Information Dissemination in Large-Scale Wireless Networks with Unreliable Links (Invited Paper) *

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ABSTRACT

We study connectivity and information dissemination in large-scale wireless networks with unreliable links from a percolation-based perspective. We first examine static models, where each link of the network is functional with some probability, independently of all other links. We then examine dynamic models, where each link is active or inactive according to a Markov on-off process. We show that a phase transition exists in such dynamic networks, and the critical density for this model is the same as the one for static networks. Furthermore, due to the dynamic behavior of links, a delay is incurred for any information dissemination process even when propagation delay is ignored. We study the behavior of this delay and show that (ignoring propagation delay) the delay scales linearly with the Euclidean distance between the sender and the receiver when the network is in the subcritical phase, and the delay scales sub-linearly with the distance if the network is in the supercritical phase. We then show that when taking propagation delay into account, the delay of information dissemination always scales linearly with the Euclidean distance between the sender and the receiver.

Categories and Subject Descriptors

C.2.4 [**Computer-Communication Networks**]: Distributed Systems; G.3 [**Probability and Statistics**]: Stochastic Processes

General Terms

Performance, theory

Keywords

Information dissemination, first passage percolation, subadditive ergodic theorem

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1. INTRODUCTION

Large-scale wireless networks for the gathering, processing, and dissemination of information have become an important part of the modern communication infrastructure. Traditionally, the performance of wireless networks has been examined under the assumption of maintaining full connectivity (or k-connectivity). Here, the system ensures that any pair of nodes in the network are connected by a path (or k paths). In large-scale wireless networks exposed to severe natural hazards, enemy attacks, and resource depletion, however, this full connectivity criterion may be overly restrictive or impossible to achieve.

In this paper, we view the connectivity of large-scale wireless networks from a different perspective. One simple measure of the network functionality is the fraction of nodes in the largest connected component of the network: nodes in that component can communicate with an extensive portion of the network, while those in smaller components can communicate only with at most a few other nodes. On the other hand, if after many sensor failures, the sensor network breaks down into isolated parts where even the largest component can reach only a few sensors, then the network is not considered to be functional. From this perspective, the characterization of network connectivity corresponds to the study of the qualitative and quantitative properties of the largest component. A powerful technique for this study comes from the mathematical theory of *percolation* [1–3]. Recently, percolation theory, especially continuum percolation theory, has become a useful tool for the analysis of coverage, connectivity, capacity and latency in large-scale wireless networks [4–6].

To intuitively understand percolation processes in large-scale wireless networks, consider the following example. Suppose a set of nodes are uniformly and independently distributed at random over an area. All nodes have the same transmission radius, and two nodes within a transmission radius of each other are assumed to communicate directly. At first, the nodes are distributed according to a very small density. This results in isolation and no communication among nodes. As the density increases, some clusters in which nodes can communicate with one another directly or indirectly (via multi-hop relay) emerge, though the sizes of these clusters are still small compared to the whole network. As the density continues to increase, at some critical point a huge cluster containing a large portion of the network forms. This phenomenon of a sudden and drastic change in the global structure is called a *phase transition*. The density at which phase transition takes place is called the *critical den-*

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sity [1–3]. A fundamental result of continuum percolation concerns such a phase transition effect whereby the macroscopic behavior of the system is very different for densities below and above the critical density λ_c. For $\lambda < \lambda_c$ (subcritical), the connected component containing the origin (or any other fixed node) contains a finite number of points almost surely. For $\lambda > \lambda_c$ (supercritical), the connected component containing the origin (or any other fixed node) contains an infinite number of points with a positive probability [1–3].

Due to noise, fading and multi-user interference, communication links in wireless networks are unreliable. Even when two nodes lie within each other's transmission range, a viable communication link may not exist between the two nodes due to path-loss. Furthermore, the link quality may switch between the active and inactive states due to fading. To capture this effect, we study percolation processes in wireless networks with dynamic links, where each link of the network is functional (active) according to some Markov on-off process, independently of all other network links. We show that a phase transition exists in these dynamic networks under certain conditions, and the critical density for this model is the same as the one for static networks with the same parameters. Due to the dynamic behavior of links, a delay is incurred for any information dissemination even when propagation delay is ignored. We study the behavior of this delay by modelling the problem as a first passage percolation [7,8] process on random geometric graphs. We show that ignoring propagation delay, the delay of information dissemination scales linearly with the Euclidean distance between the sender and the receiver when the dynamic network is in the subcritical phase, and the delay scales sub-linearly with the distance if the dynamic network is in the supercritical phase. We further show that when taking propagation delay into account, the delay of information dissemination always scales linearly with the Euclidean distance between the sender and the receiver.

This paper is organized as follows. In Section 2, we outline some preliminary results for random geometric graphs and continuum percolation. In Section 3, we study wireless networks with *static* unreliable links. In Section 4, we introduce a model for wireless networks with *dynamic* unreliable links, and study percolation-based connectivity and information dissemination delay performance in such dynamic networks. Finally, in Section 5, we conclude the paper.

2. RANDOM GEOMETRIC GRAPHS AND CONTINUUM PERCOLATION

We use random geometric graphs to model wireless networks. That is, we assume that the network nodes are randomly placed over some area or volume, and a communication link exists between two (randomly placed) nodes if the distance between them is sufficiently small, so that the received power is large enough for successful decoding. A mathematical model for this is as follows. Let $\|\cdot\|$ be the Euclidean norm, and $f(\cdot)$ be some probability density function (p.d.f.) on $\mathbb{R}^d$. Let $\mathbf{X}_1, \mathbf{X}_2, ..., \mathbf{X}_n$ be independent and identically distributed (i.i.d.) d-dimensional random variables with common density $f(\cdot)$, where $\mathbf{X}_i$ denotes the random location of node i in $\mathbb{R}^d$. The ensemble of graphs with undirected links connecting all those pairs $\{\mathbf{x}_i, \mathbf{x}_j\}$ with $\|\mathbf{x}_i - \mathbf{x}_j\| \leq r, r > 0$, is called a *random geometric graph* [3], denoted by $G(\mathcal{X}_n, r)$. The parameter r is called the characteristic radius.

In the following, we consider random geometric graphs $G(\mathcal{X}_n, r)$ in $\mathbb{R}^2$, with $\mathbf{X}_1, \mathbf{X}_2, ..., \mathbf{X}_n$ distributed i.i.d. according to a uniform distribution in a square area $\mathcal{A} = [0, \sqrt{\frac{n}{\lambda}}]^2$. Let $A = |\mathcal{A}|$ be the area of $\mathcal{A}$. There exists a link between two nodes i and j if and only if i lies within a circle of radius r around $\mathbf{x}_j$. As n and A both become large with the ratio $\frac{n}{A} = \lambda$ kept constant, $G(\mathcal{X}_n, r)$ converges in distribution to an (infinite) random geometric graph $G(\mathcal{H}_\lambda, r)$ induced by a homogeneous Poisson point process with density $\lambda > 0$. Due to the scaling property of random geometric graphs [2,3], in the following, we focus on $G(\mathcal{H}_\lambda, 1)$.

Let $\mathcal{H}_{\lambda,\mathbf{0}} = \mathcal{H}_\lambda \cup \{\mathbf{0}\}$, i.e., the union of the origin and the infinite homogeneous Poisson point process with density λ. Note that in a random geometric graph induced by a homogeneous Poisson point process, the choice of the origin can be arbitrary. As discussed before, a phase transition takes place at the critical density. More formally, we have the following definition [2]:

Definition 1. *For $G(\mathcal{H}_{\lambda,\mathbf{0}}, 1)$, let $W_\mathbf{0}$ be the component of $G(\mathcal{H}_{\lambda,\mathbf{0}}, 1)$ containing $\mathbf{0}$. Define the following critical densities: $\lambda_\# \triangleq \inf\{\lambda : \Pr(|W_\mathbf{0}| = \infty) > 0\}$, $\lambda_N \triangleq \inf\{\lambda : E[|W_\mathbf{0}|] = \infty\}$, $\lambda_c \triangleq \inf\{\lambda : \Pr(d(W_\mathbf{0}) = \infty) > 0\}$, $\lambda_D \triangleq \inf\{\lambda : E[d(W_\mathbf{0})] = \infty\}$, where $|W_\mathbf{0}|$ is the cardinality—the number of nodes—of $W_\mathbf{0}$, and $d(W_\mathbf{0}) \triangleq \sup\{||\mathbf{x} - \mathbf{y}|| : \mathbf{x}, \mathbf{y} \in W_\mathbf{0}\}$.*

As shown in Theorem 3.4 and Theorem 3.5 in [2], these four critical densities are identical. According to the theory of continuum percolation $0 < \lambda_c < \infty$. Furthermore, when $\lambda > \lambda_c$, there exists one unique infinite component in $G(\mathcal{H}_{\lambda,\mathbf{0}}, 1)$ with probability 1 (w.p.1), and when $\lambda < \lambda_c$, there is no infinite component in $G(\mathcal{H}_{\lambda,\mathbf{0}}, 1)$ w.p.1 [2].

3. WIRELESS NETWORKS WITH STATIC UNRELIABLE LINKS

Random geometric graphs are good simplified models for wireless networks. However, due to noise, fading, and interference, wireless communication links between two nodes are usually unreliable. We use the bond percolation model on random geometric graphs to study percolation-based connectivity of large-scale wireless networks with static unreliable links. Given a random geometric graph $G(\mathcal{H}_\lambda, 1)$, let each link of $G(\mathcal{H}_\lambda, 1)$ be active (independent of all other links) with probability $p_e(d)$ which may depend on d, where $d = \|\mathbf{x}_i - \mathbf{x}_j\| \leq 1$ is the length of the link (i, j). The resulting graph consisting of all active links and their end nodes is denoted by $G(\mathcal{H}_\lambda, 1, p_e(\cdot))$. This model is a specific example of the *random connection model* in continuum percolation theory [2].

Definition 2. *For $G(\mathcal{H}_{\lambda,\mathbf{0}}, 1, p_e(\cdot))$, let $W_\mathbf{0}$ be the component of $G(\mathcal{H}_{\lambda,\mathbf{0}}, 1, p_e(\cdot))$ containing $\mathbf{0}$. We define four critical densities: $\lambda_\#(p_e(\cdot)) \triangleq \inf\{\lambda : \Pr(|W_\mathbf{0}| = \infty) > 0\}$, $\lambda_N(p_e(\cdot)) \triangleq \inf\{\lambda : E[|W_\mathbf{0}|] = \infty\}$, $\lambda_c(p_e(\cdot)) \triangleq \inf\{\lambda : \Pr(d(W_\mathbf{0}) = \infty) > 0\}$, $\lambda_D(p_e(\cdot)) \triangleq \inf\{\lambda : E[d(W_\mathbf{0})] = \infty\}$, where $|W_\mathbf{0}|$ is the cardinality—the number of nodes—of $W_\mathbf{0}$, and $d(W_\mathbf{0}) \triangleq \sup\{||\mathbf{x} - \mathbf{y}|| : \mathbf{x}, \mathbf{y} \in W_\mathbf{0}\}$.*

Proposition 1. *For $G(\mathcal{H}_{\lambda,\mathbf{0}}, 1, p_e(\cdot))$, we have*

$$\lambda_\#(p_e(\cdot)) = \lambda_N(p_e(\cdot)) = \lambda_c(p_e(\cdot)) = \lambda_D(p_e(\cdot)). \quad (1)$$

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Proof: The identity $\lambda_{\#}(p_e(\cdot)) = \lambda_N(p_e(\cdot))$ is given by Theorem 6.2 in [2].

We now show $\lambda_{\#}(p_e(\cdot)) = \lambda_c(p_e(\cdot))$. The proof method is similar to the one used for Theorem 3.4 in [2]. Suppose $\lambda > \lambda_{\#}(p_e(\cdot))$. Then for some $\delta > 0$, $\Pr(|W_{\mathbf{0}}| = \infty) = \delta > 0$. For every $h > 0$, the box $B(h) = [-h, h]^2$ contains at most a finite number of nodes of $G(\mathcal{H}_{\lambda,\mathbf{0}}, 1, p_e(\cdot))$ w.p.1. Thus, $\Pr(|W_{\mathbf{0}} \cap B(h)^c| = \infty) = \delta > 0$. However, $\{|W_{\mathbf{0}} \cap B(h)^c| = \infty\}$ implies $\{|W_{\mathbf{0}} \cap B(h)^c| > 0\}$, so that $d(W_{\mathbf{0}}) \geq h$. Hence we have $\Pr(d(W_{\mathbf{0}}) \geq h) = \delta > 0$. Since this holds for all $h > 0$, we have $\lambda > \lambda_c(p_e(\cdot))$. Therefore, $\lambda_{\#}(p_e(\cdot)) \geq \lambda_c(p_e(\cdot))$. To show $\lambda_{\#}(p_e(\cdot)) \leq \lambda_c(p_e(\cdot))$, note that $d(W_{\mathbf{0}}) \leq |W_{\mathbf{0}}| - 1$, where equality is obtained when $W_{\mathbf{0}}$ is a chain and the distance between any two adjacent nodes equals 1. Thus, $\{|W_{\mathbf{0}}| < \infty\}$ implies $\{d(W_{\mathbf{0}}) < \infty\}$. This proves $\lambda_{\#}(p_e(\cdot)) = \lambda_c(p_e(\cdot))$.

Finally we show $\lambda_D(p_e(\cdot)) = \lambda_N(p_e(\cdot))$. Since $d(W_{\mathbf{0}}) \leq |W_{\mathbf{0}}| - 1$, $\{E[d(W_{\mathbf{0}})] = \infty\}$ implies $\{E[|W_{\mathbf{0}}|] = \infty\}$. Thus we have $\lambda_D(p_e(\cdot)) \geq \lambda_N(p_e(\cdot))$. On the other hand, if $\lambda > \lambda_N(p_e(\cdot))$, then $\lambda > \lambda_c(p_e(\cdot))$, i.e., $\Pr(d(W_{\mathbf{0}}) = \infty) > 0$. As a consequence, $E[d(W_{\mathbf{0}})] = \infty$, which implies $\lambda_N(p_e(\cdot)) \geq \lambda_D(p_e(\cdot))$. Therefore, $\lambda_D(p_e(\cdot)) = \lambda_N(p_e(\cdot))$. □

Since the four critical densities are identical, in the remainder of this paper, we state our results with respect to $\lambda_c(p_e(\cdot))$, which can be interchanged with $\lambda_{\#}(p_e(\cdot))$, or $\lambda_N(p_e(\cdot))$, or $\lambda_D(p_e(\cdot))$.

It is known that when $\lambda > \lambda_c(p_e(\cdot))$, $G(\mathcal{H}_\lambda, 1, p_e(\cdot))$ is percolated, i.e., there exists one unique infinite component in $G(\mathcal{H}_\lambda, 1)$ consisting of active links and their end nodes w.p.1, and when $\lambda < \lambda_c(p_e(\cdot))$, $G(\mathcal{H}_\lambda, 1, p_e(\cdot))$ is not percolated, i.e., there is no infinite component in $G(\mathcal{H}_\lambda, 1)$ consisting of active links and their end nodes w.p.1 [2].

The following proposition asserts that when the random connection model is in the subcritical phase, the probability that the origin and any node are connected decays exponentially with respect to their distance. This is analogous to similar results in traditional continuum percolation (Theorem 2.4 in [2]) and discrete percolation (Theorem 5.4 in [1]). The proof is omitted here due to space limitations.

Proposition 2. *Given $G(\mathcal{H}_{\lambda,\mathbf{0}}, 1, p_e(\cdot))$ with $\lambda < \lambda_c(p_e(\cdot))$, let $B(h) = [-h, h]^2$, $h \in \mathbb{R}^+$. Then there exist constants $c_1, c_2 > 0$, such that $\Pr(\mathbf{0} \leftrightsquigarrow B(h)^c) \leq c_1 e^{-c_2 h}$, where $\{\mathbf{0} \leftrightsquigarrow B(h)^c\}$ denotes the event that the origin and some node in $B(h)^c$ are connected, i.e., the origin and some node outside $B(h)$ are in the same component.*

4. WIRELESS NETWORKS WITH DYNAMIC UNRELIABLE LINKS

4.1 Percolation-based Connectivity

For large-scale wireless networks with static unreliable links, we assumed that the structure of the graph does not change with time. Once a link is active, it remains active forever. In wireless networks, however, the link quality usually varies with time due to shadowing and multi-path fading. In order to study percolation-based connectivity of wireless networks with time-varying links, we investigate a more sophisticated model. Formally, given a wireless network modelled by $G(\mathcal{H}_\lambda, 1)$, we associate a stationary on-off state process $\{W_{ij}(d_{ij}, t); t \geq 0\}$ with each link (i, j), where d_{ij} is the length of the link, such that $W_{ij}(d_{ij}, t) = 0$ if link (i, j) is inactive at time t, and $W_{ij}(d_{ij}, t) = 1$ if link (i, j) is active at time t. In discrete lattices, a similar problem has been studied in [9]. Our model can be viewed as a dynamic bond percolation in random geometric graphs.

For such dynamic networks, we will show that there exists a phase transition, and the critical density for this model is the same as the one for static networks with the same parameters. To simplify matters, assume that $\{W_{ij}(d_{ij}, t)\}$ is probabilistically identical for all links with the same length. Use $\{W(d, t)\}$ to denote the process for a link with length d when no ambiguity arises. Assume that $\{W(d, t)\}$ is a Markov on-off process with i.i.d. inactive periods $Y_k(d), k \geq 1$, and i.i.d. active periods $Z_k(d), k \geq 1$, where $E[Y_k(d) + Z_k(d)] < \infty$, $\Pr(Z_k(d) > 0) = 1$ and $\Pr(Y_k(d) > 0) = 1$. That is both active and inactive periods are always nonzero.

Under the above assumptions, the stationary distribution of $\{W(d, t)\}$ is given by [10]

$$\eta_1(d) \triangleq \Pr(W(d, t) = 1) = \frac{E[Z_k(d)]}{E[Z_k(d)] + E[Y_k(d)]}, \quad (2)$$

$$\eta_0(d) \triangleq \Pr(W(d, t) = 0) = \frac{E[Y_k(d)]}{E[Z_k(d)] + E[Y_k(d)]}. \quad (3)$$

We call $\eta_1(d)$ the active ratio for a link with length d.

Let the sampled graph at time t be $G(\mathcal{H}_\lambda, 1, W(d, t))$. That is, $G(\mathcal{H}_\lambda, 1, W(d, t))$ consists of all active links at time t, along with their associated end nodes. The following theorem establishes a phase transition phenomena with respect to connectivity in a wireless network with dynamic unreliable links $G(\mathcal{H}_\lambda, 1, W(d, t))$. It also asserts that the critical density is the same as the one for the static network $G(\mathcal{H}_\lambda, 1, \eta_1(d))$, i.e, the network in which each link is active with probability $\eta_1(d)$. The proof is omitted due to space limitations.

Theorem 3. *Let $\lambda_c(\eta_1(d))$ be the critical density for the static model $G(\mathcal{H}_\lambda, 1, \eta_1(d))$. Then $G(\mathcal{H}_\lambda, 1, W(d, t))$ is percolated for all $t > 0$ if $\lambda > \lambda_c(\eta_1(d))$ and not percolated at any $t > 0$ if $\lambda < \lambda_c(\eta_1(d))$.*

4.2 Delay of Information Dissemination

We have shown that there exists a critical density $\lambda_c(\eta_1(d))$ such that when $\lambda > \lambda_c(\eta_1(d))$, $G(\mathcal{H}_\lambda, 1, W(d, t))$ is percolated for all time. When $G(\mathcal{H}_\lambda, 1, W(d, t))$ is percolated, if one node inside the infinite component of $G(\mathcal{H}_\lambda, 1, W(d, t))$ broadcasts a message to the whole network, then ignoring propagation delay, all the nodes in the infinite component of $G(\mathcal{H}_\lambda, 1, W(d, t))$ receive this message instantaneously. On the other hand, the nodes in the infinite component of $G(\mathcal{H}_\lambda, 1)$ but not in the infinite component of $G(\mathcal{H}_\lambda, 1, W(d, t))$ cannot receive this message instantaneously. However, we will show that even when $\lambda < \lambda_c(\eta_1(d))$ and $G(\mathcal{H}_\lambda, 1, W(d, t))$ is never percolated, if two nodes u and v are in the infinite component of $G(\mathcal{H}_\lambda, 1)$, information can eventually be transmitted from u to v over multi-hop relays. The main question we address here is the nature of this information dissemination delay.

This problem is similar to the *first passage percolation* problem in lattices [1, 7]. Related continuum models were considered in [5, 8, 11]. In [8], the author study continuum growth model for a spreading infection. In [5] and [11], the authors consider wireless sensor networks where each sensor has independent or degree-dependent dynamic behavior, which can be modelled by an independent or a degree-dependent dynamic site percolation on random geometric

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graphs, respectively. The main tool is the Subadditive Ergodic Theorem [12]. We will use this technique to analyze our problem.

In the following, we will show that in a large-scale wireless network with dynamic unreliable links, the message delay scales linearly with the Euclidean distance between the sender and the receiver if the resulting network is in the subcritical phase, and the delay scales sub-linearly with the distance if the resulting network is in the supercritical phase.

To begin, let $T_{ij}(d_{ij})$ be a random variable associated with link (i, j) having length d_{ij}, such that

$$\begin{cases} \Pr(T_{ij}(d_{ij}) = 0) &= \eta_1(d_{ij}), \\ \Pr(T_{ij}(d_{ij}) > t) &= \eta_0(d_{ij}) P_{d_{ij}}(t), \end{cases} \quad (4)$$

where $P_{d_{ij}}(t) = \Pr(W_{ij}(d_{ij}, t') = 0, \forall t' \in [0, t) | W_{ij}(d_{ij}, 0) = 0)$, and $(\eta_1(d), \eta_0(d))$ is the stationary distribution of $\{W(d, t)\}$ given by (2) and (3).

Let $d(u, v) \triangleq ||\mathbf{X}_u - \mathbf{X}_v||$ and

$$T(u, v) = T(\mathbf{X}_u, \mathbf{X}_v) \triangleq \inf_{l(u,v) \in \mathcal{L}(u,v)} \left\{ \sum_{(i,j) \in l(u,v)} T_{ij}(d_{ij}) \right\}, \quad (5)$$

where $l(u, v)$ is a path from node u to node v, and $\mathcal{L}(u, v)$ is the set of all such paths. Hence, $T(u, v)$ is the message delay on the path from u to v with the smallest delay.[1]

Theorem 4. *Given $G(\mathcal{H}_\lambda, 1, W(d, t))$ with $\lambda > \lambda_c$, there exists a constant $\gamma < \infty$ and $\gamma > 0$ w.p.1, such that for any $u, v \in \mathcal{C}(G(\mathcal{H}_\lambda, 1))$, where $\mathcal{C}(G(\mathcal{H}_\lambda, 1))$ denotes the infinite component of $G(\mathcal{H}_\lambda, 1)$,*

(i) if $G(\mathcal{H}_\lambda, 1, W(d, t))$ is in the subcritical phase, i.e., $\lambda < \lambda_c(\eta_1(d))$, then for any $\epsilon > 0, \delta > 0$, there exists $d_0 < \infty$ such that for any u, v with $d(u, v) > d_0$,

$$\Pr\left(\left| \frac{T(u, v)}{d(u, v)} - \gamma \right| < \epsilon \right) > 1 - \delta; \quad (6)$$

(ii) if $G(\mathcal{H}_\lambda, 1, W(d, t))$ is in the supercritical phase, i.e., $\lambda > \lambda_c(\eta_1(d))$, then for any $\epsilon > 0, \delta > 0$, there exists $d_0 < \infty$ such that for any u, v with $d(u, v) > d_0$,

$$\Pr\left(\frac{T(u, v)}{d(u, v)} < \epsilon \right) > 1 - \delta. \quad (7)$$

Before proceeding, we introduce some new notation. Let

$$\tilde{\mathbf{X}}_i \triangleq \underset{\mathbf{X}_j \in \mathcal{C}(G(\mathcal{H}_\lambda, 1))}{\operatorname{argmin}} \{||\mathbf{X}_j - (i, 0)||\}, \quad (8)$$

$$T_{l,m} \triangleq T(\tilde{\mathbf{X}}_l, \tilde{\mathbf{X}}_m), \text{ for } ||\tilde{\mathbf{X}}_l - \tilde{\mathbf{X}}_m|| < \infty, 0 \le l \le m. \quad (9)$$

The proof for Theorem 4-(i) is based on the following lemma:

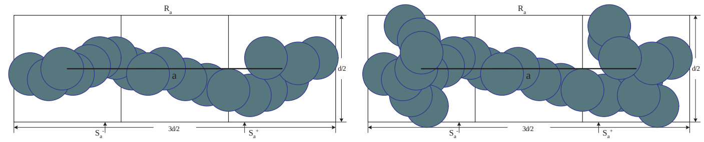
Lemma 5. *Let*

$$\gamma \triangleq \lim_{m \to \infty} \frac{E[T_{0,m}]}{m}. \quad (10)$$

Then, $\gamma = \inf_{m \ge 1} \frac{E[T_{0,m}]}{m}$, and $\lim_{m \to \infty} \frac{T_{0,m}}{m} = \gamma$ w.p.1.

To show Lemma 5, we use the following Subadditive Ergodic Theorem by Liggett [12].

[1]Note that the path with the smallest delay may be different from the shortest path (in terms of number of links) from node u to node v.



(a) Good Rectangle (b) Open Rectangle

Figure 1: Examples of good and open rectangles

Theorem 6 (Liggett [12]). *Let $\{S_{l,m}\}$ be a collection of random variables indexed by integers $0 \le l < m$. Suppose $\{S_{l,m}\}$ has the following properties: (i) $S_{0,m} \le S_{0,l} + S_{l,m}$, $0 \le l \le m$; (ii) $\{S_{(m-1)k,mk}, m \ge 1\}$ is a stationary process for each k; (iii) $\{S_{l,l+k}, k \ge 0\} = \{S_{l+1,l+k+1}, k \ge 0\}$ in distribution for each l; (iv) $E[|S_{0,m}|] < \infty$ for each m. Then (a) $\alpha \triangleq \lim_{m\to\infty} \frac{E[S_{0,m}]}{m} = \inf_{m \ge 1} \frac{E[S_{0,m}]}{m}$; $S \triangleq \lim_{m\to\infty} \frac{S_{0,m}}{m}$ exists w.p.1 and $E[S] = \alpha$. Furthermore, if (v) the stationary process in (ii) is ergodic, then (b) $S = \alpha$ w.p.1.*

To show Lemma 5, we need to verify that the sequence $\{T_{l,m}, l \le m\}$ satisfies conditions (i)–(v) of Theorem 6. It is easy to see that (i) is satisfied, since $T_{0,m}$ is the delay of the path with the smallest delay from $\tilde{\mathbf{X}}_0$ to $\tilde{\mathbf{X}}_m$ and $T_{0,l} + T_{l,m}$ is the delay on a particular path from $\tilde{\mathbf{X}}_0$ to $\tilde{\mathbf{X}}_l$ (it has the smallest delay from $\tilde{\mathbf{X}}_0$ to $\tilde{\mathbf{X}}_l$, and from $\tilde{\mathbf{X}}_l$ to $\tilde{\mathbf{X}}_m$). Furthermore, because all nodes are distributed according to a homogeneous Poisson point process, the geometric structure is stationary and hence (ii) and (iii) are guaranteed. We need only to show conditions (iv) and (v) also hold for $\{T_{l,m}, l \le m\}$. To accomplish this, we first show property (iv) holds for $\{T_{l,m}, l \le m\}$.

Lemma 7. *Let $r_0 = ||\tilde{\mathbf{X}}_0 - (0, 0)||$, then $r_0 < \infty$ w.p.1.*

Proof: We consider a mapping between $G(\mathcal{H}_\lambda, 1)$ and a square lattice $\mathcal{L} = d \cdot \mathbb{Z}^2$, where d is the edge length. The vertices of $\mathcal{L}$ are located at $(d \times i, d \times j)$ where $(i, j) \in \mathbb{Z}^2$. For each horizontal edge a, let the two end vertices be $(d \times a_x, d \times a_y)$ and $(d \times a_x + d, d \times a_y)$.

For edge a in $\mathcal{L}$, define event $A_a(d)$ as the set of outcomes for which the following condition holds: the rectangle $R_a = [a_x d - \frac{d}{4}, a_x d + \frac{5d}{4}] \times [a_y d - \frac{d}{4}, a_y d + \frac{d}{4}]$ is crossed[2] from left to right by a connected component in $G(\mathcal{H}_\lambda, 1)$. If $A_a(d)$ occurs, we say that rectangle R_a is a *good* rectangle, and edge a is a *good* edge. Let $p_g(d) \triangleq \Pr(A_a(d))$. Define $A_a(d)$ similarly for all vertical edges by rotating the rectangle by 90°. An example of a good rectangle and a good edge is illustrated in Figure 1-(a).

Further define event $B_a(d)$ for edge a in $\mathcal{L}$ as the set of outcomes for which both of the following hold: (i) $A_a(d)$ occurs; (ii) the left square $S_a^- = [a_x d - \frac{d}{4}, a_x d + \frac{d}{4}] \times [a_y d - \frac{d}{4}, a_y d + \frac{d}{4}]$ and the right square $S_a^+ = [a_x d + \frac{3d}{4}, a_x d + \frac{5d}{4}] \times [a_y d - \frac{d}{4}, a_y d + \frac{d}{4}]$ are both crossed from top to bottom by connected components in $G_1(\mathcal{H}_\lambda, 1)$.

If $B_a(d)$ occurs, we say that rectangle R_a is an *open* rectangle, and edge a is an *open* edge. Let $p_o(d) \triangleq \Pr(B_a(d))$.

[2]Here, a rectangle $R = [x_1, x_2] \times [y_1, y_2]$ being crossed from left to right by a connected component in $G(\mathcal{H}_\lambda, 1)$ means that there exists a sequence of nodes $v_1, v_2, ..., v_m \in G(\mathcal{H}_\lambda, 1)$ contained in R, with $||\mathbf{x}_{v_i} - \mathbf{x}_{v_{i+1}}|| \le 1, i = 1, ..., m - 1$, and $0 < x(v_1) - x_1 < 1, 0 < x_2 - x(v_m) < 1$, where $x(v_1)$ and $x(v_m)$ are the x-coordinates of nodes v_1 and v_m, respectively. A rectangle being crossed from top to bottom is defined analogously.

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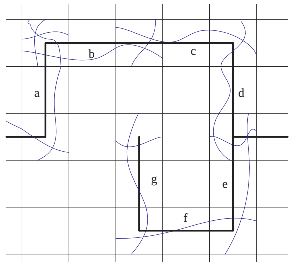


Figure 2: A path of open edges in $\mathcal{L}$ implies a path of connected nodes in $G(\mathcal{H}_\lambda, 1)$

Define $B_a(d)$ similarly for all vertical edges by rotating the rectangle by 90°. Examples of an open rectangle and an open edge are illustrated in Figure 1-(b).

Suppose edges b and c are vertically adjacent to edge a, then it is clear that when events $A_a(d)$, $A_b(d)$ and $A_c(d)$ occur, event $B_a(d)$ occurs. Moreover, since events $A_a(d)$, $A_b(d)$ and $A_c(d)$ are increasing events[3], by the FKG inequality [1–3], $p_o(d) = \Pr(B_a(d)) \geq \Pr(A_a(d) \cap A_b(d) \cap A_c(d)) \geq \Pr(A_a(d)) \Pr(A_b(d)) \Pr(A_c(d)) = (p_g(d))^3$.

According to Corollary 4.1 in [2], the probability $p_g(d)$ converges to 1 as $d \to \infty$ when $G(\mathcal{H}_\lambda, 1)$ is in the supercritical phase. In this case, $(p_g(d))^3$ converges to 1 as $d \to \infty$ as well. Hence, $p_o(d)$ converges to 1 as $d \to \infty$ when $G(\mathcal{H}_\lambda, 1)$ is in the supercritical phase.

Note that in our model, events $\{B_a(d)\}$ are not independent in general. However, if two edges a and b are not adjacent, i.e., they do not share any common end vertices, then $B_a(d)$ and $B_b(d)$ are independent. Furthermore, when edges a and b are adjacent, $B_a(d)$ and $B_b(d)$ are increasing events and thus positively correlated. Consequently, our model is a 1-dependent bond percolation model. It is known that there exists $p_{\text{1-dep}}^{\text{bond}} < 1$ such that any 1-dependent model with $p > p_{\text{1-dep}}^{\text{bond}}$ is percolated, where p is the probability of an edge being open [13].

Now define

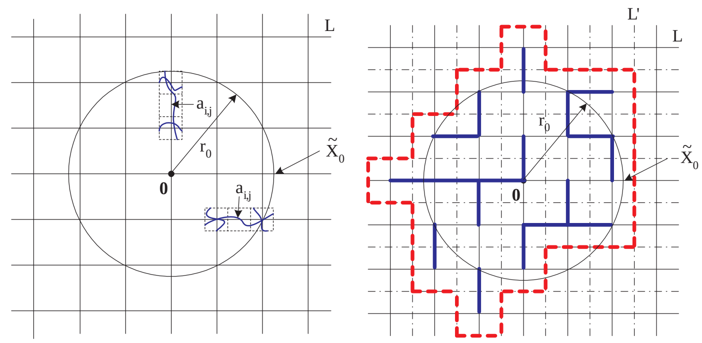
$$d_0 \triangleq \inf\left\{d' > 1 : p_o(d') > \max\left\{\frac{8}{9}, p_{\text{1-dep}}^{\text{bond}}\right\}\right\}, \qquad (11)$$

and choose the edge length of $\mathcal{L}$ to be $d > d_0$. Then there is an infinite cluster consisting of open edges and their end vertices in $\mathcal{L}$. Denote this infinite cluster by $\mathcal{C}(\mathcal{L})$.

From Figure 2, it is easy to see that all the nodes along the crossings in R_a and all the nodes along the crossings in R_b for any $a, b \in \mathcal{C}(\mathcal{L})$ are connected. Since the infinite component of $G(\mathcal{H}_\lambda, 1)$ is unique, all the nodes along the crossings in R_a for each $a \in \mathcal{C}(\mathcal{L})$ must belong to $\mathcal{C}(G(\mathcal{H}_\lambda, 1))$.

By definition, no node of $G(\mathcal{H}_\lambda, 1)$ strictly inside $\mathcal{A}(\mathbf{0}, r_0)$ belongs to $\mathcal{C}(G(\mathcal{H}_\lambda, 1))$. This implies that no edge of $\mathcal{L}$ strictly inside $\mathcal{A}(\mathbf{0}, r_0)$ belongs to $\mathcal{C}(\mathcal{L})$. To see this, suppose edge $a_{i,j}$ of $\mathcal{L}$ is strictly inside $\mathcal{A}(\mathbf{0}, r_s0)$ and belongs to $\mathcal{C}(\mathcal{L})$. The nodes along the crossings in $R_{a_{i,j}}$ belong to $\mathcal{C}(G(\mathcal{H}_\lambda, 1))$. As shown in Figure 3-(a), when $d > 1$ and $r_0 \gg 1$, no matter what direction the edge $a_{i,j}$ has, there are some nodes along the crossings in $R_{a_{i,j}}$ (therefore belonging to $\mathcal{C}(G(\mathcal{H}_\lambda, 1))$) which are strictly inside $\mathcal{A}(\mathbf{0}, r_0)$. These

[3] An event A is called increasing if $I_A(G) \leq I_A(G')$ whenever graph G is a subgraph of G', where I_A is the indicator function of A. An event A is called decreasing if A^c is increasing. For details, please see [1–3].



(a) (b)

Figure 3: (a) Two possibilities for $a_{i,j}$ in $\mathcal{L}$. (b) A closed circuit in $\mathcal{L}'$ containing all edges of $\mathcal{L}$ strictly inside $\mathcal{A}(\mathbf{0}, r_0)$

nodes then have strictly smaller distance to $\mathbf{0}$ than node $\tilde{\mathbf{X}}_0$. This contradiction ensures that no edge of $\mathcal{L}$ strictly inside $\mathcal{A}(\mathbf{0}, r_0)$ belongs to $\mathcal{C}(\mathcal{L})$.

Consider the *dual lattice* $\mathcal{L}'$ of $\mathcal{L}$. The construction of $\mathcal{L}'$ is as follows: let each vertex of $\mathcal{L}'$ be located at the center of a square of $\mathcal{L}$. Let each edge of $\mathcal{L}'$ be open if and only if it crosses an open edge of $\mathcal{L}$, and closed otherwise. It is clear that each edge in $\mathcal{L}'$ is open also with probability $p_o(d)$. Let $q = 1 - p_o(d) < \frac{1}{9}$. Choose $2m$ edges in $\mathcal{L}'$. Since the states (open or closed) of any set of non-adjacent edges are independent, we can choose m edges among the $2m$ edges such that their states are independent. As a result, $\Pr(\text{all the } 2m \text{ edges are closed}) \leq q^m$.

Now a key observation is that if no edge of $\mathcal{L}$ strictly inside $\mathcal{A}(\mathbf{0}, r_0)$ belongs to $\mathcal{C}(\mathcal{L})$, for which the event is denoted by $E_{\mathcal{L}}$, then there must exist a closed circuit in $\mathcal{L}'$ (a circuit consisting of closed edges) containing all edges of $\mathcal{L}$ strictly inside $\mathcal{A}(\mathbf{0}, r_0)$, for which the event is denoted by $E_{\mathcal{L}'}$, and vice versa. This is demonstrated in Figure 3-(b). Hence $\Pr(E_{\mathcal{L}}) = 1 \Longleftrightarrow \Pr(E_{\mathcal{L}'}) = 1$.

Any closed circuit in $\mathcal{L}'$ containing all edges of $\mathcal{L}$ strictly inside $\mathcal{A}(\mathbf{0}, r_0)$ has length greater than or equal to $2l$, where $l \triangleq 2\lfloor \frac{r_0}{d} \rfloor$. Thus we have $\Pr(E_{\mathcal{L}'}) = \sum_{m=l}^{\infty} \Pr(\exists \mathcal{O}_c(2m)) \leq \sum_{m=l}^{\infty} \gamma(2m) q^m$, where $\mathcal{O}_c(2m)$ is a closed circuit having length $2m$ in $\mathcal{L}'$ containing all edges of $\mathcal{L}$ strictly inside $\mathcal{A}(\mathbf{0}, r_0)$, and $\gamma(2m)$ is the number of such circuits. By a similar argument to the one used in [1], we can show $\gamma(2m) = \frac{4}{27}(m-1)3^{2m}$ so that

$$\begin{aligned} \sum_{m=l}^{\infty} \gamma(2m) q^m &\leq \sum_{m=l}^{\infty} \frac{4}{27}(m-1)(9q)^m \\ &= \frac{4}{27}\left[\sum_{m=l}^{\infty} m(9q)^m - \sum_{m=l}^{\infty} (9q)^m\right] \\ &= \frac{4[l-1-(l-2)9q]}{27(1-9q)^2}(9q)^l. \end{aligned} \qquad (12)$$

Since $q < \frac{1}{9}$, we have $\Pr(E_{\mathcal{L}'}) \to 0$ as $l = 2\lfloor \frac{r_0}{d} \rfloor \to \infty$. That is, as r_0 goes to infinity, w.p.1, there is some edge of $\mathcal{L}$ strictly inside $\mathcal{A}(\mathbf{0}, r_0)$ belonging to $\mathcal{C}(\mathcal{L})$. Hence, w.p.1, there is some node of $G(\mathcal{H}_\lambda, 1)$ strictly inside $\mathcal{A}(\mathbf{0}, r_0)$ belonging to $\mathcal{C}(G(\mathcal{H}_\lambda, 1))$. This contradiction implies that r_0 is finite w.p.1. □

Let $r_m = ||\tilde{\mathbf{X}}_m - (m, 0)||$, due to stationarity, we have

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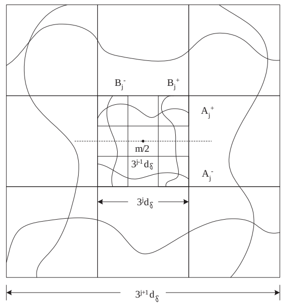

Figure 4: Square annuli

$r_m < \infty$ w.p.1, for any m.

LEMMA 8. *Let $L(\tilde{\mathbf{X}}_0, \tilde{\mathbf{X}}_m)$ be the shortest path (in terms of the number of links) from $\tilde{\mathbf{X}}_0$ to $\tilde{\mathbf{X}}_m$), and let $|L(\tilde{\mathbf{X}}_0, \tilde{\mathbf{X}}_m)|$ denote the number of links on such a path. If $||\tilde{\mathbf{X}}_0 - \tilde{\mathbf{X}}_m|| < \infty$, then $|L(\tilde{\mathbf{X}}_0, \tilde{\mathbf{X}}_m)| < \infty$, and $E[T^L_{0,m}] < \infty$, where $T^L_{0,m}$ denotes the delay on path $L(\tilde{\mathbf{X}}_0, \tilde{\mathbf{X}}_m)$.*

Proof: We use the same mapping as the one for the proof of Lemma 7. For any given $\sqrt[4]{\frac{8}{9}} < \delta < 1$, define

$$d_\delta = \max\{\inf\{d' : p_g(d') \geq \delta\}, ||\tilde{\mathbf{X}}_0 - \tilde{\mathbf{X}}_m||\}. \qquad (13)$$

Then, for any $d > d_\delta$, we have $p_g(d) \geq \delta$.

Now, consider a fractal structure as shown in Figure 4: first a square $S(d_\delta)$ is constructed with edge length d_δ centered at $\frac{\tilde{\mathbf{X}}_0 + \tilde{\mathbf{X}}_m}{2}$. Then, a second square $S(3d_\delta)$ is constructed with edge length $3d_\delta$ also centered at $\frac{\tilde{\mathbf{X}}_0 + \tilde{\mathbf{X}}_m}{2}$. The construction proceeds in the same manner, i.e., at step j, a square $S(3^{j-1}d_\delta)$ is constructed with edge length $3^{j-1}d_\delta$ centered at $\frac{\tilde{\mathbf{X}}_0 + \tilde{\mathbf{X}}_m}{2}$. Thus, we have the initial square and a sequence of square annuli that do not overlap.

Denote the square annulus with inside edge length $3^{j-1}d_\delta$ ($j \geq 2$) and outside edge length $3^j d_\delta$ by $D(3^j d_\delta)$. Let A_j^+ be the event that the upper horizontal rectangle of $D(3^j d_\delta)$—$[\frac{m}{2} - \frac{3^j}{2}d_\delta, \frac{m}{2} + \frac{3^j}{2}d_\delta] \times [\frac{3^{j-1}}{2}d_\delta, \frac{3^j}{2}d_\delta]$ is good, i.e., it is crossed by a connected component in $G(\mathcal{H}_\lambda, 1)$ from left to right. Since the length of the corresponding lattice edge of the upper horizontal rectangle of $D(3^j d_\delta)$ is $2 \cdot 3^{j-1}d_\delta > d_\delta$, we have $\Pr\{A_j^+\} \geq \delta$. Similarly define A_j^-, B_j^+ and B_j^- to be the events that the lower, right and left rectangles are good, respectively. Then $\Pr\{A_j^-\} \geq \delta$, $\Pr\{B_j^+\} \geq \delta$ and $\Pr\{B_j^-\} \geq \delta, \forall j \geq 1$.

Let E_j be the event that there exists a circuit of connected nodes in $G(\mathcal{H}_\lambda, 1)$ within $D(3^j d_\delta)$. Once A_j^+, A_j^-, B_j^+ and B_j^- all occur, E_j must also occur. Although A_j^+, A_j^-, B_j^+ and B_j^- are not independent, they are increasing events. By the FKG inequality, we have $\Pr(E_j) \geq \Pr(A_j^+ \cap A_j^- \cap B_j^+ \cap B_j^-) \geq \Pr(A_j^+)\Pr(A_j^-)\Pr(B_j^+)\Pr(B_j^-) \geq \delta^4$.

When E_j occurs, $\tilde{\mathbf{X}}_0$ and $\tilde{\mathbf{X}}_m$ are contained in $S(3^{j-1}d_\delta)$ and there is a circuit of connected nodes in $G(\mathcal{H}_\lambda, 1)$ contained in the square annulus $D(3^j d_\delta)$. If the shortest path between $\tilde{\mathbf{X}}_0$ and $\tilde{\mathbf{X}}_m$, $L(\tilde{\mathbf{X}}_0, \tilde{\mathbf{X}}_m)$, were to go outside $S(3^j d_\delta)$, it would intersect the closed circuit contained by $D(3^j d_\delta)$ and we could construct a shorter path from $\tilde{\mathbf{X}}_0$ to $\tilde{\mathbf{X}}_m$. This implies that $L(\tilde{\mathbf{X}}_0, \tilde{\mathbf{X}}_m)$ must be contained in $S(3^j d_\delta)$.

Suppose u, v and w are three consecutive nodes along $L(\tilde{\mathbf{X}}_0, \tilde{\mathbf{X}}_m)$. Then $||\mathbf{X}_u - \mathbf{X}_w|| > 1$, since otherwise v would not belong to the shortest path. Hence, if we draw circles with radius $\frac{1}{2}$, centered at $\mathbf{X}_u$ and $\mathbf{X}_w$, respectively, then the two circles do not overlap. Consequently, if the length of $L(\tilde{\mathbf{X}}_0, \tilde{\mathbf{X}}_m)$ is $|L| \triangleq |L(\tilde{\mathbf{X}}_0, \tilde{\mathbf{X}}_m)|$, then we must be able to draw at least $\lceil \frac{|L|}{2} \rceil$ circles with radius $\frac{1}{2}$ centered at alternating nodes along $L(\tilde{\mathbf{X}}_0, \tilde{\mathbf{X}}_m)$. All of these circles are contained in the square with edge length $3^j d_\delta + 1$. Such a square contains at most $\lceil (3^j d_\delta + 1)^2 / [\pi(\frac{1}{2})^2] \rceil$ non-overlapping circles with radius $\frac{1}{2}$. Therefore, $|L| \leq 2\lceil 4(3^j d_\delta + 1)^2/\pi \rceil < \infty$.

Now if $|L| > 2\lceil 4(3^j d_\delta + 1)^2/\pi \rceil$, then $|L| > 2\lceil 4(3^i d_\delta + 1)^2/\pi \rceil$ for all $i = 1, 2, ..., j$. By the above argument, none of the events $E_1, E_2, ...E_j$ can occur. Thus

$$\Pr\left(|L| > 2\left\lceil \frac{4}{\pi}(3^j d_\delta + 1)^2 \right\rceil\right) \leq \prod_{i=1}^{j} \Pr(E_i^c) \leq (1 - \delta^4)^j.$$

Let $M = 2\left\lceil \frac{4}{\pi}(3d_\delta + 1)^2 \right\rceil$, then we have

$$\begin{aligned} E[|L|] &= \sum_{k=0}^{M} \Pr(|L| > k) + \sum_{k=M+1}^{\infty} \Pr(|L| > k) \\ &\leq M + \sum_{j=1}^{\infty} \left\lceil \frac{4}{\pi}(3^{j+1}d_\delta + 1)^2 \right\rceil \Pr\left(|L| > \left\lceil \frac{4}{\pi}(3^j d_\delta + 1)^2 \right\rceil\right) \\ &\leq M + \sum_{j=1}^{\infty} \left(\frac{4}{\pi}(3^{j+1}d_\delta + 1)^2 + 1\right)(1 - \delta^4)^j \\ &= M + \sum_{j=1}^{\infty} \left(\frac{4}{\pi}(9 \cdot 9^j d_\delta^2 + 6 \cdot 3^j d_\delta + 1) + 1\right)(1 - \delta^4)^j \quad (14) \end{aligned}$$

When $\delta > \sqrt[4]{\frac{8}{9}}$, we have $(1-\delta^4)^j < 9^{-j}$. Thus, $E[|L|] < \infty$.

Let $\Lambda_{W(d,t)} \triangleq \max_{0<d\leq 1}\{\eta_0(d)E[Y_k(d)]\} < \infty$, then we have $E[T^L_{0,m}||L|] = \sum_{i=1}^{|L|} \eta_0^{(i)}(d)E[Y_k^{(i)}(d)] \leq |L|\Lambda_{W(d,t)}$, where $\eta_0^{(i)}(d)$ and $E[Y_k^{(i)}(d)]$ are the stationary probability of the inactive state, and the expected inactive period of the i-th link with length d on $L(\tilde{\mathbf{X}}_0, \tilde{\mathbf{X}}_m)$, respectively. Hence $E[T^L_{0,m}] = E[E[T^L_{0,m}||L|]] \leq E[|L|]\Lambda_{W(d,t)} < \infty$. □

To show property (v), we show $\{T_{(m-1)j,mj}, m \geq 1\}$ is strong mixing.[4]

LEMMA 9. *The sequence $\{T_{(m-1)k,mk}, m \geq 1\}$ is strong mixing, so that it is ergodic.*

Proof: From the proof of Lemma 7, we have $\Pr(E_j) \geq \delta^4$ for all $j = 1, 2, ...$ Summing over j yields $\sum_{j=1}^{\infty} \Pr(E_j) \geq \sum_{j=1}^{\infty} \delta^4 = \infty$. Since E_j are independent events, by the Borel-Cantelli Lemma, w.p.1 there exists $j' < \infty$ such that $E_{j'}$ occurs.

We now construct squares A_1 and A_2 centered at $\frac{\tilde{\mathbf{X}}_{(m-1)j} + \tilde{\mathbf{X}}_{mj}}{2}$ and $\frac{\tilde{\mathbf{X}}_{(m+k-1)j} + \tilde{\mathbf{X}}_{(m+k)j}}{2}$ with edge length $3^{j'}d_\delta$ and $3^{j''}d_\delta$ respectively, such that the path with the smallest delay from $\tilde{\mathbf{X}}_{(m-1)j}$ to $\tilde{\mathbf{X}}_{mj}$, and the path from $\tilde{\mathbf{X}}_{(m+k-1)j}$ to $\tilde{\mathbf{X}}_{(m+k)j}$ are contained in A_1 and A_2, respectively. Let E be the event that $j' < \infty$ and $j'' < \infty$. Then $\Pr(E) = 1$.

When finite j' and j'' exist, due to stationarity, j' and j'' are independent of k. Hence, as $k \to \infty$, A_1 and A_2 become

[4] A measure preserving transformation H on $(\Omega, \mathcal{F}, P)$ is called strong mixing if for all measurable sets A and B, $\lim_{m\to\infty} |P(A \cap H^{-m}B) - P(A)P(B)| = 0$. A sequence $\{X_n, n \geq 0\}$ is called strong mixing if the shift on sequence space is strong (weak) mixing. Every strongly-mixing system is ergodic [14].

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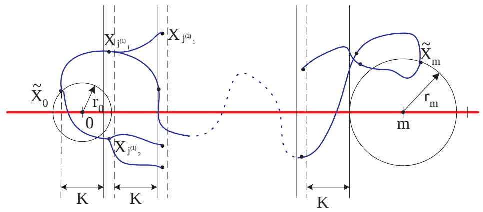

Figure 5: Path segments.

non-overlapping so that the paths inside A_1 and A_2 do not share any common nodes of $G(\mathcal{H}_\lambda, 1)$. Hence $T_{(m-1)j,mj}$ and $T_{(m+k-1)j,(m+k)j}$ are independent of each other as $k \to \infty$.

Therefore

$$\begin{aligned}
&\lim_{k\to\infty} \Pr(\{T_{(m-1)j,mj} < t\} \cap \{T_{(m+k-1)j,(m+k)j} < t'\}) \\
=& \lim_{k\to\infty} \Pr(\{T_{(m-1)j,mj} < t\} \cap \{T_{(m+k-1)j,(m+k)j} < t'\} | E) \Pr(E) \\
+& \lim_{k\to\infty} \Pr(\{T_{(m-1)j,mj} < t\} \cap \{T_{(m+k-1)j,(m+k)j} < t'\} | E^c) \Pr(E^c) \\
=& \Pr(T_{(m-1)j,mj} < t | E) \Pr(T_{(m-1)j,mj} < t' | E) \\
=& \Pr(T_{(m-1)j,mj} < t) \Pr(T_{(m-1)j,mj} < t'), \qquad (15)
\end{aligned}$$

This implies that sequence $\{T_{(m-1)k,mk}, m \geq 1\}$ is strong mixing, so that it is ergodic. $\square$

Now, we present the proof for Lemma 5.

Proof of Lemma 5: Conditions (i)–(iii) of Theorem 6 have been verified. The validation of (iv) is provided by Lemma 8. Let $L(\tilde{\mathbf{X}}_0, \tilde{\mathbf{X}}_m)$ be the shortest path from $\tilde{\mathbf{X}}_0$ to $\tilde{\mathbf{X}}_m$. Since $L(\tilde{\mathbf{X}}_0, \tilde{\mathbf{X}}_m)$ is a particular path, we have $T_{0,m} \leq T^L_{0,m}$ so that $E[T_{0,m}] \leq E[T^L_{0,m}]$, where $T^L_{0,m}$ denotes the delay on path $L(\tilde{\mathbf{X}}_0, \tilde{\mathbf{X}}_m)$. By Lemma 8, we have $E[T^L_{0,m}] < \infty$ and therefore $E[T_{0,m}] < \infty$. Furthermore, due to Lemma 9, $\{T_{(m-1)k,mk}, m \geq 1\}$ is ergodic, thus the results (a) and (b) of Theorem 6 hold. $\square$

Remark: Using the proof for condition (iv), we can show that for any two nodes u and v in the infinite component of $G(\mathcal{H}_\lambda, 1)$ which are within finite Euclidean distance of each other, i.e., $u, v \in \mathcal{C}(G(\mathcal{H}_\lambda, 1))$ with $d(u, v) < \infty$, $E[T(u, v)] < \infty$.

LEMMA 10. *Suppose $G(\mathcal{H}_\lambda, 1, p_e(\cdot))$ is in the supercritical phase, i.e, $\lambda > \lambda_c(p_e(\cdot))$. Let $v \notin \mathcal{C}(G(\mathcal{H}_\lambda, 1, p_e(\cdot)))$ and define $w \triangleq \operatorname{argmin}_{i \in \mathcal{C}(G(\mathcal{H}_\lambda, 1, p_e(\cdot)))} d(i, v)$, i.e., w is the node in the infinite component of $G(\mathcal{H}_\lambda, 1, p_e(\cdot))$ with the smallest Euclidean distances to node v. Then, $d(w, v) < \infty$ w.p.1.*

The idea behind the proof for this lemma is similar to that for the proof for Lemma 7. The difference is that the probability of a good event is now defined with respect to $G(\mathcal{H}_\lambda, 1, p_e(\cdot))$ instead of $G(\mathcal{H}_\lambda, 1)$. Due to space limitations, the proof is omitted here.

LEMMA 11. *Let γ be defined as (10). (i) If $G(\mathcal{H}_\lambda, 1, W(d, t))$ is in the subcritical phase, i.e., $\lambda < \lambda_c(\eta_1(d))$, then $\gamma < \infty$, and $\gamma > 0$ w.p.1. (ii) If $G(\mathcal{H}_\lambda, 1, W(d, t))$ is in the supercritical phase, i.e., $\lambda > \lambda_c(\eta_1(d))$, then $\gamma = 0$ w.p.1.*

Proof: To show (i), note that $\gamma < \infty$ follows directly from $\gamma = \inf_{m \geq 1} \frac{E[T_{0,m}]}{m} \leq E[T_{0,1}] < \infty$, where the last inequality is shown above in the proof for Lemma 5.

To see why γ is positive, suppose the node at $\tilde{\mathbf{X}}_0$ disseminates a message at time $t = t_0$ and consider $G(\mathcal{H}_\lambda, 1, W(d, t_0))$. Choose K large enough such that $c_1 e^{-c_2 K} < \frac{1}{2}$, where c_1 and c_2 are the constants given in Proposition 2. Let $q = \lfloor \frac{m}{2(K+1)} \rfloor$. When $m > 2(K+1)$, $q \geq 1$.

Let $S_h = \{(x, y) \in \mathbb{R}^2 : K + (h-1)(K+1) \leq x - x(\tilde{\mathbf{X}}_0) < h(K+1)\}$ for $h = 1, 2, ...$, where $x(v)$ is the x-coordinate of node v. Since $\tilde{\mathbf{X}}_0$ and $\tilde{\mathbf{X}}_m$ are both in $\mathcal{C}(G(\mathcal{H}_\lambda, 1))$, there exists at least one path from $\tilde{\mathbf{X}}_0$ to $\tilde{\mathbf{X}}_m$. Moreover, since each strip S_h has width 1, at least one node of $\mathcal{C}(G(\mathcal{H}_\lambda, 1))$ lies inside each S_h.

Let $\{\mathbf{X}^{(1)}_l, l = 1, 2, ...\}$ be the nodes of $\mathcal{C}(G(\mathcal{H}_\lambda, 1))$ which lie inside S_1. Since $G(\mathcal{H}_\lambda, 1, W(d, t_0))$ is in the subcritical phase, by Proposition 2, the probability that there exists a path consisting of only active links from $\tilde{\mathbf{X}}_0$ to any $\mathbf{X}^{(1)}_l$, $l = 1, 2, ...$, is less than or equal to $c_1 e^{-c_2 K} < \frac{1}{2}$. In other words, with probability strictly greater than $\frac{1}{2}$, there exists at least one inactive link at time $t = t_0$ on any path from $\tilde{\mathbf{X}}_0$ to $\mathbf{X}^{(1)}_l$, $l = 1, 2, ...$ Let $T^{(1)} = \inf_l \{T(\tilde{\mathbf{X}}_0, \mathbf{X}^{(1)}_l)\}$. Let $\Gamma_{W(d,t)} \triangleq \min_{0<d\leq 1} \{\eta_0(d) E[Y_k(d)]\} > 0$, then $E[T^{(1)}] > \frac{1}{2}\Gamma_{W(d,t)} > 0$.

Let $\{\mathbf{X}^{(h+1)}_{l'}, l' = 1, 2, ...\}$ be the nodes of $\mathcal{C}(G(\mathcal{H}_\lambda, 1))$ which lie inside S_{h+1}, for $h \geq 1$. By the same argument as above, the probability that there exists a path consisting of only active links from any node in S_h to any node in S_{h+1} is less than or equal to $c_1 e^{-c_2 K} < \frac{1}{2}$. In other words, with probability strictly greater than $\frac{1}{2}$, there exists at least one inactive link on any path from any node in S_h to any node in S_{h+1}. Let $T^{(h+1)} = \inf_{l,l'} \{T(\mathbf{X}^{(h)}_l, \mathbf{X}^{(h+1)}_{l'})\}$. Then $E[T^{(h+1)}] > \frac{1}{2}\Gamma_{W(d,t)} > 0$. The path segments are illustrated in Figure 5.

Since $||\tilde{\mathbf{X}}_0 - \tilde{\mathbf{X}}_m|| \geq m - r_0 - r_m$, when $\frac{m}{2} > r_0 + r_m$, any path from $\tilde{\mathbf{X}}_0$ to $\tilde{\mathbf{X}}_m$ has at least $\lfloor \frac{m}{2(K+1)} \rfloor = q$ segments and the delay on each segment is strictly greater than $\frac{1}{2}\Gamma_{W(d,t)} > 0$. Hence, $E[T_{0,m}] > \frac{1}{2} q \Gamma_{W(d,t)}$ when $\frac{m}{2} > r_0 + r_m$. Since both r_0 and r_m are finite w.p.1, $\frac{m}{2} > r_0 + r_m$ holds w.p.1 as $m \to \infty$.

Since K is finite and $\Gamma_{W(d,t)}$ is positive and independent of m, we have $\gamma = \lim_{m\to\infty} \frac{E[T_{0,m}]}{m} > \lim_{m\to\infty} \frac{q}{m} \frac{1}{2} \Gamma_{W(d,t)} > \lim_{m\to\infty} \left(\frac{1}{2(K+1)} - \frac{1}{m} \right) \frac{1}{2} \Gamma_{W(d,t)} > 0$ w.p.1, where we used the fact that $q > \frac{m}{2(K+1)} - 1$.

For (ii), suppose $G(\mathcal{H}_\lambda, 1, W(d, t))$ is in the supercritical phase. To simplify notation, let $\mathcal{C}(t)$ be the infinite component of $G(\mathcal{H}_\lambda, 1, W(d, t))$. Let t' be the first time when some node in $\mathcal{C}(t')$ receives $\tilde{\mathbf{X}}_0$'s message, and let $w_1 \triangleq \operatorname{argmin}_{i \in \mathcal{C}(t')} d(\mathbf{X}_i, \tilde{\mathbf{X}}_0)$, and $w_2 \triangleq \operatorname{argmin}_{i \in \mathcal{C}(t')} d(\mathbf{X}_i, \tilde{\mathbf{X}}_m)$. If node $\tilde{\mathbf{X}}_0$ is in $\mathcal{C}(t_0)$, then $t' = t_0$ and $w_1 = \tilde{\mathbf{X}}_0$. If at time t', node v is in $\mathcal{C}(t')$, then $w_2 = \tilde{\mathbf{X}}_m$.

Since both w_1 and w_2 belong to $\mathcal{C}(t')$, $T(w_1, w_2) = 0$. The distances $d(\tilde{\mathbf{X}}_0, \mathbf{X}_{w_1})$ and $d(\mathbf{X}_{w_2}, \tilde{\mathbf{X}}_m)$ are finite w.p.1 by Lemma 10. Clearly, $d(\tilde{\mathbf{X}}_0, \mathbf{X}_{w_1})$ is independent of m. By stationarity, $d(\mathbf{X}_{w_2}, \tilde{\mathbf{X}}_m)$ is also independent of m. Hence by the proof of Lemma 5, $E[T(\tilde{\mathbf{X}}_0, \mathbf{X}_{w_1})] < \infty$, $E[T(\mathbf{X}_{w_2}, \tilde{\mathbf{X}}_m)] < \infty$ w.p.1 for any m, and $E[T(\tilde{\mathbf{X}}_0, \mathbf{X}_{w_1})]$ and $E[T(\mathbf{X}_{w_2}, \tilde{\mathbf{X}}_m)]$ are independent of m. Moreover,

$$0 \leq \frac{T_{0,m}}{m} \leq \frac{T(\tilde{\mathbf{X}}_0, \mathbf{X}_{w_1}) + T(w_1, w_2) + T(\mathbf{X}_{w_2}, \tilde{\mathbf{X}}_m)}{m}.$$

Hence $\gamma = \lim_{m\to\infty} \frac{E[T_{0,m}]}{m} = 0$ w.p.1. $\square$

Proof of Theorem 4: Assume node u disseminates a message at time $t = t_0$. Take $\mathbf{X}_u$ as the origin, and the line

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$\mathbf{X}_u\mathbf{X}_v$ as the x-axis. By definition $u, v \in \mathcal{C}(G(\mathcal{H}_\lambda, 1))$. Since node u is the origin, $\mathbf{X}_u = \tilde{\mathbf{X}}_0$. Let m be the closest integer to $x(v)$—the x-axis coordinate of node $\mathbf{X}_v$. Now $T_{0,m} = T(\mathbf{X}_u, \tilde{\mathbf{X}}_m)$. If $\mathbf{X}_v = \tilde{\mathbf{X}}_m$, $T(u,v) = T_{0,m}$.

Note that $m - 1 < d(u,v) < m+1$, Thus, for any $m > 1$, we have

$$\frac{T_{0,m}}{m+1} < \frac{T(u,v)}{d(u,v)} < \frac{T_{0,m}}{m-1}. \quad (16)$$

On the other hand, if $\mathbf{X}_v \neq \tilde{\mathbf{X}}_m$, then $\tilde{\mathbf{X}}_m$ must be adjacent to $\mathbf{X}_v$. This is because $||(m,0) - \mathbf{X}_v|| \leq \frac{1}{2}$ (m is the closest integer to $x(v)$) and $||(m,0) - \tilde{\mathbf{X}}_m|| \leq \frac{1}{2}$ ($\tilde{\mathbf{X}}_m$ is the closest node to $(m,0)$). Consequently, $T_{0,m} - T(\tilde{\mathbf{X}}_m, \mathbf{X}_v) \leq T(u,v) \leq T_{0,m} + T(\tilde{\mathbf{X}}_m, \mathbf{X}_v)$. Thus, for any $m > 1$, we have

$$\frac{T_{0,m} - T(\tilde{\mathbf{X}}_m, \mathbf{X}_v)}{m+1} < \frac{T(u,v)}{d(u,v)} < \frac{T_{0,m} + T(\tilde{\mathbf{X}}_m, \mathbf{X}_v)}{m-1}. \quad (17)$$

Since $\tilde{\mathbf{X}}_m$ is adjacent to $\mathbf{X}_v$, $T(\tilde{\mathbf{X}}_m, \mathbf{X}_v) < \infty$ w.p.1. Therefore in both cases, by Lemma 5 and a typical ϵ-δ argument, we have for any $\epsilon > 0, \delta > 0$, there exists $d_0 < \infty$, such that if $d(u,v) > d_0$ then (6) holds. When $G(\mathcal{H}_\lambda, 1)$ is in the subcritical phase, by Lemma 11, we have $0 < \gamma < \infty$ w.p.1.

On the other hand, when $G(\mathcal{H}_\lambda, 1)$ is in the supercritical phase, by Lemma 11, we have $\gamma = 0$ w.p.1. Then, by a typical ϵ-δ argument, we have for any $\epsilon > 0, \delta > 0$, there exists $d_0 < \infty$, such that if $d(u,v) > d_0$ then (7) holds. □

4.3 Effects of Propagation Delay

Up to this point, we have ignored propagation delays. We now take this type of delay into account. Suppose the propagation delay is $0 < \tau < \infty$ for any link independent of length. We assume the following mechanism is enforced for a transmission from node i to node j: a packet is successfully received by node j if the length of the active period on link (i,j), during which the packet is being transmitted, is greater than or equal to τ; node i retransmits a packet to node j until the packet is successfully received by j.

If node i initiates transmission on (i,j) at time $t = 0$ and link (i,j) is on at time 0 with $Z_1(d) \geq \tau$, then the transmission delay $T^\tau_{ij}(d)$ on (i,j) is τ. However, if link (i,j) is on at time 0 with $Z_1(d) < \tau$, or if (i,j) is off at time $t = 0$, then the delay on (i,j) is less straightforward to calculate. In this case, we need to capture the behavior of retransmissions. Let $K(d) = \text{argmin}_{k\geq 1}\{Z_k(d) \geq \tau\}$. Then, $K(d)$ is a stopping time for the sequence $\{Z_k(d), k \geq 1\}$. Now we have

$$\begin{cases} T^\tau_{ij} = \sum_{i=1}^{K-1}(Y_i + Z_i) + Y_K + \tau, & W(d,0) = 0, \\ T^\tau_{ij} = \sum_{i=1}^{K-1}(Y_i + Z_i) + \tau, & W(d,0) = 1, \end{cases} \quad (18)$$

where we abbreviate $T^\tau_{ij}(d)$, $K(d)$, $Y_i(d)$ and $Z_i(d)$ as T^τ_{ij}, K, Y_i and Z_i, respectively.

Let

$$T^\tau(u,v) = T^\tau(\mathbf{X}_u, \mathbf{X}_v) \triangleq \inf_{l(u,v)\in\mathcal{L}(u,v)} \left\{ \sum_{(i,j)\in l(u,v)} T^\tau_{ij}(d_{ij}) \right\}, \quad (19)$$

where $T^\tau_{ij}(d_{ij})$ is given by (18). Then, $T^\tau(u,v)$ is the message delay on the path from u to v with the smallest delay, including propagation delays.

Corollary 12. *Given $G(\mathcal{H}_\lambda, 1, W(d,t))$ with $\lambda > \lambda_c$ and propagation delay $0 < \tau < \infty$, there exists a constant $\gamma(\tau) < \infty$ with $\gamma(\tau) > \tau$ (w.p.1), such that for any $u, v \in \mathcal{C}(G(\mathcal{H}_\lambda, 1))$, and any $\epsilon > 0, \delta > 0$, there exists $d_0 < \infty$ such that for any u, v with $d(u,v) > d_0$,*

$$\Pr\left(\left|\frac{T^\tau(u,v)}{d(u,v)} - \gamma(\tau)\right| < \epsilon\right) > 1 - \delta. \quad (20)$$

Moreover, when $G(\mathcal{H}_\lambda, 1, W(d,t))$ is in the subcritical phase, as $\tau \to 0$, $\gamma(\tau) \to \gamma$ w.p.1, where γ is defined in Theorem 4. When $G(\mathcal{H}_\lambda, 1, W(d,t))$ is in the supercritical phase, as $\tau \to 0$, $\gamma(\tau) \to 0$ w.p.1.

To prove this corollary, we need the following two lemmas.

Lemma 13. *Given any $0 < \tau < \infty$, for all $0 < d \leq 1$, the expected delay on each link (i,j) is positive and finite, i.e., $0 < E[T^\tau_{ij}] < \infty$.*

Proof: By (18), we have

$$\begin{aligned} E[T^\tau_{ij}] &= \eta_0 E[T^\tau_{ij}|W(d,0)=0] + \eta_1 E[T^\tau_{ij}|W(d,0)=1] \\ &= \eta_0 E\left[\sum_{i=1}^{K-1}(Y_i + Z_i) + Y_K + \tau | Z_i < \tau\right] \\ &\quad + \eta_1 E\left[\sum_{i=1}^{K-1}(Y_i+Z_i) + \tau | Z_i < \tau\right] \\ &= \tau + \eta_0 E[Y_K] + E\left[\sum_{i=1}^{K-1}(Y_i+Z_i)|Z_i < \tau\right] \\ &< E[K]\tau + \eta_0 E[Y_i] + (E[K]-1)E[Y_i], \end{aligned} \quad (21)$$

where in the last equality, we used the fact that Y_i and Z_i are i.i.d. and $Z_i < \tau$ for $i = 1, 2, ...K-1$, and the Wald's Equality for stopping time K.

Since $0 < \tau < \infty$, $0 < \eta_0 < 1$, and $0 < E[Y_i] < \infty$, in order to show $0 < E[T^\tau_{ij}] < \infty$, it suffices to show $1 \leq E[K] < \infty$. By definition, $K \geq 1$ so that $E[K] \geq 1$. Thus, we need only to show $E[K] < \infty$. For any $k \geq 1$, $\Pr(K = k) = \Pr(Z_1 < \tau, ..., Z_{k-1} < \tau, Z_k \geq \tau) = F_Z(\tau)^{k-1}(1 - F_Z(\tau))$, where $F_Z(\cdot) = \Pr(Z_i \leq \tau)$. Then

$$E[K] = \sum_{k=1}^{\infty} kF_Z(\tau)^{k-1}(1 - F_Z(\tau)) = \frac{1}{1 - F_Z(\tau)}. \quad (22)$$

Therefore, we have $E[K] < \infty$. □

Lemma 14. *Given $G(\mathcal{H}_\lambda, 1, W(d,t))$ with $\lambda > \lambda_c$ and no propagation delay, let $L_{0,m}$ be the path from $\tilde{\mathbf{X}}_\mathbf{0}$ to $\tilde{\mathbf{X}}_\mathbf{m}$ that attains $T_{0,m}$ and has the smallest number of links (in case there exist multiple paths attaining $T_{0,m}$). Then $|L_{0,m}| < \infty$ w.p.1 for each m, where $|L_{0,m}|$ is the number of links along $L_{0,m}$.*

Proof: By the proof of Lemma 8, we have $E[T_{0,m}] < \infty$. Note that $E[T_{0,m}] = E[E[T_{0,m}||L_{0,m}|]]$, where $E[T_{0,m}||L_{0,m}|] = \sum_{i=1}^{|L_{0,m}|}\eta_0^{(i)}(d)E[Y_k^{(i)}(d)] \geq |L_{0,m}|\Gamma_{W(d,t)}$, where $\eta_0^{(i)}(d)$ and $E[Y_k^{(i)}(d)]$ are the stationary probability of inactive state, and the expected inactive period of the i-th link on $L_{0,m}$ respectively, and $\Gamma_{W(d,t)} = \min_{0<d\leq 1}\{\eta_0(d)E[Y_k(d)]\} > 0$. Thus, we have $E[|L_{0,m}|]\Gamma_{W(d,t)} < \infty$. This implies $E[|L_{0,m}|] < \infty$, which further implies $|L_{0,m}| < \infty$ w.p.1. □

Proof of Corollary 12: Let $T^\tau_{l,m} = T^\tau(\tilde{\mathbf{X}}_l, \tilde{\mathbf{X}}_m)$, for $||\tilde{\mathbf{X}}_l - \tilde{\mathbf{X}}_m|| < \infty$, $0 \leq l \leq m$, where $\tilde{\mathbf{X}}_i$ is defined as in (8).

Clearly, the relationship $T^\tau_{0,m} \leq T^\tau_{0,l} + T^\tau_{l,m}$ still holds for any $0 \leq l \leq m$. Hence condition (i) of Theorem 6 holds. Since the propagation delay does not affect the stationarity

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of the geometric structure of the network, conditions (ii) and (iii) of Theorem 6 also hold.

By the same argument as that in the proof of Lemma 8, we have $E[|L|] < \infty$, where $|L| \triangleq |L(\tilde{\mathbf{X}}_0, \tilde{\mathbf{X}}_m)|$ and $L(\tilde{\mathbf{X}}_0, \tilde{\mathbf{X}}_m)$ is the shortest path from $\tilde{\mathbf{X}}_0$ to $\tilde{\mathbf{X}}_m$. Let $T^{\tau,L}_{0,m}$ be the delay on this path. Then, $E[T^{\tau,L}_{0,m}||L|] = \sum_{i=1}^{|L|} E[T^{\tau}_i(d_i)] \leq |L|\Lambda_{W^{\tau}(d,t)}$, where $T^{\tau}_i(d_i)$ is the delay on the i-th link with length d_i on the path $L(\tilde{\mathbf{X}}_0, \tilde{\mathbf{X}}_m)$, as given by (18), and $\Lambda_{W^{\tau}(d,t)} \triangleq \max_{0<d\leq 1} E[T^{\tau}_i(d_i)] < \infty$. By Lemma 13, $0 < E[T^{\tau}_i(d_i)] < \infty$ for all $0 < d_i \leq 1$, so that $\Lambda_{W^{\tau}(d,t)} < \infty$. Hence $E[T^{\tau,L}_{0,m}] = E[E[T^{\tau,L}_{0,m}||L|]] \leq E[|L|]\Lambda_{W^{\tau}(d,t)} < \infty$, which implies $E[T^{\tau}_{0,m}] < \infty$. This ensures that condition (iv) of Theorem 6 holds.

Furthermore, the propagation delay does not affect the strong mixing property of $\{T^{\tau}_{l,m}, 0 \leq l \leq m\}$. Therefore the result of Lemma 5 holds for $\{T^{\tau}_{l,m}, 0 \leq l \leq m\}$. Let $\gamma(\tau) \triangleq \lim_{m\to\infty} \frac{E[T^{\tau}_{0,m}]}{m}$, then $\gamma(\tau) = \inf_{m\geq 1} \frac{E[T^{\tau}_{0,m}]}{m}$, and $\lim_{m\to\infty} \frac{T^{\tau}_{0,m}}{m} = \gamma(\tau)$ w.p.1.

Then applying the same proof for Theorem 4, we can show that for any $\epsilon > 0, \delta > 0$, there exists $d_0 < \infty$, such that if $d(u, v) > d_0$ then (20) holds.

To see why $\gamma(\tau) < \infty$, note that $\gamma(\tau) = \inf_{m\geq 1} \frac{E[T^{\tau}_{0,m}]}{m} \leq E[T^{\tau}_{0,1}] < \infty$. Moreover, since the shortest path between nodes $\tilde{\mathbf{X}}_0$ and $\tilde{\mathbf{X}}_m$ has at least $\lfloor ||\tilde{\mathbf{X}}_0 - \tilde{\mathbf{X}}_m|| \rfloor \geq \lfloor m - r_0 - r_m \rfloor$ links, $T^{\tau}_{0,m} \geq \tau \lfloor m - r_0 - r_m \rfloor$. Since r_0 and r_m are both finite w.p.1, we have $\gamma(\tau) \geq \tau$ w.p.1.

We now show that as $\tau \to 0$, $\gamma(\tau) \to \gamma$ w.p.1 when $G(\mathcal{H}_\lambda, 1)$ is in the subcritical phase, and $\gamma(\tau) \to 0$ w.p.1 when $G(\mathcal{H}_\lambda, 1)$ is in the supercritical phase. Observe that $T_{0,m} \leq T^{\tau}_{0,m} \leq \sum_{i=1}^{|L_{0,m}|} T^{\tau}_i(d_i)$, where $L_{0,m}$ is defined in Lemma 14, and $T^{\tau}_i(d_i)$ is the delay on the i-th link with length d_i along $L_{0,m}$, as given by (18). By Lemma 14, $|L_{0,m}| < \infty$ w.p.1. Thus $E[T_{0,m}] \leq E[T^{\tau}_{0,m}] \leq \sum_{i=1}^{|L_{0,m}|} E[T^{\tau}_i(d_i)]$ w.p.1. Since (21) and $E[T_{0,m}] = \sum_{i=1}^{|L_{0,m}|} \eta_0(d_i)E[Y_k(d_i)]$, we have

$$E[T_{0,m}] \leq E[T^{\tau}_{0,m}] \leq E[T_{0,m}] + |L_{0,m}|E[K]\tau + \sum_{i=1}^{|L_{0,m}|} (E[K]-1)E[Y_k(d_i)] \quad (23)$$

w.p.1. From (22), we know that as $\tau \to 0$, $E[K] \to 1$. Therefore, as $\tau \to 0$, we have $|L_{0,m}|E[K]\tau + \sum_{i=1}^{|L_{0,m}|}(E[K] - 1)E[Y_k(d_i)] \to 0$ w.p.1. This, combined with (23) implies $\lim_{\tau\to 0} E[T^{\tau}_{0,m}] = E[T_{0,m}]$ w.p.1. Therefore,

$$\begin{aligned} \lim_{\tau\to 0} \gamma(\tau) &= \lim_{\tau\to 0} \lim_{m\to\infty} \frac{E[T^{\tau}_{0,m}]}{m} \\ &= \lim_{m\to\infty} \lim_{\tau\to 0} \frac{E[T^{\tau}_{0,m}]}{m} \\ &= \lim_{m\to\infty} \frac{E[T_{0,m}]}{m} \\ &= \gamma \end{aligned} \quad (24)$$

w.p.1, where the interchanging of limitation operations is justified by $E[T^{\tau}_{0,m}] < \infty$. Consequently, as $\tau \to 0$, $\gamma(\tau) \to \gamma$ w.p.1 when $G(\mathcal{H}_\lambda, 1)$ is in the subcritical phase, and $\gamma(\tau) \to 0$ w.p.1 when $G(\mathcal{H}_\lambda, 1)$ is in the supercritical phase. □

An interesting observation of this corollary is when the propagation delay is large, the message delay cannot be improved too much by transforming the network from the subcritical phase to the supercritical phase. However, as the propagation delay becomes negligible, the message delay scales almost sub-linearly ($\gamma(\tau) \approx 0$) when the network is in the supercritical phase, while the delay scales linearly ($\gamma(\tau) \approx \gamma$) when the network is in the subcritical phase.

5. CONCLUSIONS

In this paper, we studied percolation-based connectivity and information dissemination latency in large-scale wireless networks with unreliable links. We first studied static models, where each link of the network is functional (or active) with some probability, independently of all other links. We then studied wireless networks with dynamic unreliable links, where each link is active or inactive according to Markov on-off processes. We showed that a phase transition exists in such dynamic networks, and the critical density for this model is the same as the one for static networks under some mild conditions. We further investigated the delay performance in such networks by modelling the problem as a first passage percolation process on random geometric graphs. We showed that without propagation delay, the delay of information dissemination scales linearly with the Euclidean distance between the sender and the receiver when the resulting network is in the subcritical phase, and the delay scales sub-linearly with the distance if the resulting network is in the supercritical phase. We further showed that when taking propagation delay into account, the delay of information dissemination always scales linearly with the Euclidean distance between the sender and the receiver.

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