Stability Regions of Two-Way Relaying with Network Coding

(Invited Paper)

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ABSTRACT

We consider a pair of nodes with stochastic traffic flows who wish to communicate in a bi-directional communication scenario using intermediate relays in two-hop fashion. Intermediate relays are capable of XOR network coding. Transmission scheduling is done by tailoring the backpressure algorithm to the problem at hand. Two main alternatives for network operation are either to have queues at the relays (hop-by-hop scheduling) or no queues at the relays (immediate forwarding). In this two-way network with stochastic flows, we formulate and show that the resulting stability regions of these two approaches are identical.

Categories and Subject Descriptors

C.2.1 [**Computer-Communication Networks**]: Network Architecture and Design—*wireless communications, store and forward networks, network communications*

General Terms

Theory, Performance

Keywords

Two-way relaying, network stability, network coding

1. INTRODUCTION

In multi-hop networks, where intermediate relay nodes assist source node(s) [3, 8, 7], cross-layer approaches in which physical layer decisions are made jointly with higher layers [13, 4] have recently been attracting considerable attention. To that end, one essential issue is to consider the stochastic nature of the traffic to be communicated and to determine the power levels and the rates allocated for nodes according to the queue states as well as the channel states and allowable transmission modes [4, 13, 14]. Another topic of recently growing interest is network coding, where data is mixed at intermediate nodes in the network. Network coding has been shown to maximize multicast rate in single source scenarios [1].

A number of "throughput optimal" network control policies that ensure bounded queues whenever arrival rates lie in the stability region of the network have been developed [13, 4, 14, 9]. The most common of such approaches is the backpressure policy, also known as the Maximum Differential Backlog (MDB) algorithm, which has the desirable property of not requiring any a priori information on the input traffic statistics [13, 4]. MDB has recently been extended for cooperative communication scenarios as well [14].

Many wireless applications such as ad-hoc networks and peer-to-peer systems are based on two-way traffic. As a result, there has been considerable attention in understanding and exploiting the bi-directional nature of the information flow with intermediate relays [9, 10, 11, 6, 12, 5]. The nature of network coding is suitable for such scenarios [11].

In [2], we found that the backpressure policy tailored to the two-way four-node diamond network offers a diverse variety of transmission protocols and queueing options. In this work, we consider a pair of nodes with stochastic flows that communicate with each other in a bi-directional fashion, via two-hops where the intermediate relays utilize network coding. We consider slotted operation and two scheduling techniques. One option is hop-by-hop scheduling, with queueing at the relay(s). The other is immediate forwarding, where the relays immediately forward the received information at the relay nodes without queueing. In this work, we formulate the stability regions for the two-way network with network coding at the relay nodes and show that the resulting stability regions of these two approaches are identical.

2. SYSTEM MODEL

For clarity of exposition, we first consider a two-hop bi-directional network G with three nodes $N = \{1,R,2\}$ as shown in Figure 1. The end nodes $\{1,2\}$ are sources which aim to communicate with each other. No direct link exists between the end nodes, and, thus the relay node enables

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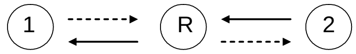

Figure 1: Single relay two-way network.

communication. Decode-and-forward relaying is used [3]. We extend our analysis to the two-way diamond network with sources {*1,2*} and two relays {*3,4*} in Section V.

As in references [14, 2], within a time slot, we enforce a half-duplex constraint, i.e., nodes cannot transmit and receive information simultaneously. Rate allocation decisions are made in each slot using the maximum differential backlog algorithm [13], tailored to the problem at hand.

The bi-directional information flow is carried out in two phases. In the first phase, information is transmitted to the relay node. In the second phase the relay node R transmits to the end nodes.

Traffic arriving at node i is assumed to be an ergodic process. Packet lengths $\{L_i\}$ of traffic at node i are assumed to be i.i.d. with $E[L_i] < \infty$ and $E[L_i^2] < \infty$. We assume infinite buffers. Due to the bi-directional nature of the model, we differentiate queues at relays according to their final destination. We thus define "forward" and "reverse" queues associated with transmission to node 2 (from 1) and 1 (from 2), respectively. All nodes know all channel coefficients and queue states. We assume the channels are static. The power constraint is P for all nodes, the noise variance and bandwidth are normalized to one.

3. RATE REGIONS AND THROUGHPUT OPTIMAL RATE ALLOCATION

For hop-by-hop scheduling, one of the following two transmission phases is selected for the time slot.

3.1 Phase I

In the first phase, where data is transmitted to the relay node, we have a multiple access channel whose capacity region is given by:

$$R_{1R} \leq \log(1 + h_{1R}P) \tag{1}$$

$$R_{2R} \leq \log(1 + h_{2R}P) \tag{2}$$

$$R_{1R} + R_{2R} \leq \log(1 + (h_{1R} + h_{2R})P). \tag{3}$$

where R_{iR} is the rate from node i to relay node R and $\sqrt{h_{iR}}$ denotes the channel gain from node i to node R.

3.2 Phase II

In this phase, the relay broadcasts to the end receivers. Note that, due to the bi-directional nature of communication, the end nodes 1 and 2 can subtract their own information from the received broadcast message [9]. Thus, we can assume that the operation is interference-free since all packets originate from the end nodes. In the bi-directional network, we exploit network coding following a similar approach to reference [11], by having the relay nodes transmit the exclusive-OR (XOR) of the information destined to nodes 1 and 2. In this case, the overall rate must be selected as the minimum of rates achievable between the relay and both end nodes. Both end nodes would be able to decode the overall codeword, obtaining the desired information by an XOR operation. If the queue for data to be combined for one of the end nodes empties before the end of the allocated transmission duration, zero padding is applied. In such a scenario, the remaining part of the codeword consists of data for one end. Hence, the effective rate region for Phase II is given by

$$R_{R1} \leq min(\log(1 + h_{1R}P), \log(1 + h_{2R}P)) \tag{4}$$

$$R_{R2} \leq min(\log(1 + h_{1R}P), \log(1 + h_{2R}P)). \tag{5}$$

3.3 Immediate Forwarding

An alternative transmission scheme for bi-directional communication is to divide the time slot into two phases and immediately forward the traffic received in the first phase by the relay nodes to the end nodes in the second phase [9, 10, 6] without queues at the relay nodes. References [9, 10] consider three-node networks with immediate forwarding with superposition coding at the relay. Within a time-slot, now end-to-end rate regions are defined:

$$\Re_{e-e} = \Delta(\Re_{Phase1}) \bigcap (1 - \Delta)(\Re_{Phase2}), \tag{6}$$

where $\Delta \in [0, 1]$is the time sharing parameters of the relay receive phase, and $\Re_{Phase1}$, $\Re_{Phase2}$ are rate regions corresponding to Phase I and Phase II.

Having discussed the achievable rate regions for different transmission stages, we next present the throughput optimal network control policy.

3.4 Network Control Policy

Our aim is to ensure the bi-directional network to operate according to a policy, where the queue backlogs remain bounded for any rate arrival vector that lies within the stability region of the network. The stability region of a network is defined as the closure of the set of all arrival rate vectors such that there exists some feasible joint rate allocation and routing policy in the network that guarantees that all queues in the network are stable. For the bi-directional network, at each time slot, active link selection and the corresponding rate allocation is done in accordance with the maximum differential backlog policy, also known as the backpressure approach [13]. We note that for the case of multiple relays cooperating, more advanced backpressure algorithms based on the cooperative maximum differential backlog algorithm(CMDB) [14] can be used. The optimal rate allocation at each time slot is given by the solution of the following optimization problem:

$$\max_{R \in C} \sum_{(i,j) \in L} w_{ij}^* R_{ij}, \tag{7}$$

where

$$w_{ij}^* = \max_{k \in K} \; q_i^k - q_j^k, \tag{8}$$

with q_i^k denoting the queue length associated with destination k at node i in bits. In our problem, for hop-by-hop scheduling, due to the half-duplex constraints, one of the two activation sets defines the allowable rates for the capacity regions depending on whether the relay is receiving or transmitting:

$$C_M \rightarrow \{R_{1R}, R_{2R}, 0, 0\} \tag{9}$$

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$$C_B \rightarrow \{0, 0, R_{R2}, R_{R1}\}. \tag{10}$$

The instantaneous link capacity region, i.e., the set of feasible link rates in any time slot is defined as $C = C_M \bigcup C_B$. The weight vector is given by $\vec{w^*} = (q_1 - q_{Rf}, q_2 - q_{Rr}, q_{Rf}, q_{Rr})$, where subscripts "$f$" denote "forward", "$r$" denote "reverse" for the relay queues. Accordingly, the resulting rate allocation is given by the configuration maximizing either

$$\bigcup_{R \in C_M} \{\ R_{1R}(q_1 - q_{Rf}) + R_{2R}(q_2 - q_{Rr})\ \} \tag{11}$$

or

$$\bigcup_{R \in C_B} \{\ R_{R2}(q_{Rf}) + R_{R1}(q_{Rr})\ \}, \tag{12}$$

Similar to [14, 2], for either the multi-access or the broadcast case, the rate allocation is determined in order to maximize the weighted sum of the two rate terms. The optimal operating points are found for the multi-access and broadcast case, and under hop-by-hop scheduling the policy selects the optimal rate allocation as the operating point yielding the maximum weighted rate. Either a Phase I or Phase II is scheduled each time slot. The rate allocation for Phase I defines the decoding order to be followed by the relays. In particular, data of the end node with the higher weight is decoded after the data of the other node.

For immediate forwarding, rate allocation is done in accordance with the backpressure approach as well. Due to the absence of relay queues, the backlogs reduce to the queue lengths for the two end nodes since the opposite edge of the link is assumed to be the other end node. The optimization problem reduces into the maximization of the inner product of two end-to-end rate terms and two queue terms at 1 and 2, i.e.,

$$\max_{(R_1, R_2) \in \Re_{e-e}} q_1 R_1 + q_2 R_2. \tag{13}$$

4. STABILITY REGIONS

In this section, we formulate and compare the stability regions for the hop-by-hop scheduling and immediate forwarding cases for the two-way single-relay network with network coding.

4.1 Hop-by-hop Scheduling

The stability region is the set of all arrival rates such that there exists non-negative flow variables which the arrival rates are supported relative to a weighted graph defined by the long-term rates in the convex hull of C, $conv(C)$. In the bidirectional network at hand, this corresponds to the set of source rate vectors (ρ_1^2, ρ_2^1) that can be supported. For the two-hop bidirectional network, in the hop-by-hop case (Fig.2), the following flow conservation relations must be satisfied:

$$\rho_1^2 = f_{1R} \tag{14}$$

$$\rho_2^1 = f_{2R} \tag{15}$$

for source nodes, and

$$0 = f_{R2} - f_{1R} \tag{16}$$

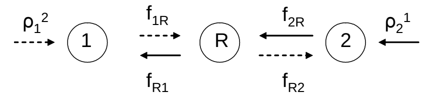

Figure 2: Hop-by-hop scheduling flows.

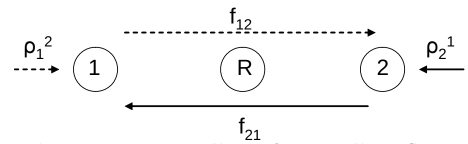

Figure 3: Immediate forwarding flows.

$$0 = f_{R1} - f_{2R} \tag{17}$$

for the relay node, where f_{ij} denotes flows from node i to node j. Additionally,

$$\rho_1^2 = f_{R2} \tag{18}$$

$$\rho_2^1 = f_{R1} \tag{19}$$

for the conservation of traffic specific to each commodity, that is the arrival rate of a specific commodity is equal to the departure rate. Moreover, $f \in conv(C)$. In hop-by-hop scheduling, $conv(C)$ is the set of convex combinations between $C_M = (R_{1R}, R_{2R}, 0, 0)$ and $C_B = (0, 0, R_{R2}, R_{R1})$, where C_M and C_B were defined in Section III.

In other words,

$$conv(C) = (\bigcup_{\Delta \in [0,1]} (\Delta R_{1R}, \Delta R_{2R}, (1-\Delta)R_{R2}, (1-\Delta)R_{R1})). \tag{20}$$

The flow conservation equations dictate

$$\rho_1^2 = f_{1R} = f_{R2} \tag{21}$$

$$\rho_2^1 = f_{2R} = f_{R1}, \tag{22}$$

and $(f_{1R}, f_{2R}, f_{R2}, f_{R1}) \in conv(C)$. In order to ensure that the flow conservation equations hold, some hop flows should be equal to each other. In the broad sense, arrival vectors in the stability region can be characterized by arrival rate vectors that are equal to the flow values such that $\Delta R_{1R} = (1-\Delta)R_{R2}$ and $\Delta R_{2R} = (1-\Delta)R_{R1}$. Points from the original (R_{1R}, R_{2R}) region are scaled as well as points from (R_{R2}, R_{R1}) for a specific Δ. The overall convex hull is the collection of all points for every Δ. For each Δ, flow variables satisfying the flow conservation equalities are valid.

4.2 Immediate Forwarding

For immediate forwarding (Fig.3), we have

$$\rho_1^2 = f_{12} \tag{23}$$

$$\rho_2^1 = f_{21} \tag{24}$$

for source nodes and $f \in conv(C)$. C is now two-dimensional, and for a specific Δ, is defined as

$$C_\Delta = (min(\Delta R_{1R}, (1-\Delta)R_{R2}), min(\Delta R_{2R}, (1-\Delta)R_{R1})), \tag{25}$$

where the individual phase rates $R_{1R}, R_{2R}, R_{R1}, R_{R2}$ are defined similarly as in the hop-by-hop case.

Here, $conv(C)$ is defined as the set of convex combinations between all C_Δ. The requirements due to the flow conservation equations are that the input arrival rate vectors are supported by the overall $conv(C)$. We have individual hop rates from the regions defined for (R_{1R}, R_{2R}) and

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(R_{R2}, R_{R1}). These rate pairs are scaled for a specific Δ time sharing parameter. Next, the minimum of hop components towards each direction are determined as end-to-end rates. Finally, the overall convex hull is defined by convex combinations between these regions for specific Δ. The set of allowable arrival rate pairs that can be supported are defined by the resulting convex hull region.

4.3 Comparison of Stability Regions

For hop-by-hop scheduling, the following flow conservation equations should be satisfied:

$$\Delta R_{1R} = (1-\Delta)R_{R2} \tag{26}$$

$$\Delta R_{2R} = (1-\Delta)R_{R1}. \tag{27}$$

This means that we need

$$\frac{R_{1R}}{R_{2R}} = \frac{R_{R2}}{R_{R1}}. \tag{28}$$

For any (R_{1R}, R_{2R}) pair on the Phase I boundary, we can find a corresponding (R_{R2}, R_{R1}) pair on the Phase II boundary.

Furthermore,

$$\frac{\Delta}{(1-\Delta)} = \frac{R_{R2}}{R_{1R}} = \frac{R_{R1}}{R_{2R}} \tag{29}$$

$$\frac{1}{\Delta} = \frac{R_{1R}}{R_{R2}} + 1 = \frac{R_{1R}+R_{R2}}{R_{R2}} = \frac{R_{2R}+R_{R1}}{R_{R1}} \tag{30}$$

$$\Delta = \frac{R_{R2}}{R_{1R}+R_{R2}} = \frac{R_{R1}}{R_{2R}+R_{R1}}. \tag{31}$$

Again, for each Phase I boundary point there is a corresponding Phase II point and a resulting Δ. Moreover, the resulting arrival rates supported are given by

$$\rho_1^2 = f_{1R} = \Delta R_{1R} = \frac{R_{1R}R_{R2}}{R_{1R}+R_{R2}} \tag{32}$$

$$\rho_2^1 = f_{2R} = \Delta R_{2R} = \frac{R_{2R}R_{R1}}{R_{2R}+R_{R1}}. \tag{33}$$

On the other hand, for immediate forwarding, for a specific Δ,

$$C_\Delta = (min(\Delta R_{1R}, (1-\Delta)R_{R2}), min(\Delta R_{2R}, (1-\Delta)R_{R1})), \tag{34}$$

where the individual phase rates $R_{1R}, R_{2R}, R_{R1}, R_{R2}$ are defined similarly as in the hop-by-hop scheduling case. Next, $conv(C)$ is defined as the set of convex combinations between all C_Δ.

First, we show that rate pairs supported resulting from the flow conservation equations of hop-by-hop scheduling are also supported by immediate forwarding. Indeed, as will be shown, these equations define rate pairs such that both rates cannot be increased simultaneously for a given Phase I or Phase II boundary point.

Proposition 1. *For immediate forwarding, the following equations define rates and flows such that:*
a)For any rate pair (R_{1R}, R_{2R}) from the Phase I boundary, both resulting rate components can not be increased simultaneously.
b)For any rate pair (R_{R1}, R_{R2}) from the Phase II boundary, both resulting rate components can not be increased simultaneously.

$$\Delta R_{1R} = (1-\Delta)R_{R2} \tag{35}$$

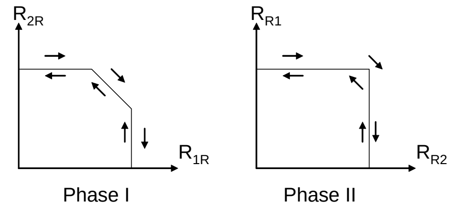

Figure 4: Moving on the Phase Rate Region Boundaries.

$$\Delta R_{2R} = (1-\Delta)R_{R1}. \tag{36}$$

Solving the equations, we have

$$\Delta = \frac{R_{R2}}{R_{1R}+R_{R2}} = \frac{R_{R1}}{R_{2R}+R_{R1}}, \tag{37}$$

and obtain the Δ s maximizing the individual rates, and the resulting flows are

$$\rho_1^2 = f_{12} = \Delta R_{1R} = \frac{R_{1R}R_{R2}}{R_{1R}+R_{R2}} \tag{38}$$

$$\rho_2^1 = f_{21} = \Delta R_{2R} = \frac{R_{2R}R_{R1}}{R_{2R}+R_{R1}}. \tag{39}$$

Proof. First, note that rates from Phase I (R_{1R}, R_{2R}) are coupled with each other. In a similar fashion, rates from Phase II (R_{R2}, R_{R1}) are coupled. To see whether we can have any rate improvement, first, for a fixed (R_{1R}, R_{2R}), vary Δ, (R_{R1}, R_{R2}). Note that while moving on the Phase II boundary (Fig. 4), while we increase one rate the other either remains the same or decreases. Similarly, if we decrease one rate, the other one either remains the same or increases. That is, we cannot increase or decrease both rates simultaneously. To accommodate for this behavior, in the following analysis, we note that γ_1, γ_2 cannot be of the opposite sign. Assume $\epsilon > 0$.

(i) Δ to $\Delta+\epsilon$, (R_{R1}, R_{R2}) to $(R_{R1} \pm \gamma_1, R_{R2} \mp \gamma_2)$.

$$\begin{aligned} &min((\Delta+\epsilon)R_{1R}, (1-\Delta-\epsilon)(R_{R2} \mp \gamma_2)) = \\ &min(\Delta R_{1R} + \epsilon R_{1R}, (1-\Delta)R_{R2} - \epsilon R_{R2} \mp \gamma_2(1-\Delta-\epsilon)) \end{aligned} \tag{40}$$

$$\begin{aligned} &min((\Delta+\epsilon)R_{2R}, (1-\Delta-\epsilon)(R_{R1} \pm \gamma_1)) = \\ &min(\Delta R_{2R} + \epsilon R_{2R}, (1-\Delta)R_{R1} - \epsilon R_{R1} \pm \gamma_1(1-\Delta-\epsilon)). \end{aligned} \tag{41}$$

In either subcase, the second term of one of the two rates decreases, hence one of the rates is reduced and no overall improvement on the rates is possible.

(ii) Δ to $\Delta-\epsilon$, (R_{R1}, R_{R2}) to $(R_{R1} \pm \gamma_1, R_{R2} \mp \gamma_2)$.

$$\begin{aligned} &min((\Delta-\epsilon)R_{1R}, (1-\Delta+\epsilon)(R_{R2} \mp \gamma_2)) = \\ &min(\Delta R_{1R} - \epsilon R_{1R}, (1-\Delta)R_{R2} + \epsilon R_{R2} \mp \gamma_2(1-\Delta+\epsilon)) \end{aligned} \tag{42}$$

$$\begin{aligned} &min((\Delta-\epsilon)R_{2R}, (1-\Delta+\epsilon)(R_{R1} \pm \gamma_1)) = \\ &min(\Delta R_{2R} - \epsilon R_{2R}, (1-\Delta)R_{R1} + \epsilon R_{R1} \pm \gamma_1(1-\Delta+\epsilon)). \end{aligned} \tag{43}$$

The first terms of both expressions decrease, hence the both rates are reduced and no overall improvement on the rates is possible.

When we change Δ while (R_{R1}, R_{R2}) is fixed for a fixed (R_{1R}, R_{2R}), note that both rates decrease. For fixed Δ, adjusting (R_{R1}, R_{R2}) results in either the decrease of one of the rates while the other remains the same, or both rates remain the same.

Next, for a fixed (R_{R1}, R_{R2}), consider varying Δ, (R_{1R}, R_{2R}). Again, while moving on the Phase I boundary(Fig.4), while we increase one rate either the other remains the same or

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decreases. Likewise, if we decrease one rate the other one either remains the same or increases. That is, we cannot increase or decrease both rates simultaneously. Again, we note that γ_1, γ_2 cannot be of the opposite sign in the following analysis. Assume $\epsilon > 0$.

(i) Δ to $\Delta + \epsilon$, (R_{1R}, R_{2R}) to $(R_{1R} \pm \gamma_1, R_{2R} \mp \gamma_2)$.

$$min((\Delta+\epsilon)(R_{1R} \pm \gamma_1), (1-\Delta-\epsilon)R_{R2} = \\ min(\Delta R_{1R} + \epsilon R_{1R} \pm \gamma_1(\Delta+\epsilon), (1-\Delta)R_{R2} - \epsilon R_{R2}) \tag{44}$$

$$min((\Delta+\epsilon)(R_{2R} \mp \gamma_2), (1-\Delta-\epsilon)R_{R1} = \\ min(\Delta R_{2R} + \epsilon R_{2R} \mp \gamma_2(\Delta+\epsilon), (1-\Delta)R_{R1} - \epsilon R_{R1}). \tag{45}$$

The second terms of both expressions decrease, hence both rates are reduced and no overall improvement on the rates is possible.

(ii) Δ to $\Delta - \epsilon$, (R_{1R}, R_{2R}) to $(R_{1R} \pm \gamma_1, R_{2R} \mp \gamma_2)$.

$$min((\Delta-\epsilon)(R_{1R} \pm \gamma_1), (1-\Delta+\epsilon)R_{R2} = \\ min(\Delta R_{1R} - \epsilon R_{1R} \pm \gamma_1(\Delta-\epsilon), (1-\Delta)R_{R2} + \epsilon R_{R2}) \tag{46}$$

$$min((\Delta-\epsilon)(R_{2R} \mp \gamma_2), (1-\Delta+\epsilon)R_{R1} = \\ min(\Delta R_{2R} - \epsilon R_{2R} \mp \gamma_2(\Delta-\epsilon), (1-\Delta)R_{R1} + \epsilon R_{R1}). \tag{47}$$

In either subcase, the first term of one of the two rates is reduced and no overall improvement on the rates is possible.

When we change Δ while (R_{1R}, R_{2R}) is fixed for a fixed (R_{R1}, R_{R2}), note that both rates decrease. For fixed Δ, adjusting (R_{1R}, R_{2R}) results in either the decrease of one of the terms while the other remains the same, or both remain the same.

Hence, pairs of $(R_{1R}, R_{2R}, \Delta, R_{R1}, R_{R2})$ given by the two equalities are pairs maximizing both rates in the sense that:
(i) For any point (R_{1R}, R_{2R}) on the Phase I boundary, no other combination of $\Delta, (R_{R1}, R_{R2})$ results in higher rates.
(ii)For any point (R_{R1}, R_{R2}) on the Phase II boundary, no other combination of $(R_{1R}, R_{2R}), \Delta$ results in higher rates.

So, both rates cannot be simultaneously increased apart from rate pairs given by these equations. □

Note that these equations are essentially identical to the flow conservation equations of the hop-by-hop case. So, immediate forwarding supports all arrival rate pairs supported by hop-by-hop scheduling.

One possible question is whether we can support extra flow pairs with immediate forwarding by increasing one of the rates while not necessarily maximizing the other. Note that it is obvious that rates are individually maximized when $\Delta R_{1R} = (1-\Delta)R_{R2}$ and $\Delta R_{2R} = (1-\Delta)R_{R1}$.

Proposition 2. *Assume that one of the rates is maximized, that is one of the equalities $\Delta R_{1R} = (1-\Delta)R_{R2}$ and $\Delta R_{2R} = (1-\Delta)R_{R1}$ is tight, while the other rate is simply the value which results from choosing the parameters to maximize the former rate. All such rate pairs can also be supported by the hop-by-hop flow conservation equations.*

PROOF. Given in the Appendix. □

We have shown that all arrival rate pairs supported by immediate forwarding can also be supported by hop-by-hop scheduling as well. As a result, we have proven the following theorem:

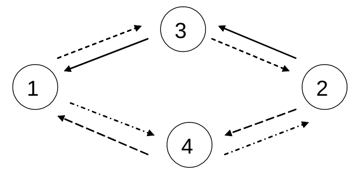

Figure 5: Two-way diamond network.

Theorem 1. *Every arrival rate pair in the hop-by-hop scheduling case can be supported by the immediate forwarding case and every arrival rate pair in the immediate forwarding case can be supported by the hop-by-hop scheduling case. The resulting stability regions are identical.*

5. EXTENSION TO THE TWO-WAY DIAMOND NETWORK

Next, we generalize our results on the stability regions for the single relay two-way network to the bi-directional diamond network with two relays {*3,4*} (Fig.5). First, we consider the model where one of the two relays receives or transmits data transmitted at a given time instant. In the hop-by-hop scheduling case, each time slot either sources transmit to one of the two relays, or one of the relays forwards previously received data via XOR network coding. On the other hand, in immediate forwarding, at each time slot, one relay is determined, and within the time slot the relay first receives data from the sources and immediately forwards it within the same slot.

5.1 Hop-by-hop Scheduling

As in the single relay case, flow conservation equations need to be satisfied for the sources and relays. The stability region corresponds to the set of the source rate vectors (ρ_1^2, ρ_2^1) that can be supported. In particular, the following flow conservation relations must be satisfied:

$$\rho_1^2 = f_{13} + f_{14} \tag{48}$$

$$\rho_2^1 = f_{23} + f_{24} \tag{49}$$

for the source nodes, and

$$0 = f_{32} - f_{13}, 0 = f_{31} - f_{23} \tag{50}$$

$$0 = f_{42} - f_{14}, 0 = f_{41} - f_{24} \tag{51}$$

for the relay nodes, where f_{ij} denotes flows from node i to node j. Additionally,

$$\rho_1^2 = f_{32} + f_{42} \tag{52}$$

$$\rho_2^1 = f_{31} + f_{41} \tag{53}$$

for the conservation of traffic specific to commodities. Again, $f \in conv(C)$. For the diamond network, using hop-by-hop scheduling, $conv(C)$ is the set of convex combinations between $C_{M3} = (R_{13}, R_{23}, 0, 0, 0, 0, 0, 0)$,
$C_{M4} = (0, 0, R_{14}, R_{24}, 0, 0, 0, 0)$, $C_{B3} = (0, 0, 0, 0, R_{32}, R_{31}, 0, 0)$ and $C_{B4} = (0, 0, 0, 0, 0, 0, R_{42}, R_{41})$ where phase rates are defined similarly as in Section III, with nodes *3* or *4* replacing R.

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In other words,

$$conv(C) = (\bigcup_{\sum \Delta_i = 1} (\Delta_1 R_{13}, \Delta_1 R_{23}, \Delta_2 R_{14}, \Delta_2 R_{24}, \Delta_3 R_{31}, \Delta_3 R_{32}, \Delta_4 R_{41}, \Delta_4 R_{42})), \tag{54}$$

and $(f_{13}, f_{23}, f_{14}, f_{24}, f_{31}, f_{32}, f_{41}, f_{42}) \in conv(C)$. The arrival rate vectors are equal to:

$$\rho_1^2 = f_{13} + f_{14} = \Delta_1 R_{13} + \Delta_2 R_{14} \tag{55}$$

$$\rho_2^1 = f_{23} + f_{24} = \Delta_1 R_{23} + \Delta_2 R_{24}, \tag{56}$$

where $\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 = 1$.

According to the flow conservation equations at the relays, we have the following requirements:

$$\Delta_1 R_{13} = \Delta_3 R_{32}, \Delta_1 R_{23} = \Delta_3 R_{31} \tag{57}$$

$$\Delta_2 R_{14} = \Delta_4 R_{42}, \Delta_2 R_{24} = \Delta_4 R_{21}. \tag{58}$$

Note that when we consider the flow conservation equations related with each relay node, they are in similar form as the single relay case. So, we have

$$\tfrac{R_{13}}{R_{23}} = \tfrac{R_{32}}{R_{31}} \tag{59}$$

$$\tfrac{R_{14}}{R_{24}} = \tfrac{R_{42}}{R_{41}}. \tag{60}$$

For any rate pair on the Phase I boundary of each relay, we can find a corresponding pair on the Phase II boundary of the same relay.

However, one difference from the single relay case is in the calculation of the resulting time sharing parameters:

$$\tfrac{\Delta_1}{\Delta_3} = \tfrac{R_{32}}{R_{13}} = \tfrac{R_{31}}{R_{23}} \tag{61}$$

$$\tfrac{\Delta_2}{\Delta_4} = \tfrac{R_{42}}{R_{14}} = \tfrac{R_{41}}{R_{24}}. \tag{62}$$

We do not have explicit expressions, but rather have individual relations between time sharing parameters related with each relay. Let us define $\tau = \Delta_1 + \Delta_3$. As a result, $\Delta_2 + \Delta_4 = 1 - \tau$.
Now let us focus on f_{13}:

$$f_{13} = \Delta_1 R_{13} = \tau(\tfrac{\Delta_1}{\tau}) R_{13} \tag{63}$$

$$\tfrac{\Delta_1}{\tau} = \tfrac{1}{1+\frac{\Delta_3}{\Delta_1}} = \tfrac{R_{32}}{R_{32}+R_{13}} \tag{64}$$

$$f_{13} = \tau \tfrac{R_{13} R_{32}}{R_{32}+R_{13}}. \tag{65}$$

Note that $R_{13}R_{32}/(R_{32} + R_{13})$ is equal to the arrival rate that would have resulted if only relay *3* was operating. We denote that as f'_{13}. Following a similar logic for the remaining flow variables, we have:

$$\begin{aligned} \rho_1^2 &= \tau f'_{13} + (1-\tau) f'_{14} \\ \rho_2^1 &= \tau f'_{23} + (1-\tau) f'_{24} \end{aligned} \tag{66}$$

for all $\tau \in [0, 1]$, since $\Delta_1 + \Delta_3$ can take any value from 0 to 1. The main observation is that the overall stability region of the relay selection based two-way diamond network is a time-sharing between all points in the stability regions of two single-relay two-way networks.

5.2 Immediate Forwarding

The following flow conservation equations needed to be satisfied at the source nodes: $\rho_1^2 = f_{12}$ and $\rho_2^1 = f_{21}$, where $(f_{12}, f_{12}) \in conv(C)$. The main difference from the single relay case is conv(C), which is now the convex combination of two separate single-relay immediate forwarding systems:

$$conv(C) = (\bigcup_{\Delta\in[0,1],\gamma\in[0,1],\tau\in[0,1]} (\tau(min(\Delta R_{13}, (1-\Delta)R_{32})) + (1-\tau)(min(\gamma R_{14}, (1-\gamma)R_{42}))), (\tau(min(\Delta R_{23}, (1-\Delta)R_{31})) + (1-\tau)(min(\gamma R_{24}, (1-\gamma)R_{41})))). \tag{67}$$

It is easy to notice that the expression is equivalent to the time sharing between all points in the stability regions resulting from using the relays separately. Hence, the stability regions resulting from hop-by-hop scheduling and immediate forwarding are equivalent. Note that while making this statement, we have used the fact that the stability regions of using only one of the relays is identical for hop-by-hop scheduling and immediate forwarding.

Remark: With multiple relays, another possibility is to beamform data to be forwarded. Even though applying coherent beamforming simultaneously to both directions via multiple relays becomes challenging when the XORed information is to be transmitted, for the case when all channel conditions are symmetric this can be done. In such a scenario, sources transmit to both relays and both relays forward data. The cooperative maximum differential backlog algorithm [14] is suitable for such networks for rate allocation. The rate regions are modified, however the analysis regarding the equivalence of the stability regions of hop-by-hop scheduling and immediate forwarding applies, hence the stability regions of these two approaches are also identical for such scenarios as well.

6. NUMERICAL RESULTS

In order to evaluate the performance of various strategies, we next present simulation results. Input traffic to the two nodes are independent Poisson processes. Figure 6 demonstrates queue evolution for a symmetric network with normalized channel gains and power levels with a single relay node. We observe that hop-by-hop scheduling and immediate forwarding result in similar performance. Figure 7 demonstrates empirical stability regions for various strategies, with $h_{13}P = h_{24}P = 1$ and $h_{14}P = h_{23}P = 2$, which models a scenario where one relay is closer to one source and the other relay is closer to the other source. The advantage of relay selection multiple relays is seen, as the overall operation is equivalent to time sharing between the individual relays, leading to a larger stability region than both individual cases. Again, we observe that hop-by-hop scheduling and immediate forwarding result in similar performance.

7. CONCLUSIONS

In this paper, we considered the stability region for two-hop bi-directional communication between a pair of nodes with stochastic flows. The two main possible options for operation are immediately forwarding data from the relay node or storing the data received from the end nodes in the relay buffers, and scheduling the next transmission. The relay nodes use XOR network coding as the forwarding scheme. The backpressure policy guarantees that the system will be

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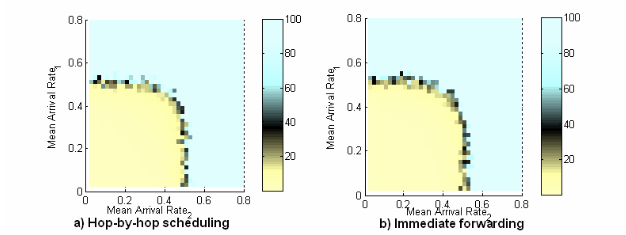

Figure 6: Mean queue backlogs vs. load for single relay with symmetric channel conditions.

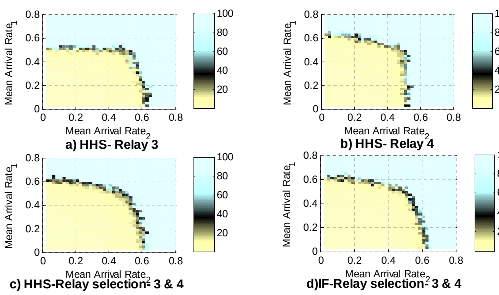

Figure 7: Mean queue backlogs vs. load for the diamond network.

stable for any arrival vector within the stability regions for both configurations.

We have shown that hop-by-hop forwarding and immediate forwarding result in identical stability regions, both for the two-way single relay network and the two-way diamond network. To achieve this performance, immediate forwarding involves time-slot optimization and more complex scheduling of two hops jointly, whereas hop-by-hop scheduling necessitates buffers at the relays.

This work considers static channel gains and decode and forward relaying. Incorporating the effect of time-varying channels on stability regions and considering other relaying strategies as well is of future interest.

APPENDIX

(*Proof of Proposition 2*) Recall that for the hop-by-hop scheduling case, the flow conservation equations lead to points $(R_{1R}, R_{2R}, \Delta, R_{R2}, R_{R1})$ such that $\frac{R_{1R}}{R_{2R}} = \frac{R_{R2}}{R_{R1}}$, $\Delta = \frac{R_{R2}}{R_{1R}+R_{R2}} = \frac{R_{R1}}{R_{2R}+R_{R1}}$, resulting in arrival rates $\rho_1^2 = \Delta R_{1R} = (1-\Delta)R_{R2}$, $\rho_2^1 = \Delta R_{2R} = (1-\Delta)R_{R1}$ with the ratio $\frac{\rho_1^2}{\rho_2^1} = \frac{R_{1R}}{R_{2R}} = \frac{R_{R2}}{R_{R1}}$.

Let us consider the four such possible cases explained in the proposition for immediate forwarding, and for each case we are interested in whether these arrival rate pairs are supported by the hop-by-hop scheduling as well:

(i) Rate 1 selected such that $\Delta_1 R_{1R} = (1-\Delta_1)R_{R2}$, rate 2 results in $min(\Delta_1 R_{2R}, (1-\Delta_1)R_{R1})$ which is equal to $\Delta_1 R_{2R}$ in this case. Consequently, $\frac{R_{1R}}{R_{2R}} > \frac{R_{R2}}{R_{R1}}$. The resulting arrival rates supported are $(\Delta_1 R_{1R}, \Delta_1 R_{2R})$.

We aim to find $(R'_{1R}, R'_{2R}, \Delta', R'_{R1}, R'_{R2})$ supported by hop-by-hop scheduling such that $\Delta' R'_{1R} = (1-\Delta')R'_{R2}$, $\Delta' R'_{2R} = (1-\Delta')R'_{R1}$ and $\frac{R'_{1R}}{R'_{2R}} = \frac{R'_{R2}}{R'_{R1}} = \frac{R_{1R}}{R_{2R}}$, which is equal to the ratio of the arrival rates supported by immediate forwarding for this case. These equalities are satisfied for $(R'_{1R}, R'_{2R}) = (R_{1R}, R_{2R})$ and associated (R'_{R2}, R'_{R1}) satisfying either:

a) $(R'_{R2} > R_{R2})$ and $(R'_{R1} < R_{R1})$. These rates lead to $\Delta' = \frac{R'_{R2}}{R_{1R}+R'_{R2}} > \Delta_1$. Finally, the arrival rates supported $= (\Delta' R_{1R}, \Delta' R_{2R})$ which is greater than $(\Delta_1 R_{1R}, \Delta_1 R_{2R})$.

b) $(R'_{R2} > R_{R2})$ and $(R'_{R1} = R_{R1})$. These rates lead to $\Delta' = \frac{R'_{R2}}{R_{1R}+R'_{R2}} > \Delta_1$. Finally, the arrival rates supported $= (\Delta' R_{1R}, \Delta' R_{2R})$ which is greater than $(\Delta_1 R_{1R}, \Delta_1 R_{2R})$.

c) $(R'_{R2} = R_{R2})$ and $(R'_{R1} < R_{R1})$. These rates lead to $\Delta' = \frac{R'_{R2}}{R_{1R}+R'_{R2}} = \Delta_1$. Finally, the arrival rates supported $= (\Delta' R_{1R}, \Delta' R_{2R})$ which is equal to $(\Delta_1 R_{1R}, \Delta_1 R_{2R})$.

Hence, those arrival rates supported can also be supported by hop-by-hop scheduling.

(ii) Rate 1 selected such that $\Delta_1 R_{1R} = (1-\Delta_1)R_{R2}$, rate 2 results in $min(\Delta_1 R_{2R}, (1-\Delta_1)R_{R1})$ which is equal to $(1-\Delta_1)R_{R1}$ in this case. Consequently, $\frac{R_{1R}}{R_{2R}} < \frac{R_{R2}}{R_{R1}}$. The resulting arrival rates supported are $((1-\Delta_1)R_{R2}, (1-\Delta_1)R_{R1})$.

We find $(R'_{1R}, R'_{2R}, \Delta', R'_{R1}, R'_{R2})$ such that $\Delta' R'_{1R} = (1-\Delta')R'_{R2}$, $\Delta' R'_{2R} = (1-\Delta')R'_{R1}$ and $\frac{R'_{1R}}{R'_{2R}} = \frac{R'_{R2}}{R'_{R1}} = \frac{R_{R2}}{R_{R1}}$, which is equal to the ratio of arrival rates supported by immediate forwarding. These equalities are satisfied for $(R'_{R1}, R'_{R2}) = (R_{R1}, R_{R2})$ and associated (R'_{1R}, R'_{2R}) satisfying either:

a)$(R'_{1R} > R_{1R})$ and $(R'_{2R} < R_{2R})$ leading to $\Delta' = \frac{R_{R2}}{R'_{1R}+R_{R2}} < \Delta_1$. Finally, the arrival rates supported $= ((1-\Delta')R_{R2}, (1-\Delta')R_{R1})$ which is greater than $((1-\Delta_1)R_{R2}, (1-\Delta_1)R_{R1})$.

b)$(R'_{1R} = R_{1R})$ and $(R'_{2R} < R_{2R})$ leading to $\Delta' = \frac{R_{R2}}{R_{1R}+R_{R2}} = \Delta_1$. Finally, the arrival rates supported $= ((1-\Delta')R_{R2}, (1-\Delta')R_{R1})$ which is equal to $((1-\Delta_1)R_{R2}, (1-\Delta_1)R_{R1})$.

c)$(R'_{1R} > R_{1R})$ and $(R'_{2R} = R_{2R})$ leading to $\Delta' = \frac{R_{R2}}{R'_{1R}+R_{R2}} < \Delta_1$. Finally, the arrival rates supported $= ((1-\Delta')R_{R2}, (1-\Delta')R_{R1})$ which is greater than $((1-\Delta_1)R_{R2}, (1-\Delta_1)R_{R1})$.

Hence, those arrival rates supported can also be supported by hop-by-hop scheduling.

(iii) Rate 2 selected such that $\Delta_2 R_{2R} = (1-\Delta_2)R_{R1}$, rate 1 results in $min(\Delta_2 R_{1R}, (1-\Delta_2)R_{R2})$ which is equal to $\Delta_2 R_{1R}$ in this case. This is shown similarly as (i).

(iv) Rate 2 selected such that $\Delta_2 R_{2R} = (1-\Delta_2)R_{R1}$, rate 1 results in $min(\Delta_2 R_{1R}, (1-\Delta_2)R_{R2})$ which is equal to $(1-\Delta_2)R_{R2}$ in this case. This is shown similarly as (ii).

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