Comparison of Steady-State Methods Computing Markov Modulated Fluid Models

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1. INTRODUCTION

The Markov-modulated fluid models (MMFM) are widely used in networks modeling. They assimilate the network traffic to a fluid generated by sources at a rate controlled by a background Markov process. Bursts of data are considered as continuous fluid flows and variations of cell inter-arrival times are assumed too small compared with the burst times for traffic evolving periodically in different timescales.

The buffer content distribution in fluid models is governed by a linear differential system that can be solved using various techniques such as spectral analysis, Laplace transforms, orthogonal polynomials or recurrence relations. The purpose of this work is to compare between the resolution techniques in term of computational complexity and accuracy. In literature, many articles were interested in the resolution techniques of the MMFM but too few of them focused on computing and complexity aspects, which are the main concerns of this work.

2. THE MATHEMATICAL MODEL

A fluid buffer is controlled by a background homogeneous Markov chain $(X_t)_{t\geq 0}$ described by a state space S with N states and an irreducible arbitrary infinitesimal generator $\mathbf{A} = (a_{ij})_{i,j\in S}$. Let Q_t be the amount of fluid in the buffer at the instant t and let d_i be the effective input rate of the buffer for the state i. When the system is stable, let us denote by $F_i(x) = P(Q_\infty \leq x, X_\infty = i)$. The distribution $F_i(x)$ is governed by the following differential system:

$$\mathbf{F}(x)\mathbf{A} = \frac{d\mathbf{F}(x)}{dx}\mathbf{D} \quad (1)$$

where $\mathbf{D} = diag(d_i, i \in S)$ and $\mathbf{F}$ is the vector of F_i $i \in S$.

3. TIME AND SPACE COMPLEXITY

We compare between three different methods: AMS (spectral method) [1], Scheinhardt (polynomial) [3] and Nabli (analytic) [2] on the basis of the implementation of the algorithms they use to obtain $\mathbf{F}(x)$.

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For the spectral and polynomial methods, the time complexity of the methods is about $\mathcal{O}(N^3)$ with a shorter execution time for AMS method thanks to the explicit computation of eigenvalues. The complexity of Nabli's method depends both on N and N_∞, which is the necessary number of iterations to reach the limit of the recursive sequence depending on the target accuracy ε. The total number of operations is then $\mathcal{O}(K.N_\infty(N_\infty + 1)/2)$. The number of iterations N_∞ grows when ε is small and when the system has a high traffic intensity. It depends also on $d = \min\{d_i, i \in S^+\}$. We studied the effect of the greatest and the smallest negative eigenvalues the execution time but experimental results proved that these parameters have no influence.

For AMS method, the global space complexity is about N^2 whereas for Scheinhardt, it is about $4N^2$. As for Nabli method, the total size of the necessary structures is about $N(2N_\infty - 1)$ which is usually greater than N^2.

4. STABILITY AND ACCURACY

The AMS method presents some unstability issues due to the expressions of combinations and small values in the denominators leading to overflows and errors. The unstable part of the Scheinhardt algorithm is the computation of eigenvalues. The analytic methods do not have particularly unstable parts because they compute sums and products of positive numbers smaller than 1. The analytic methods are stable and adapted for systems with a big number of sources.

We compared the precision of the methods and obtained overlapping curves for small N less accurate results for big values of buffer size and number of sources. We verified also that the stopping condition for the algorithm is sufficient to ensure that the limit of the considered sequence is reached towithin ε. We show that Kosten's approximation is valid for big values of x. It reduces computational complexity with an acceptable accuracy.

5. REFERENCES

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