the system capacity for multi-hop cellular systems and found that the optimal relay location was relevant with the relay number and transmission schemes.

In this paper, we study the effect caused by the location of relays on the performance of opportunistic cooperative networks for DF protocols. The main contributions of this work include: 1) multiple relay scenario is studied and the effect of the direct link is taken into consideration; 2) the closed-form expression of the optimal relay location is derived and verified through numerical analysis; 3) the impact of the relay number, power configuration and pathloss exponent on the optimal relay location has been evaluated.

II. SYSTEM MODEL

Our model consists of a single source S, single destination D pair and N relays $\{R_k\}_{k=1}^N$ as shown in Fig.1. Assume that the links between the source and the relays, the relays and the destination, the source and the destination are independent, quasi-static Rayleigh fading. We use h_{SD}, h_{Sk} and h_{kD} to denote the instantaneous channel gains of the source-destination, source-relay k and relay k-destination, respectively. The channel coefficients are constant over a transmission and are independent from one transmission to another. There is also independent, zero-mean additive white Gaussian noise with unit variance at each receiver. It is assumed that the relays locate close to each other, so the pathloss for the source-relay and relay-destination links are almost the same. This assumption is practical in real systems, because deploying the relays at the same site can decrease the cost and facilitate the management, and this assumption has been used in other

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literatures [11]. Based on the assumption above, we use a basic location model as shown in Fig.2.

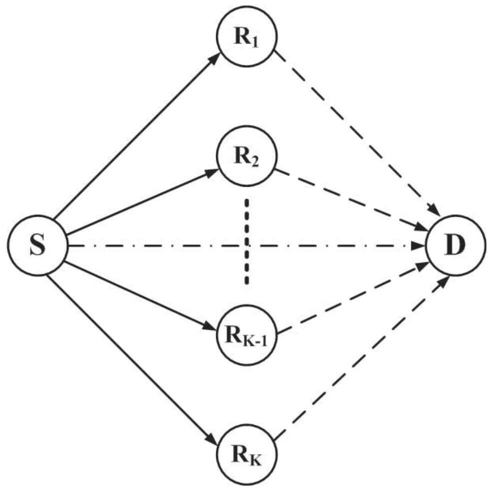

Fig. 1. Illustration of the cooperative relay system model.

In our analysis, we normalize the distance between the source and the destination, and assume that the relays are located between the source and the destination, on the straight line connecting them. We denote the source-relay distance as d and the relay-destination distance as $1-d$, where $0 < d < 1$. Due to the Rayleigh fading, the amplitude squares of the channel coefficients, denoted by $g_{SD} = |h_{SD}|^2$, $g_{Sk} = |h_{Sk}|^2$ and $g_{kD} = |h_{kD}|^2$, are exponentially distributed random variables with means λ_{SD}, λ_{Sk} and λ_{kD}, respectively. The means are determined by the pathloss across the corresponding link and from Fig.2 we can deduce that $\lambda_{SD} = 1$, $\lambda_{Sk} = d^{-\alpha}$ and $\lambda_{kD} = (1-d)^{-\alpha}$, where α is the pathloss exponent dependent on the propagation environments: for propagation that approximately follows a free-space or two-ray model α is set to 2 or 4, respectively.

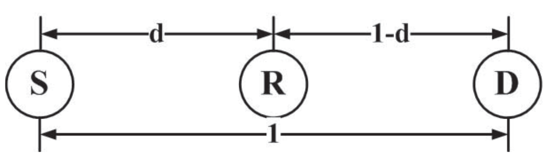

Fig. 2. The model for the location of the source, the relay and the destination.

For simplicity, protocols studied in this paper are based on time division schemes as in [3], where only one node is transmitting at each time slot. We consider half-duplex relays that can not transmit and receive simultaneously, and study opportunistic cooperation based on DF. The total time of the source is divided into two equal portions. In the first half, the source transmits and all the relays can listen, and in the second half, the $'best'$ relay which is selected according to a certain criterion forwards the message to the destination. The target rate of the communication between the source and the destination is denoted as R, the let $D(S)$ denote the decoding set of source S, which is consists of all the relays that can successfully decode the source's messages. Since all the relays in the decoding set can successfully decode the message, and the transmission rate for a two-hop network is constrained by the minimum of the rates on each hop, the transmission will outage if and only if the destination can not successfully decode the message.

III. OUTAGE PERFORMANCE ANALYSIS

A. Without direct link

The decoding set $D(S)$ is dependent on the normalized source transmit power P_S, the channel power gains of the source-relay links g_{Sk} and the target rate R, i.e., a specific relay $R_k \in D(S)$ if the capacity of the source-relay channel exceeds the target rate R:

$$\frac{1}{2}\log_2(1 + P_S g_{Sk}) \geq R \tag{1}$$

Where the factor of $1/2$ models the required spectral efficiency per hop due to the half-duplex constraint. Then we have the probability of $D(S)$ as (2). Let η denote 2^{2R-1}, which stands for the target received SNR, then (2) can be transformed as (3).

Similar to [6], we define a new random variable X as

$$X = \max_{k \in D(S)} P_R g_{kD} \tag{4}$$

where P_R is the normalized transmit power of relay, then due to the independence of g_{kD}, $\forall k$, the cumulative distribution function of X can be deduced as

$$\begin{aligned} F_X(x) &= \Pr\{X \leq x\} \\ &= \prod_{j \in D(S)} \left[1 - \exp\left(-\frac{x}{P_R}(1-d)^\alpha\right)\right] \\ &= \left[1 - \exp\left(-\frac{x}{P_R}(1-d)^\alpha\right)\right]^{|D(S)|} \end{aligned} \tag{5}$$

The channel mutual information for the relay-destination link is

$$I_{RD} = \frac{1}{2}\log_2(1 + X) \tag{6}$$

Since an non-zero decoding set exists, the outage happens only when I_{RD} is less than target rate R, then the conditional outage probability is

$$\begin{aligned} &\Pr\{outage\,|D(S)\} \\ &= F_X(\eta) = \left[1 - \exp\left(-\frac{\eta}{P_R}(1-d)^\alpha\right)\right]^{|D(S)|} \end{aligned} \tag{7}$$

The outage probability for opportunistic decode-and-forward cooperation without direct link can be obtained by the total probability law as (8).

B. With direct link

In this case, the destination can hear from the source and the $'best'$ relay, and the perfect channel state information is available at the receiver, so MRC can be implemented. We define another new random variable Y as

$$Y = P_S h_{SD} \tag{9}$$

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$$\Pr\{D(S)\} = \prod_{j\in D(S)} \Pr\left\{\frac{1}{2}\log_2(1+P_S g_{Sj}) \geq R\right\} \prod_{j\notin D(S)} \Pr\left\{\frac{1}{2}\log_2(1+P_S g_{Sj}) < R\right\}$$
$$= \prod_{j\in D(S)} \exp\left(-\frac{2^{2R-1}}{P_S}d^\alpha\right) \prod_{j\notin D(S)} \left[1-\exp\left(-\frac{2^{2R-1}}{P_S}d^\alpha\right)\right] \quad (2)$$

$$\Pr\{D(S)\} = \prod_{j\in D(S)} \exp\left(-\frac{\eta}{P_S}d^\alpha\right) \prod_{j\notin D(S)} \left[1-\exp\left(-\frac{\eta}{P_S}d^\alpha\right)\right] \quad (3)$$

$$\Pr\{I_{RD} < R\} = \sum_{D(S)} \Pr\{D(S)\} \Pr\{I_{RD} < R\,|D(S)\}$$
$$= \sum_{D(S)} \prod_{j\in D(S)} \exp\left(-\frac{\eta}{P_S}d^\alpha\right) \prod_{j\notin D(S)} \left[1-\exp\left(-\frac{\eta}{P_S}d^\alpha\right)\right] \left[1-\exp\left(-\frac{\eta}{P_R}(1-d)^\alpha\right)\right]^{|D(S)|}$$
$$= \sum_{i=0}^{N} \binom{N}{i} \left[\exp\left(-\frac{\eta}{P_S}d^\alpha\right)\right]^i \left[1-\exp\left(-\frac{\eta}{P_S}d^\alpha\right)\right]^{N-i} \left[1-\exp\left(-\frac{\eta}{P_R}(1-d)^\alpha\right)\right]^i \quad (8)$$

Then the combined SNR can be expressed as $X+Y$. The channel mutual information is thus

$$I_{SD} = \frac{1}{2}\log(1+X+Y) \quad (10)$$

Once an non-zero decoding set exists, an outage happens if and only if I_{SD} is less than the target rate R, and the conditional probability is

$$\Pr\{outage\,|D(S)\}$$
$$= \Pr\{X+Y<\eta\} \quad (11)$$
$$= \int_0^\eta \left[1-\exp\left(-\frac{\eta-x}{P_R}(1-d)^\alpha\right)\right]^{|D(S)|} \frac{\exp\left(-\frac{x}{P_S}\right)}{P_S}dx$$

Similarly, from the total probability law, we can get the outage probability for decode-and-forward opportunistic cooperation with direct link as (12).

IV. OPTIMAL RELAY LOCATION

In this section, we analyze how the relative position of the relays impacts the end-to-end outage performance. In order to find the relationship between the location and the outage probability, approximate analysis is widely used.

A. Without direct link

For very large power, we have the following approximations:

$$\exp\left(-\frac{\eta}{P}d^\alpha\right) \approx 1-\frac{\eta}{P}d^\alpha \approx 1 \quad (13)$$

$$1-\exp\left(-\frac{\eta}{P}d^\alpha\right) \approx \frac{\eta}{P}d^\alpha \quad (14)$$

Then (8) can be simplified as

$$\Pr\{I_{RD}<R\}$$
$$\approx \eta^N \sum_{i=0}^{N}\binom{N}{i}\left(\frac{d^\alpha}{P_S}\right)^{N-i}\left[\frac{(1-d)^\alpha}{\mathrm{P}_R}\right]^i$$
$$= \eta^N\left(\frac{d^\alpha}{P_S}+\frac{(1-d)^\alpha}{\mathrm{P}_R}\right)^N \quad (15)$$

Due to the fact that x^N is an increasing function for any non-negative x and positive integer N, the optimal location of the relay for opportunistic decode-and-forward cooperative network can be expressed as

$$d^* = \arg\min_{0<d<1} \frac{d^\alpha}{P_S}+\frac{(1-d)^\alpha}{\mathrm{P}_R} \quad (16)$$

Solve $P'_{out}(d)=0$ and we can obtain the optimal relay location for decode-and-forward protocol without direct link as

$$d^* = \frac{1}{1+\left(\frac{P_R}{P_S}\right)^{\frac{1}{\alpha-1}}} \quad (17)$$

B. With direct link

Bring (13) and (14) into (12) and according to the result in [7], we can get the approximation as

$$P_{out}$$
$$\approx \frac{\eta^{N+1}}{P_S}\sum_{D(S)}\frac{1}{|D(S)|+1}\left(\frac{(1-d)^\alpha}{P_R}\right)^{|D(S)|}\left(\frac{d^\alpha}{P_S}\right)^{N-|D(S)|}$$
$$\approx \frac{\eta^{N+1}}{P_S}\left[\frac{1}{N+1}\left(\frac{(1-d)^\alpha}{P_R}\right)^N+\left(\frac{d^\alpha}{P_S}\right)^N\right] \quad (18)$$

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$$
\begin{aligned}
\Pr\{I_{RD} < R\} &= \sum_{D(S)} \Pr\{D(S)\} \Pr\{I_{RD} < R | D(S)\} \\
&= \sum_{D(S)} \left[\left[\exp\left(-\frac{\eta}{P_S} d^\alpha\right) \right]^{|D(S)|} \left[1 - \exp\left(-\frac{\eta}{P_R}(1-d)^\alpha\right) \right]^{N-|D(S)|} \right] \\
&\quad \times \int_0^\eta \left[1 - \exp\left(-\frac{\eta - x}{P_R}(1-d)^\alpha\right) \right]^{|D(S)|} \frac{1}{P_S} \exp\left(-\frac{x}{P_S}\right) dx
\end{aligned} \tag{12}
$$

Solve $P'_{out}(d) = 0$, and we can get the optimal relay location for decode-and-forward protocol with direct link as

$$
d^* = \frac{1}{1 + \left[(N+1) \left(\frac{P_R}{P_S} \right)^N \right]^{\frac{1}{\alpha N - 1}}} \tag{19}
$$

V. NUMERICAL RESULTS

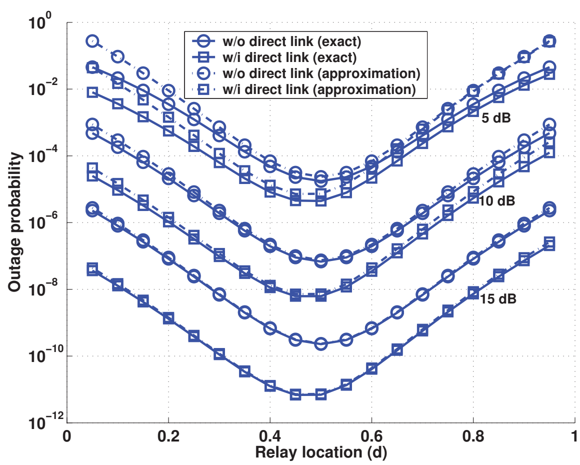

Fig. 3. The outage probability vs. relay location (relay number = 5, $P_S = P_R$, R = 1bps/Hz, $\alpha = 2$). The solid lines are for the exact analysis from (8) and (12), and the dash-dot lines are for the approximate analysis from (15) and (18).

Fig. 3 plots the outage probability vs. relay location when the transmit power of the source and the relays are the same. It is shown that the relay location has significant influence on the outage probability, and the approximation curves almost overlap the exact analysis ones, especially when the transmit power is high enough. For example, $15dB$ can make the two curves relatively close to each other. At the same time, we can see that the optimal relay location is at the middle point of the line between the source and the destination when the direct link is available, and when the direct link doesn't exist, the optimal location is nearer the the source, which can be proved by (17) and (19).

To see the effect of the relay location optimization, we plot the outage probability vs. the transmit power for the source and the relays. The searched results are obtained through exhaustive search from possible relay locations for the minimal end-to-end outage probability, while the calculated ones are obtained from (8), (12), (17) and (19), which provide the optimal relay location and the related outage probability. As

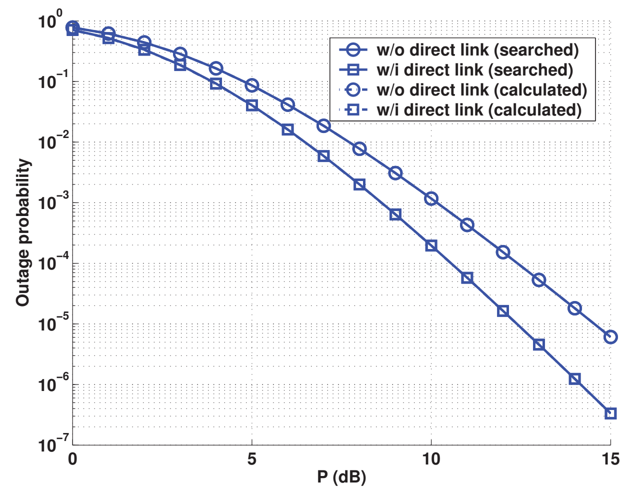

Fig. 4. The outage probability vs. transmit power (relay number = 5, $P_S = P_R = 15dB$, R = 1bps/Hz, $\alpha = 2$). The solid lines are obtained by exhaustive search, and the dash-dot lines are obtained by calculation.

shown in Fig. 4, the searched results match perfectly with the calculated ones, even in a lower transmit power region, which is due to the effective approximation at the impact of the optimal relay location.

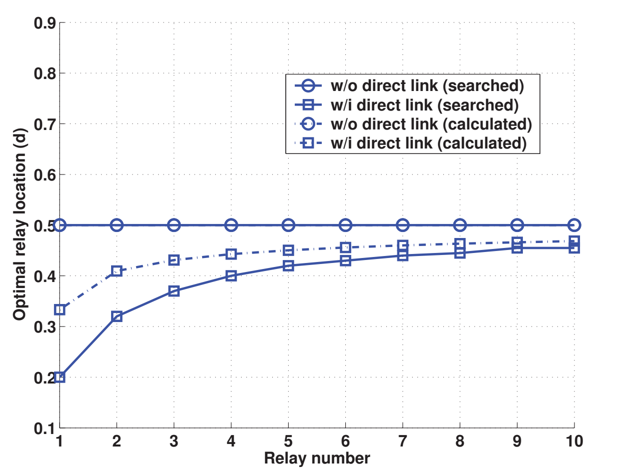

Fig. 5. The optimal relay location (d) vs. relay number ($P_S = P_R = 15dB$, R = 1bps/Hz, $\alpha = 2$). The solid lines are obtained by exhaustive search, and the dash-dot lines are obtained by calculation.

Fig. 5 illustrates the variation of the optimal relay location with respect to the number of relays. It is proved by (17) and (19) that when the direct link is unavailable, the optimal

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relay location is not related to the relay number, but when the direct link exists, the optimal relay location is connected to the relay number, which is also verified by Fig. 5. When the relay number is small, the searched results don't meet the calculated ones well due to the approximation of (18) is under the condition that the relay number is large. Therefore, as the relay number grows, the two results converge.

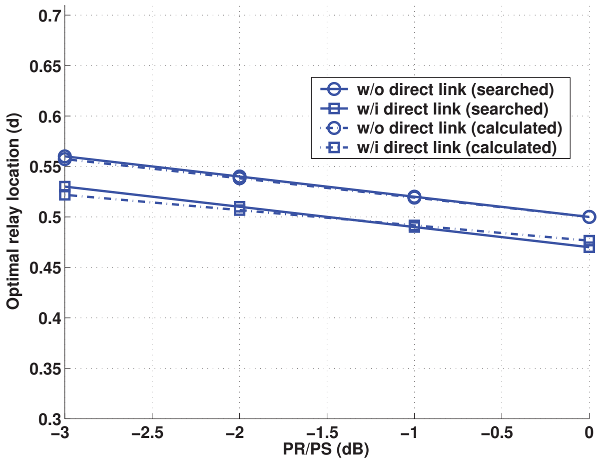

Fig. 6. The optimal relay location (d) vs. power ratio of the source and the relay (relay number = 5, $P_S = 15dB$, R = 1bps/Hz, $\alpha = 2$). The solid lines are obtained by exhaustive search, and the dash-dot lines are obtained by calculation.

Fig. 6 displays the optimal relay location vs. the transmit power ratio of the source and the relays. We can see that the power configuration has limited effect on the optimal relay location. As the power of the relay grows, the optimal relay location moves towards the source, and when the direct link exists, the optimal location is closer to the source.

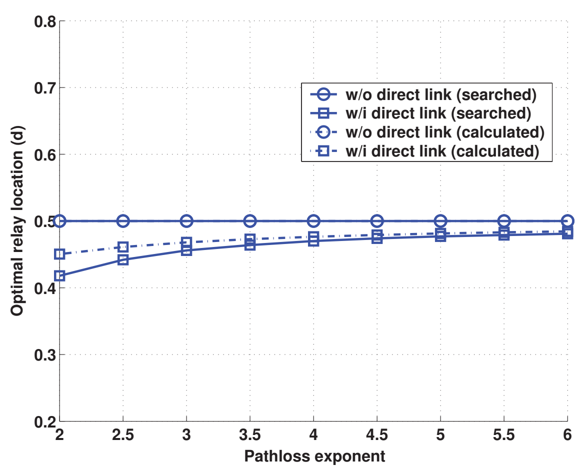

Fig. 7. The optimal relay location (d) vs. pathloss exponent α (relay number = 5, $P_S = P_R = 15dB$, R = 1bps/Hz). The solid lines are obtained by exhaustive search, and the dash-dot lines are obtained by calculation.

The curves for the optimal relay location vs. pathloss exponent are presented in Fig. 7. It is interesting to note that as the pathloss exponent increases, the optimal relay location is closer to the middle point of the line between the source and the destination. It is because that a large pathloss exponent can cause significant energy loss of the signal, which relatively reduces the difference between the power of the source and the relay. From (17) and (19) we can see that when the difference between the power is smaller, the optimal relay location is closer to the middle point.

VI. CONCLUSIONS

In this paper, the optimal location for decode-and-forward opportunistic cooperative networks is studied. The closed-form solution of the optimal location is obtained taking the availability of the direct link into consideration. The results show that many factors, including the power configuration, the pathloss exponent, the total number of the relays and the availability of the direct link have impact on the optimal relay location. Besides, with location optimization, the minimal outage probability can be obtained.

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