A Slotted ALOHA Protocol with Cooperative Diversity*

Ramiro Samano-Robles
Instituto de Telecomunicações
Aveiro - Portugal
ramiro@av.it.pt

Atilio Gameiro
Instituto de Telecomunicações
Aveiro - Portugal
amg@av.it.pt

ABSTRACT

Over the last few years the area of cooperative communications has regained attention within the physical layer community. However, existing works on cooperative random access protocols are relatively scarce and biased towards their physical layer properties, thus leaving unattended important problems of the medium access control (MAC) sublayer such as calculation of the backlog delay, design of appropriate back-off retransmission strategies, and stability evaluation, among many others. This paper partially fills this gap by studying the general performance of a symmetrical Slotted ALOHA protocol in which a cooperative relaying phase is enabled in order to improve the decoding probability of collision-free transmissions. Infinite and finite user schemes are used, and for the latter, Bernoulli and Markov models are further employed to study the steady- and the dynamic-state properties of the protocol, respectively. A stochastic reception model is presented which fairly describes the underlying physical layer events from the perspective of the MAC sublayer, including correct packet decoding probabilities, relay node availability, and error detection capabilities. Important results regarding the boundaries for optimum performance of cooperative relaying schemes and useful guidelines for the design of optimum relaying strategies are here derived and discussed.

Categories and Subject Descriptors

C.2.1 [**Computer-Communication Networks**]: Network architecture and design- *packet switching networks, wireless communications.*

General Terms

Algorithms.

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Keywords

Cooperative diversity, signal processing in random access, cross-layer design, ALOHA protocol.

1. INTRODUCTION

A useful method to counteract the degrading effects of interference and fading in wireless communications is by collecting and appropriately combining independent received copies of the transmitted signal. Such copies are commonly obtained from diversity sources that span frequency, code, time or spatial domains. The latter is a particular feature of wireless networks that has been typically exploited by means of multiple antennas located either at the receiver, at the transmitter, or at both locations [2]. However, due to size limitations of modern mobile terminals, the minimum spacing between antennas required to achieve diversity might be unfeasible. As an alternative approach, the area of cooperative communications attempts to obtain such diversity gains by means of user cooperation schemes that efficiently exploit the spatial configuration of the network [3].

1.1 Cooperative diversity

Achieving diversity through user cooperation critically relies on the existence of a set of users who are willing to cooperate and who are located at the appropriate position in the network. These cooperative users are listening to the transmissions from the active users, and when they are requested to, they proceed to relay to the destination either a processed or an amplified version of their received packet (decode and forward (DF), or amplify and forward (AF), respectively). Finally, the destination combines the received copies of the signal thereby potentially achieving similar gains to an equivalent multi-antenna system [4].

From the physical layer perspective, the performance of cooperative diversity schemes critically depends on the characteristics of the channels between source and destination, source and relay, and relay and destination. The source-destination channel is also called direct channel, whereas the one composed of the direct and the source-relay-destination channel is called relay channel. The statistics of such relay channel determine the relaying protocol, modulation and coding schemes, and whether to activate the relaying phase or not. For example, AF relaying protocols are optimum when the relay node is close to the destination, whereas DF is optimum when the relay node is close to the source [3]. Describing all the physical layer properties of cooperative diversity is, however, out of the scope of this paper. Instead, we refer the reader to the papers of Van der Meulen in [5];

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Cover and El Gamal in [6]; and to the papers by Sendonaris in [7] and [8]. A summary of recent works can be found in the compilation made by in Fitzek and Katz in [3].

In cooperative diversity systems, variables across different layers closely interact with each other [9]. For example, when a relaying phase is activated in half-duplex systems, one or more time-slots of the system are used to transmit the relayed packets thereby affecting the delay and throughput at the medium access control (MAC) sublayer [10]. Therefore, the achieved gain in diversity should greatly compensate the loss in throughput and delay due to such relaying phases. One of the goals of this paper is assessing this minimum required gain for cooperative diversity schemes, which is an interesting cross-layer design problem.

Another example of cross-layer interaction is the selection of the relaying nodes, particularly in half-duplex systems [10]. In a real system configuration, a relay node must have traffic load of its own that needs to be transmitted in addition to the relayed packets from other users, which constitutes a complex queuing theoretical problem. In this paper we will follow the approach used in [11], where the authors propose using the inactive users of a random access network as relaying nodes, which greatly reduces the complexity of the analysis. More cross-layer issues related to cooperative communications can be found in [3], [4], and [9].

1.2 Cooperative random access protocols

Despite the extensive literature on cooperative diversity, cooperative random access protocols are relatively scarce. Existing works have been focused more on the physical layer attributes rather than considering medium access control (MAC) sublayer parameters which include stability, retransmission schemes, backlog delay and bandwidth loss due to signalling. A recent work on the analysis of the achievable maximum stable throughput (MST) of cooperative random access protocols can be found in [11], where the authors have shown that cooperative protocols over fading channels asymptotically achieve the MST of simple random access protocols over additive Gaussian noise channels. Using another perspective, a cooperative random access protocol called ALLIANCES has been proposed in [12]. The authors have used the diversity created by subsequent retransmissions from the relay nodes not only to achieve decoding gains but also to resolve packet collisions and reduce multiple access interference. This means that in a system without any diversity source other than relaying retransmissions, a collision of K users would be resolved by a succession, in a time-division manner, of at least $K-1$ relaying retransmissions plus the initial transmission. At the central node, all the collected transmissions and retransmissions are processed using source separation tools to finally obtain the original packets [12]. This is similar to the non-cooperative retransmission diversity multiple access protocol called NDMA (network assisted diversity multiple access) proposed in [13], in which the retransmissions are provided by the contending users themselves and not by the relaying nodes.

Unlike NDMA, which is affected by correlation between successive retransmissions and by channels with deep and long term fades, the ALLIANCES protocol exhibits resilience to these problems due to the diversity provided by the spatially distributed relay nodes. The relevance of NDMA-type protocols for the work presented in this paper is that the effects of relaying phases at the MAC sublayer are basically the same as the effects of subsequent retransmissions in NDMA and ALLIANCES. These effects were first recognized in [14], where the author has coupled the effects of cooperative relaying phases with the retransmission scheme of an H-ARQ (hybrid automatic repeat request) protocol.

1.3 Cross-layer random access protocols

ALLIANCES and NDMA belong the category of cross-layer random access protocols. This area has recently been subject of considerable research efforts due to two important facts: the advent of new physical layer schemes that allow multipacket reception (MPR) capabilities, and the development of stochastic reception models that accurately describe the effects of wireless channels such as fading and interuser interference [15]. This approach is different to the conventional collision model in which any packet collision represented the loss of all the transmitted information, and where successful or correct packet reception was only achieved in transmissions containing a single packet [16].

Perhaps the most relevant examples of cross-layer random access protocols are two different versions of the Slotted ALOHA (S-ALOHA) protocol with stochastic MPR capabilities. The first one, presented in [17], considers an infinite user scheme with symmetrical configurations (i.e., all users present the same reception and traffic parameters), while the second one in [18] assumes finite and asymmetrical populations. Although these works fairly characterize MPR capabilities along the space, frequency or code dimension, the first system with adaptive MPR capabilities along the time dimension was the NDMA protocol proposed in [13]. The formulation for training-based NDMA systems in [13] and later in [19] consists of useful but inaccurate stochastic reception model based on probabilities of detection and false alarm. Enhanced reception models for NDMA which are based on the model used in [18] for the S-ALOHA protocol have been proposed in [20] for the perfect user detection case and in [21] for the imperfect user detection case.

The model presented in this paper is a particular variation of the reception model used in [20] adapted to the particular case of cooperative diversity systems without MPR capabilities. As the formulation is based on the MPR models discussed in this subsection, the extension of our model to considering MPR capabilities is straightforward.

1.4 Paper contributions

This paper presents a detailed study on the general performance of a symmetrical S-ALOHA protocol in which a cooperative relaying phase is enabled for collision-free transmissions. A two hop scheme with only one relaying retransmission has been considered. To cover all the aspects of the behavior of the protocol we reuse here the tools commonly employed in the past for the analysis of the conventional ALOHA protocol (see [16]). We first use an infinite user scheme which is known to provide a fair analysis of the maximum stable throughput and queuing delay of random access protocols under the assumptions of large numbers of users and finite traffic loads. In this scheme both new incoming and previously incorrectly decoded packets (i.e., backlogged packets) are considered as new users joining the active population, thus avoiding analysis of complex buffering strategies. Additionally, we also use a finite-user scheme which provides a fair evaluation of the system performance considering low numbers of users and high traffic loads. Within

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this finite-user scheme we further use two different models. The first one, called Bernoulli model, uses a generalized probabilistic description of the queuing statistics thus being useful for steady-state analysis. The second one, called the Markov model, is useful in designing back-off retransmission strategies, in assessing stability, and in studying the dynamic behavior of the algorithm.

An interesting result is the derivation of the minimum performance required by cooperative diversity schemes in order to provide a gain in maximum stable throughput with respect to the non-cooperative version of the protocol. The analysis is carried out under different assumptions regarding the activation of the relaying phase, using a stochastic model in which the relaying mechanism can be dependent on the decoding events from the direct transmission or not. Also, this probabilistic model absorbs other random variables that may affect the activation of the relaying phase such as availability of relaying nodes. However, a detailed analysis of relay selection strategies is topic of future works.

The stochastic reception model used in this paper fairly describes the underlying physical layer events from the perspective of the MAC sublayer. In particular, it absorbs the gain provided by considering either the direct channel only, or the relay and the direct channel together. As previously mentioned, it is an extension of the model used in [20] but without considering MPR capabilities. A basic optimization of the protocol expressions reveals a close relationship between the aforementioned parameters from the physical layer and their counterparts in the MAC sublayer. Important results regarding the boundaries for optimum performance of cooperative relaying schemes and useful guidelines for the design of optimum relaying strategies are here derived and discussed. Details are given in subsequent sections.

The structure of this paper is as follows. Section 2 formulates the protocol rules and system assumptions. Section 3 presents the performance analysis, while Section 4 presents the optimization of the protocol. Finally, Section 5 presents the results and Section 6 draws conclusions.

2. SYSTEM DESCRIPTION

This section describes the assumptions made for the analysis of the system. First, subsection 2.1 describes the general operational principles of the protocol, and then subsections 2.2 and 2.3 define the population schemes to be employed.

2.1 Protocol description and reception model

Consider a slotted multi-access network with centralized infrastructure or base station (BS) towards all the direct and relayed packet transmissions are forwarded. For convenience, we will only consider a two-hop relaying scheme and a maximum of only one relaying retransmission. Whenever the users of the network are allowed to transmit at the beginning of a collision resolution period or epoch slot, they do so provided they have a packet ready to be transmitted. If it happens that more than one user is transmitting in the same time slot then we say that a collision has occurred and that all information is lost, i.e. the collision is unresolvable. On the other hand, if such time-slot presents a packet transmission with no collisions, then we assume that the BS attempts the decoding of the packet. Due to wireless channel impairments, this decoding process is prone to errors. Therefore, a packet can be correctly (or successfully) decoded with probability p_d. We will also call this term the probability of correct or successful packet decoding in the direct transmission. By correct or successful packet decoding it is meant that either the information of the decoded packet or the signal to noise and interference ratio of the user are above a prescribed quality of service threshold.

In a cooperative diversity scenario, the system should decide whether the relaying phase should be activated or not. However, in the decision process many variables come into play such as relay node availability, detection of errors in the direct transmission, availability of relay channel state information, etc. To absorb all these random effects we say that the system switches to a cooperative relaying phase with probability p_{rel}. To explicitly indicate the possible statistical dependency of the relaying mechanism with respect to the outcome of decoding process in the direct transmission, $p_{rel|co}$ and $p_{rel|in}$ will denote the probabilities of relaying dependent on the correct or incorrect packet decoding process in the direct transmission, respectively. Note that the cooperative relaying phase is activated only when the initial packet transmission has no collisions, which is a relaxed assumption as in real systems the collision detection mechanism is also prone to errors. During the relaying phase only one relay node is allowed to transmit and all the other users are requested to wait for the next time-slot. Our approach is similar to the consideration made in [11], where inactive users may act a potential relay nodes. The total probability of relaying p_{rel} is thus given by:

$$p_{rel} = p_d p_{rel|co} + \bar{p}_d p_{rel|in}, \qquad (1)$$

where $\bar{(\cdot)} = 1 - (\cdot)$. Additionally, the probability of correct packet decoding with the relaying channel is given by p_c, where the inequality $p_c > p_d$ indicates the availability of a cooperative physical layer algorithm that, in average, outperforms the decoding processes for the direct transmission. Note that here we are using a special type of enhanced decoding capability along the time dimension but no MPR capabilities are being used at all. A future research topic is to study MPR capabilities using the reception models proposed in [20] and [21].

It is worth mentioning that we have assumed statistical independence between the decoding events from the relaying and direct transmissions. This is both an optimistic and a pessimistic assumption since an incorrect decoding event in the direct transmission might reduce the decoding probability in the relay transmission, or, on the other hand, high decoding probabilities in the direct transmission do not necessarily yield high decoding probabilities in the relay channel.

Finally, we define the random variable that describes the length of the collision resolution period or epoch slot as l. From a particular user perspective, a resolution period is relevant or irrelevant depending on whether the user is transmitting or not in such an epoch, respectively. Their lengths will be denoted by the random variables l_r and l_{ir}, respectively.

2.2 Infinite user scheme

In this subsection we use an infinite population with finite and Poisson distributed traffic arrival rate with parameter λ packets/time-slot. Such traffic rate is composed of both new incoming and backlogged packets, i.e., packets that were incorrectly decoded in previous slots and need to be retransmitted with a uniform probability p_r. This means that back-

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logged packets rejoin the active population as new users thus avoiding the analysis of complex buffering schemes.

2.3 Finite user scheme

Within the finite user scheme, we will use two separate models that are useful in the analysis of different properties of the protocol.

2.3.1 Bernoulli model

In this subsection we define a network with a finite population of J active users, each one with an overall transmission probability described by a Bernoulli process with parameter p. We recall that in this model accurate steady-state throughput values can be predicted whereas the delay values are less reliable.

2.3.2 Markov model

In this model we consider a finite population of J users from which n of them are said to be in the backlog state. We remind the reader that a user in the backlog state means that the user previously attempted to transmit a packet and it was incorrectly delivered to the central node, thus the user is attempting the retransmission of such packet with a uniform probability p_r. It is considered that new arriving packets are transmitted with probability p_a and that a user in the backlog state does not receive new packets. A particular user is in the backlog state with probability P_b and in the idle state with probability P_i. In the strict sense, the complete dynamic state of the network of a buffered system is given by the vector of queue lengths. However, it has been proved that the number of backlogged users, as in our approach, represents a useful variable that fairly summarizes the dynamic state of symmetrical systems [22].

3. PERFORMANCE ANALYSIS

In this section, the main performance metrics of the protocol, i.e. throughput, delay, etc, are defined, derived and discussed.

3.1 Infinite user scheme

As it is commonly used in the analysis of infinite user schemes (e.g. [21]), it is convenient to define the matrix of transition probabilities between epoch slots of length i to epoch slots of length j, which in our case can be written as:

$$P_{i,j} = \begin{cases} 1 - p_{rel}\lambda i e^{-\lambda i}, & j = 1 \\ p_{rel}\lambda i e^{-\lambda i}, & j = 2 \end{cases}$$

where the term $\lambda i e^{-\lambda i}$ denotes the probability of arrival of exactly one packet due to the Poisson process. Therefore, the term $p_{rel}\lambda i e^{-\lambda i}$ denotes the probability of relaying provided that only one packet is present in the direct transmission. The matrix of transition probabilities in eq.(2) can be easily proved to be a positive recurrent Markov chain. This allows us to calculate the vector of steady-state epoch lengths, denoted here by $\mathbf{x} = [x_1, x_2]^T$, by solving the following eigen-value problem:

$$\mathbf{x} = \mathbf{P}\mathbf{x}, \qquad \text{subject to} \qquad \sum_l x_l = 1,$$

where the summation is taken over all values of l and where $\mathbf{P}$ is the matrix with elements given by $P_{i,j}$ in eq.(2). Once obtained the steady-state statistics, the system throughput is defined as the ratio of average correctly or successfully transmitted packets, denoted by $E[s]$, to the average length of an epoch slot, denoted by $E[l]$. This results in the following formula:

$$T = \frac{E[s]}{E[l]}, \tag{2}$$

where the numerator is given by averaging over all the possible cases as follows:

$$E[s] = \sum_l x_l p_s \lambda l e^{-\lambda l}.$$

The term p_s denotes the total probability of correct or successful packet decoding in the absence of collision in the direct transmission, and it is given by the contribution of all the possible outcomes of epoch lengths with at least one correct packet decoding event. Epochs with one time slot and correct packet decoding occur with probability $p_d\bar{p}_{rel|co}$. Epochs with two time slots, correct decoding in the direct transmission and incorrect decoding in the cooperative relaying phase occur with probability $p_d\bar{p}_c p_{r|co}$, whereas when the decoding in the direct transmission is incorrect and the decoding in the relaying phase is correct occurs with probability $\bar{p}_d p_c p_{rel|in}$. Finally, both outcomes are successful with probability $p_d p_c p_{rel|co}$. Therefore p_s is given by adding all these probabilities as follows:

$$p_s = p_d\bar{p}_{rel|co} + p_d\bar{p}_c p_{rel|co} + \bar{p}_d p_c p_{rel|in} + p_d p_c p_{rel|co},$$

which can be further simplified to:

$$p_s = p_d + \bar{p}_d p_c p_{rel|in} \tag{3}$$

Particular cases of the above formula are given when the activation of the relaying phase is independent of the outcome of the decoding process in the first transmission. In this case $p_{rel|co} = p_{rel|in} = p_{rel}$, so p_s reduces to:

$$p_s = p_d + \bar{p}_d p_c p_{rel}. \tag{4}$$

Consider now a system that has a perfect error detection mechanism and that the relaying phase can be activated exactly when the decoding of the direct transmission has failed. Therefore, we have $p_{rel|in} = 1$ and $p_{rel|co} = 0$, which results in the following expression for p_s:

$$p_s = p_d + \bar{p}_d p_c, \tag{5}$$

which is clearly larger than the value in eq.(4) and eq.(3). Finally, the denominator of eq.(2) can be calculated by averaging the epoch length over all the probabilistic space as follows:

$$E[l] = \sum_l l x_l.$$

As regards the average delay, it is defined here as the average number of time-slots that a packet takes from its arrival to the system to be correctly delivered to the BS. It consists of three terms which are simply related in the following way:

$$D = (D_b + 1)(D_s + D_q) = (D_b + 1)\sum_l x_l \cdot \left[\frac{l}{2} + D_{q|l}\right] \tag{6}$$

where D_s is the service time delay, defined as the average number of time-slots that a packet in the head of the queue takes to be processed either correctly or incorrectly by the system; and D_q is the queuing delay, defined as the average

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number of time slots that a packet takes from its arrival to the system to reach the head of the queue. It is also known that the backlog delay, defined here as the average number of time slots that a user is in the backlog state, can be approximated, when the retransmission probability is constant, by using Little's theorem as follows [16]:

$$D_b = \frac{E[s_{in}]}{E[s]} - 1, \tag{7}$$

where

$$E[s_{in}] = \sum_l x_l \lambda l$$

is the input traffic due to the Poisson traffic. Finally, the queuing delay conditioned on the length of a previous epoch can be written as:

$$D_{q|l} = p_{rel} e^{-\lambda l} + 1, \tag{8}$$

which indicates the length of the collision resolution period of a particular packet that arrived during a previous contention period and that in the current one is contending with other users with a Poisson arrival rate. The term $e^{-\lambda l}$ stands for the probability of no packet collision and the term $p_{rel} e^{-\lambda l}$ indicates the probability of activation of the relaying phase given no packet collision is present in the first time slot of the epoch.

3.2 Bernoulli model

Let us now reconsider the throughput in eq.(2) but this time in the context of the Bernoulli model. Therefore, it is possible to write the following expression for $E[s]$:

$$E[s] = Jpp_s \bar{p}^{J-1},$$

which denotes the probability that one user out of J correctly transmits a packet with probability Jpp_s and that the remaining $J-1$ users do not transmit in the same time-slot with probability $\bar{p}^{J-1}$. Finally, $E[l]$ in eq.(2) is given, in the case of the Bernoulli model, by the contribution of epochs with length two time-slots with probability $Jpp_{rel}\bar{p}^{J-1}$ and epochs with one time-slot with probability $(1 - Jpp_{rel}\bar{p}^{J-1})$:

$$E[l] = 2Jpp_{rel}\bar{p}^{J-1} + (1 - Jpp_{rel}\bar{p}^{J-1}),$$

which further reduces to:

$$E[l] = Jpp_{rel}\bar{p}^{J-1} + 1.$$

A useful expression that relates the Bernoulli parameter p with a Poisson distribution per user with parameter $\hat{\lambda}$ is given by the following balance traffic equation [23]:

$$p = \hat{\lambda} E[l] \tag{9}$$

As regards the delay we write a modified version of eq.(6) in which the formula of an M/G/1 system accounts both service and queuing delay:

$$D = (D_b + 1) D_{M/G/1}, \tag{10}$$

where D_b is derived in a similar way to eq.(7) by considering Little's theorem:

$$D_b = \frac{Jp}{E[s]} - 1, \tag{11}$$

and $D_{M/G/1}$ can be written as follows [13]:

$$D_{M/G/1} = E[l_r] + \frac{\hat{\lambda} E[l_r^2]}{2(1 - \hat{\lambda} E[l_r])} + \frac{E[l_{ir}^2]}{2E[l_{ir}]}. \tag{12}$$

The terms $E[l_r]$, $E[l_r^2]$, $E[l_{ir}]$, and $E[l_{ir}^2]$ stand for the first- and second-order moments of the length of a relevant and irrelevant epochs, respectively. We recall from [13] that, from a particular user perspective, a relevant epoch is the one in which such user transmits, whereas an irrelevant epoch is the one in which the user is idle. We also recall from [13] that eq.(12) considers that relevant and irrelevant epochs are statistically independent, which is an approximation valid only at low values of the traffic loads. The expressions for the first- and second-order moments of both kinds of epochs can be easily derived by considering that a relevant epoch has length one with probability $(1 - p_{rel}\bar{p}^{J-1})$ and length two with probability $p_{rel}\bar{p}^{J-1}$; whereas the irrelevant epoch has length one with probability $(1-(J-1)p_{rel}p\bar{p}^{J-2})$ and length two with probability $(J-1)p_{rel}p\bar{p}^{J-2}$. These distributions finally yield to the following expressions:

$$E[l_r] = p_{rel}\bar{p}^{J-1} + 1, \qquad E[l_r^2] = 3p_{rel}\bar{p}^{J-1} + 1$$

$$E[l_{ir}] = (J-1)p_{rel}p\bar{p}^{J-2} + 1, \quad E[l_{ir}^2] = 3(J-1)p_{rel}p\bar{p}^{J-2} + 1.$$

This completes the analysis related to the Bernoulli model.

3.3 Markov model

This subsection deals with the analysis of the dynamic properties of the algorithm using the well known Markov model for the backlog states of the system. First we state a modified version of the transition probabilities of a conventional S-ALOHA protocol in [22] and [16], but this time including our reception model for cooperative diversity through the parameter p_s:

$$P_{j,k} = \begin{cases} 0, & k \leq j-2 \\ j p_r p_s \bar{p}_r^j \bar{p}_a^{J-j}, & k = j-1 \\ \bar{p}_r^j (J-j) p_a p_s \bar{p}_a^{J-j-1} \\ +[1 - j p_r p_s \bar{p}_r^{j-1}] \bar{p}_a^{J-j}, & k = j \\ (J-j) p_a \bar{p}_a^{J-j-1} [1 - p_s \bar{p}_r^j], & k = j+1 \\ \begin{pmatrix} J-j \\ k-j \end{pmatrix} p_a^{k-j} \bar{p}_a^{J-k} & k \geq j+2 \end{cases},$$

which can be proved, as in the conventional protocol, to be a finite and positive recurrent Markov chain. This allows us to solve it using the following eigenvalue problem:

$$\tilde{\mathbf{x}} = \mathbf{P}\tilde{\mathbf{x}}, \qquad \text{subject to} \quad \sum_n \tilde{x}(n) = 1,$$

where $\tilde{\mathbf{x}} = [\tilde{x}(0), \ldots \tilde{x}(n)]^T$ is the vector of steady-state probabilities of the number of backlogged users or the backlog states probabilities. Let us now consider again the throughput of the system in eq.(2), where the term $E[s]$ is given, in the particular case of the Markov model, by:

$$E[s] = \sum_{n=1}^{J} \tilde{x}(n) [(J-n) p_a p_s \bar{p}_a^{J-n-1} \bar{p}_r^n + n p_r p_s \bar{p}_a^{J-n} \bar{p}_r^{n-1}]$$

$$= \sum_{n=1}^{J} \tilde{x}(n) S_{out}(n)$$

where $S_{out}(n)$ indicates the output traffic when n users are in the backlog state. Details of the derivations in this subsection have been omitted as they can be easily obtained

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following the tools and procedures of the Bernoulli model in the previous subsection. Now, $E[l]$ in eq.(2), in the context of the Markov model, is given by:

$$E[l] = \sum_{n=1}^{J} \tilde{x}(n)[(J-n)p_a p_{rel}\bar{p}_a^{J-n-1}\bar{p}_r^{n} + np_r p_{rel}\bar{p}_r^{n-1}\bar{p}_a^{J-n} + 1], \quad (13)$$

For the stability analysis we now define the drift function as the difference between the incoming and outgoing traffic:

$$f(n) = S_{in}(n) - S_{out}(n), \quad (14)$$

where $S_{in}(n) = (J-n)p_a$. As regards the backlog delay, it can be calculated using again Little's theorem as follows [22]:

$$D_b = \frac{E[n]}{E[s]},$$

and the queuing delay can be calculated using the formula of an M/G/1 queue with vacations in eq.(6). The particular terms in eq.(6), for the case studied in this subsection, are derived in a similar way to the terms in the subsection dedicated to the Bernoulli model. The final expressions are given by

$$E[l_r] = 1 + \sum_{n=1}^{J} \tilde{x}(n)\left[P_i p_{rel}\bar{p}_a^{J-n-1}\bar{p}_r^{n} + \bar{P}_i p_{rel}\bar{p}_a^{J-n}\bar{p}_r^{n-1}\right],$$

$$E[l_r^2] = 1 + \sum_{n=1}^{J} 3\tilde{x}(n)\left[P_i p_{rel}\bar{p}_a^{J-n-1}\bar{p}_r^{n} + \bar{P}_i p_{rel}\bar{p}_a^{J-n}\bar{p}_r^{n-1}\right],$$

$$E[l_{ir}] = 1 + \sum_{n=1}^{J} \tilde{x}(n)\left[P_i(J-n)p_a p_{rel}\bar{p}_a^{J-n-1}\bar{p}_r^{n} + np_r\bar{P}_i p_{rel}\bar{p}_a^{J-n}\bar{p}_r^{n-1}\right],$$

$$E[l_{ir}^2] = 1 + \sum_{n=1}^{J} \tilde{x}(n)\left[P_i(J-n)p_a p_{rel}\bar{p}_a^{J-n-1}\bar{p}_r^{n} + np_r\bar{P}_i p_{rel}\bar{p}_a^{J-n}\bar{p}_r^{n-1}\right],$$

where $P_b = \frac{n}{J}$ and $P_i = 1 - P_b$. Finally, the total delay is simply given by eq.(10), but this time using the expressions for the Markov model provided in the previous lines.

4. PROTOCOL OPTIMIZATION

This section deals with the protocol optimization with respect to the transmission parameters. For convenience, only the expressions for the Bernoulli model in subsection 3.2 will be considered. A similar approach can be used in the case of the infinite user model, but it is out of the scope of this paper. In the case of the Markov model we can use the results provided by the Bernoulli formulation by using an elementary change of variables. Consider eq.(2) for the throughput of the system. Strictly speaking the optimization should be carried out jointly with respect to the following parameters: the transmission probability p, the probability of relaying p_{rel}, and the decoding probabilities p_d and p_c, which is a complex cross-layer joint optimization problem. However, for the sake of obtaining a basic understanding of the protocol, we will only carry out the optimization with respect to the transmission probability as follows:

$$p_{opt} = \arg\max_{p} \quad T.$$

Using a simple optimization method we proceed to differentiate eq.(2) with respect to p and set it to zero as follows

$$\frac{dT}{dp} = 0.$$

Due to the simplicity of the derivation, the details have been omitted. The optimum transmission probability results to be identical to the solution for the conventional S-ALOHA protocol, i.e.:

$$p_{opt} = \frac{1}{J}.$$

By substituting this expression back in eq.(2) and taking the limit when $J \to \infty$ we obtain the following asymptotic value for the maximum stable throughput:

$$MST_{coop} = \frac{p_s e^{-1}}{p_{rel}e^{-1} + 1},$$

which further reduces to:

$$MST_{coop} = \frac{p_s}{p_{rel} + e}.$$

By substituting in the above equation the value for p_s in eq.(3) and the total probability of relaying p_{rel} in eq.(1), we further obtain:

$$MST_{coop} = \frac{p_d + \bar{p}_d p_c p_{rel|in}}{p_d p_{rel|co} + \bar{p}_d p_{rel|in} + e}. \quad (15)$$

Note that by substituting $p_{rel|in} = p_{rel|co} = 0$ in the above equation results in the MST for the conventional S-ALOHA protocol with packet decoding losses:

$$MST_{saloha} = p_d e^{-1}.$$

Let us now define the gain of the cooperative protocol with respect the conventional one as follows:

$$G_{MST} = \frac{MST_{coop}}{MST_{saloha}} = \frac{1 + \frac{\bar{p}_d}{p_d}p_c p_{rel|in}}{(p_d p_{rel|co} + \bar{p}_d p_{rel|in})e^{-1} + 1}, \quad (16)$$

and then attempt to derive the values of p_c for which this gain function is larger than one, which actually means that the cooperative component improves protocol performance:

$$G_{MST} > 1,$$

from which we can obtain the following meaningful expression for p_c:

$$p_c > \frac{(p_d p_{rel|co} + \bar{p}_d p_{rel|in})e^{-1}p_d}{\bar{p}_d p_{rel|in}}. \quad (17)$$

This formula represents the minimum packet decoding probability p_c for an underlying physical layer cooperative algorithm in order to provide some gain over a non-cooperative S-ALOHA protocol. We proceed to derive more precise boundaries for the values of p_d under which the cooperative protocol is useful for our purposes. If we substitute the value of $p_c = 1$ in the previous equation and then we obtain the value for p_d the following second-order equation is obtained:

$$p_d^2 e^{-1}(p_{rel|co} - p_{rel|in}) + p_d p_{rel|in}(e^{-1} + 1) - p_{rel|in} = 0. \quad (18)$$

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The solution for this equation denotes the value of p_d above which there is no need to do any cooperative relaying phase, no matters how much gain the cooperative physical layer algorithm can provide. Thus, higher the values for p_d in eq.(18) yield better performance of the cooperative relaying scheme. An analysis of the possible solutions of eq.(18) reveals that, on the limit, p_d equals one when $p_{rel|co}$ is zero. This means that the optimum relaying scheme occurs when $p_{rel|co} = 0$, i.e., when the relaying phase is activated only if necessary.

Now, we derive the lower bound for a system that instead of using cooperative diversity uses a simple retransmission diversity at the link layer. If in eq.(17) we substitute $p_c = p_d$ then we obtain the following inequality for p_d:

$$p_d > \frac{(e^{-1} - 1)p_{rel|in}}{(e^{-1} - 1)p_{rel|in} - p_{rel|co}e^{-1}}, \tag{19}$$

which indicates the value of p_d above which a system that simply uses retransmissions from the collision-free user stops providing gains over the original protocol.

In the case when the relaying probability is independent of the decoding process in the direct transmission, i.e. $p_{rel|in} = p_{rel|co} = p_{rel}$, the expression in eq.(17) clearly becomes:

$$p_c > \frac{e^{-1}p_d}{\bar{p}_d}. \tag{20}$$

Note that the last expression is, as expected, independent of the relaying probability. Now, using the same relaying strategy the solution for eq.(18) simplifies to:

$$p_d > \frac{1}{1 + e^{-1}} = 0.7311, \tag{21}$$

and eq.(19) reduces to:

$$p_d > 1 - e^{-1} = 0.6321. \tag{22}$$

The result for a strategy with perfect detection of packet errors, i.e. $p_{rel|co} = 0$ and $p_{rel|in} = 1$ is now derived. Under this assumption, eq.(17) becomes:

$$p_c > e^{-1}p_d. \tag{23}$$

Note that the last expression is smaller by a factor of $\bar{p}_d$ than the value for the previous relaying strategy in eq.(20). Also note that p_c does not need to be larger than p_d in order to provide a gain in MST. Using the same relaying strategy with perfect packet error detection, it happens that the solutions for eq.(18) and eq.(19) do not exist at all, which means that this relaying strategy always provides some gain over the original protocol given that the decoding probabilities comply with the last inequality in eq.(23). It is important to mention that the results presented are valid only for the operational point that gives the maximum stable throughput. At any other operational point, the optimum values for the system parameters should be obtained by a joint optimization with respect to all the variables involved. However, the results presented here and in the following section provide a good understanding of the consequences of cooperative relaying at the medium access control sublayer.

5. RESULTS

This section displays graphical results for the performance metrics of the system studied in previous sections of this paper. Fig.1 shows the results for the system throughput (T) in eq.(2) vs. traffic load ($\hat{\lambda}J$) in eq.(9) for a 16-user S-ALOHA protocol using the Bernoulli model with different values and assumptions for the decoding and relaying probabilities. The purpose of this figure is to show under which values of the decoding and relaying probabilities the protocol assisted by cooperative diversity produces a gain with respect to the protocol with no cooperation, and compare these results with the analytical formulae derived in the previous section. The figure illustrates two cases with the following decoding probabilities in the direct transmission $p_d = 0.3$, and $p_d = 0.9$, each case combined with two different decoding probabilities in the relay channel $p_c = p_d$ and $p_c = 0.95$. Each combination has been further divided into two cases, one with a relaying scheme with perfect error detection, and the other one which is independent of the decoding process with a fixed value of $p_{rel} = 0.5$. In the case where the decoding probability in the direct transmission is $p_d = 0.3$ all the relaying strategies have provided some gain over the original protocol (shown in the figure as $p_{rel} = 0$). The larger gain is obtained in the case of perfect packet error detection, i.e. $p_{rel|co} = 0$ and $p_{rel|in} = 1$, followed by the relaying strategy that is unaware of the decoding events in the direct transmission, i.e. $p_{rel} = 0.5$. However, this is not the same situation when the decoding probability in the direct transmission is equal to $p_d = 0.9$, where the relaying strategy that is unaware of the decoding events in the direct transmission has a performance even lower that the original protocol. This confirms the performance boundary for p_d derived in eq.(20) ($p_d > 0.7311$).

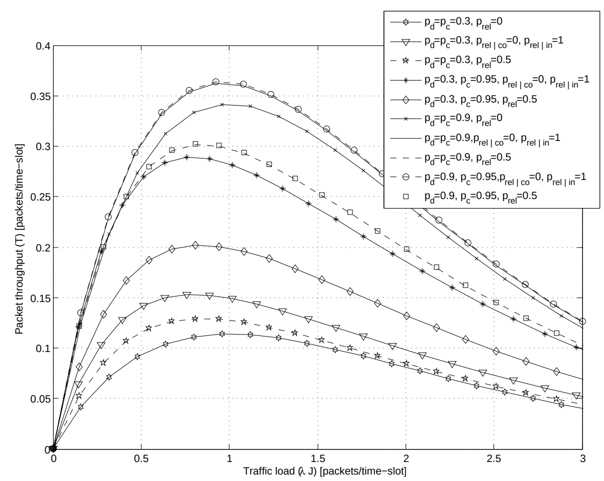

Figure 1: Throughput (T) vs. traffic load (λJ) for a 16-user S-ALOHA protocol with different relaying strategies and different decoding probabilities.

Fig.2 displays the results for the throughput (T) vs access-delay (D) of a S-ALOHA system that uses a relaying scheme with perfect packet error detection and decoding probabilities given by $p_d = 0.4$ and $p_c = 0.8$. In the same figure we present the results for the infinite user scheme, and for the Markov and Bernoulli models. All the schemes have considered a uniform retransmission probability $p_r = 0.15$. As observed in the figure, the three methods achieve more or less the same maximum stable throughput, but they differ in the delay values. As mentioned in previous sections,

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the Markov model provides the better approximation to the backlog delay, whereas the queuing delay is better approximated by the infinite model.

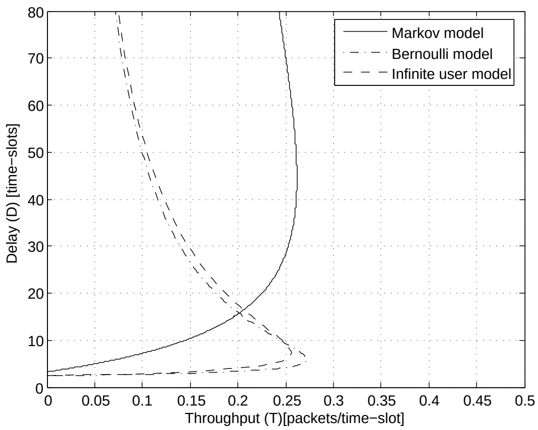

Figure 2: Throughput (T) vs. access delay (D) for a 16-user S-ALOHA protocol with the optimum relaying strategy using the infinite user scheme, and the Bernoulli and the Markov model.

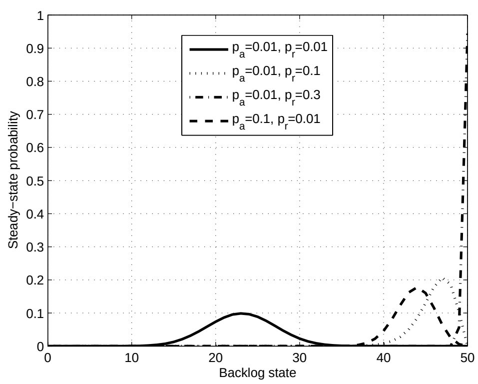

Figure 3: Backlog state distribution for a 50-user S-ALOHA protocol with the optimum relaying strategy $p_{rel|co} = 0$ and $p_{rel|in} = 1$, decoding probabilities $p_d = 0.4$ and $p_c = 0.8$, and different traffic parameters.

Fig.4 displays the results for the backlog state probability distribution using a Markov model for a 50-user S-ALOHA system with the optimum relaying scheme and decoding parameters $p_d = 0.4$ and $p_c = 0.8$. We have used different values for the arrival packet probability p_a and for the uniform retransmission probability p_r. Using the same system parameters, Fig.3 shows the drift functions for each particular case. A system with good stability properties will show a backlog state probability shifted to the left side of the figure. Also, a stable algorithm will show only one root for the drift function which has to coincide with the peak of the backlog state distribution. In Fig.4 it can be observed that the case with $p_a = 0.01$ and $p_r = 0.3$ presents a large number of backlogged users, therefore being the less stable case with a drift function also shifted towards high number of backlogged users. In comparison, the case with $p_a = 0.01$ and $p_r = 0.01$ provides better stability and, in average, lower number of backlogged users with a linear behavior of its drift function in Fig.3. The design of adaptive retransmission strategies that ensure stability represents a relevant future research topic.

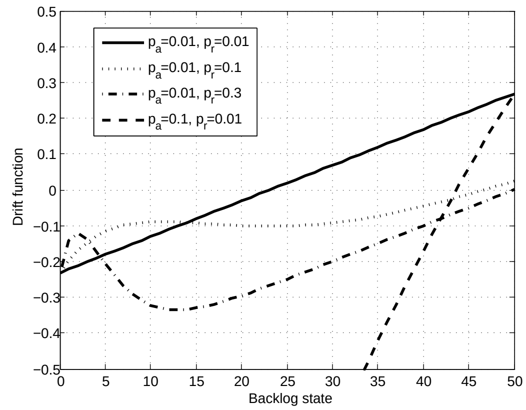

Figure 4: Drift function for a 50-user S-ALOHA protocol with the optimum relaying strategy $p_{rel|co} = 0$ and $p_{rel|in} = 1$, decoding probabilities $p_d = 0.4$ and $p_c = 0.8$, and different traffic parameters.

6. CONCLUSIONS

This paper presented a detailed analysis of a symmetrical S-ALOHA protocol with cooperative diversity. The analysis has yielded important conclusions about the boundaries and required parameters from the physical layer in order to achieve gains at the medium access control sublayer. The most important performance metrics of the protocol have been studied under different relaying strategies. It has been concluded that designing a relaying scheme that is aware of how good is the direct transmission greatly improves the performance of the protocol. It was shown that a relaying scheme with perfect error detection always improves the performance of the protocol, even if the cooperative relying phase is only deployed as a simple retransmission from the same user. Interesting future research topics include the analysis of the protocol considering correlated channel outcomes between the direct and cooperative phase, the study of imperfect collision detection, and the inclusion of MPR capabilities.

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