A Recursive Formula for Estimating the Packet Loss Rate in IP Networks

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ABSTRACT

We present a recursive formula for estimating the packet loss rate in IP networks. Specifically, we consider a single link whose capacity is shared dynamically by elastic data flows, each characterized by some peak rate, and derive the formula from the steady-state distribution of the number and peak rates of ongoing flows. The result is particularly useful for dimensioning ISP backhaul networks that aggregate the traffic of various types of users.

Categories and Subject Descriptors

G.3 [**Probability and statistics**]: Queueing theory

Keywords

Multirate systems, Kaufman-Roberts formula.

1. INTRODUCTION

The teletraffic theory derived from the Erlang formula is not applicable to IP networks whose resources are shared dynamically by elastic data flows. New results are required to predict key performance metrics (throughput, delay, loss) with respect to network capacity and traffic characteristics (intensity, peak rates). A practically interesting case is that of an ISP that aggregates the traffic of a high number of DSL and FTTH users, with peak rates ranging from 500 kbit/s to 100 Mbit/s, and needs to predict the maximum load sustainable by its backhaul network for some target packet loss rate.

In this paper, we focus on a single link whose capacity is shared by various types of elastic data flows. Specifically, we characterize each flow by some peak rate that is typically equal to the speed of the user's access line. Assuming that link capacity is shared according to balanced fairness [1], a recursive formula has been derived in [2] for evaluating the throughput of each flow. This formula is the analogue of the Kaufman-Roberts formula that has proven very useful for dimensioning circuit-switched networks [3, 4]. We here adapt the formula to derive some simple, conservative estimation of the packet loss rate.

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2. MODEL

Consider a link of capacity C shared by elastic data flows. Each flow is characterized by some peak rate. Specifically, we consider N types of flows, referred to as *classes*. Each class-i flow has peak rate c_i. Class-i flows arrive according to a Poisson process and leave the system once some random volume of data has been transferred. We denote by α_i the associate traffic intensity, defined as the product of the flow arrival rate by the mean flow size, and by $\rho_i = \alpha_i/C$ the corresponding load. The overall link load is given by:

$$\rho = \sum_{i=1}^{N} \rho_i.$$

Let x_i be the number of class-i flows. The evolution of the system state $x = (x_1, \dots, x_N)$ depends on the way link capacity is shared between ongoing flows. We assume that this sharing realizes balanced fairness [1]. Under the stability condition $\rho < 1$, the steady-state distribution of x is then insensitive to the flow size distribution beyond the mean and given by:

$$\pi(x) = \pi(0)\Phi(x)\alpha^x, \tag{1}$$

where Φ is the so-called balance function, defined by

$$\Phi(x) = \frac{1}{x!c^x} \quad \text{if } x.c \le C,$$

and

$$\Phi(x) = \frac{1}{C}\sum_{i=1}^{N} \Phi(x - e_i) \quad \text{otherwise.}$$

In the above expressions, the vectorial notation is that introduced in [2].

The packet loss rate is hard to estimate in practice since it depends on the complex packet-level dynamics induced by TCP. At the considered aggregation link, however, packet losses are most likely due to the flow-level dynamics, that is to the presence of a too high number of simultaneous flows. A simple yet reasonable appproximation then consists in considering the worst case where each ongoing flow is active at its peak rate. Thus in state x, the intensity of lost traffic is equal to $x.c - C$ if $x.c > C$ and to 0 otherwise; we deduce the following estimate for the packet loss rate:

$$L = \frac{\sum_{x:x.c>C}(x.c - C)\pi(x)}{\sum_x (x.c)\pi(x)}. \tag{2}$$

Although this expression can in principle be evaluated directly from (1), the computation is exponential in the number of classes N. The following recursive formula makes the computation linear in N.

3. RECURSIVE FORMULA

We assume as for the Kaufman-Roberts formula that the link capacity C and the rate limits $c_1, \dots, c_N$ are integers. In the rest of the paper, we denote by π the measure (1) obtained with $\pi(0) = 1$. For any integer n, let:

$$p(n) = \sum_{x: x.c = n} \pi(x).$$

We define:

$$p = \sum_{n > C} p(n) \quad \text{and} \quad q = \sum_{n > C} np(n)$$

In view of (2), we have:

$$L = \frac{q - Cp}{\sum_{n=1}^{C} np(n) + q}$$

For $n = 1, \dots, C$, we have

$$p(n) = \sum_{i=1}^{N} \frac{\alpha_i}{n} p(n - c_i), \tag{3}$$

with $p(0) = 1$ and $p(n) = 0$ for all $n < 0$. This is the analogue of the Kaufman-Roberts formula. It remains to calculate p and q, that can be derived from the values of $p(n)$ for $n = 1, \dots, C$:

PROPOSITION 1. *We have:*

$$p = \sum_{i=1}^{N} \frac{\rho_i p_i}{1 - \rho}, \quad q = \sum_{i=1}^{N} \frac{\rho_i (q_i + c_i(p + p_i))}{1 - \rho}, \tag{4}$$

where for all $i = 1, \dots, N$,

$$p_i = \sum_{C - c_i < n \le C} p(n), \quad q_i = \sum_{C - c_i < n \le C} np(n).$$

Proof. Using the fact that, for all x such that $x.c > C$,

$$\pi(x) = \sum_{i=1}^{N} \rho_i \pi(x - e_i),$$

we obtain:

$$\begin{aligned} p &= \sum_{n > C} \sum_{x: x.c = n} \pi(x), \\ &= \sum_{n > C} \sum_{x: x.c = n} \sum_{i=1}^{N} \rho_i \pi(x - e_i), \\ &= \sum_{i=1}^{N} \rho_i \sum_{n > C} p(n - c_i), \\ &= \sum_{i=1}^{N} \rho_i (p + p_i), \end{aligned}$$

and:

$$\begin{aligned} q &= \sum_{n > C} n \sum_{x: x.c = n} \pi(x), \\ &= \sum_{n > C} n \sum_{x: x.c = n} \sum_{i=1}^{N} \rho_i \pi(x - e_i), \\ &= \sum_{i=1}^{N} \rho_i \sum_{n > C} np(n - c_i), \\ &= \sum_{i=1}^{N} \rho_i \sum_{n > C} \left((n - c_i) p(n - c_i) + c_i p(n - c_i)\right), \\ &= \sum_{i=1}^{N} \rho_i (q + q_i + c_i(p + p_i), \end{aligned}$$

from which (4) follows. □

4. APPLICATION

Consider $N = 10$ flow classes with peak rates $1, 2, \dots, 10$ and equal traffic intensities. The following figure shows the maximum sustainable load ρ with respect to the link capacity C for various target loss rates:

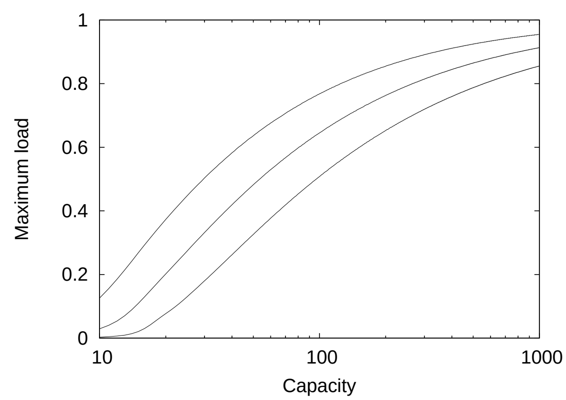

Figure 1: Maximum load with respect to capacity for target loss rates 5%, 1% and 0.1%, from top to bottom.

5. REFERENCES

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