

Investigating Feasible Tools for Swarm Pattern Transformation

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Abstract—The work reported in this paper is motivated by the need for developing swarm pattern transformation methodologies. Two methods, namely a macroscopic method and a mathematical method are investigated for pattern transformation. The first method is based on macroscopic parameters while the second method is based on both microscopic and macroscopic parameters. A formal definition to pattern transformation considering four special cases of transformation is presented. Simulations on a physics simulation engine are used to confirm the feasibility of the proposed transformation methods. A brief comparison between the two methods is also presented.

I. INTRODUCTION

PATTERN formation is a classical area of research in swarm robotics. The establishment and maintenance of patterns are considered in pattern formation studies. On the other hand, pattern transformation, which refers to the reconfiguration of swarm patterns, is a minimally considered area within swarm robotics. Transformation of patterns is an appropriate response to obstacles for unhindered motion. Patterns are reconfigured by repositioning all or a subset of agents in a swarm. A transformation may result in a change of geometric orientation of a pattern and relationships between interacting units in the pattern.

Work based on pattern transformation is reported by researchers. In [1], a stable virtual leader pattern transforms to a different pattern by the addition of a morphing force. Illustrations of transformation and mathematical notations for computation of forces in the pattern are presented. The transformation technique facilitates pattern change by allowing participating agents to find their own equilibrium. However, the morphing procedure for transforming pattern is not defined.

An algorithm reported in [2] is capable of transforming patterns in response to a command issued by a human operator. The command is issued to a single robot and causes a chain reaction in the neighbouring robots resulting in a global transformation. Pattern transformation from a parabola to a sine curve is illustrated. Though the notion of transforming patterns is presented, the transformation method remains unaddressed.

A relative distance versus orientation model for transformation is reported in [3]. The strategy involves varying the orientation value to globally transform a triangle

to a line formation. Though a positional transformation is not executed on all participating agents (one position remains unaltered), a global geometrical transformation is achieved. This strategy is specific to the scenario when a triangle to line transformation is performed.

The use of affine transformation, a mathematical tool, for transforming swarms patterns is presented in [4]. The target position of each member of the swarm is pre-determined by the mathematical transformation tool. The shortest path between the original and target positions is traced by considering the ant colony optimization [5] algorithm. Simulation results illustrate the transformation of a horizontal line pattern to a diagonal line pattern. Though a mathematical method is explored, a geometrical transformation between different shapes may not be possible using affine transformations.

In short, it is understood that researchers have not concentrated on investigating pattern transformation in swarm robotics. General methods for reconfiguring patterns are not emphasized either. The work reported in this paper is motivated towards the study of two methods that would enable geometric transformation. Firstly, a swarm macroscopic parameter oriented method based on macroscopic (group behaviour of a swarm pattern) parameters is proposed. Secondly, a mathematical tool based on macroscopic and microscopic (individual robot behaviour) parameters is proposed. The problem of transformation is formally stated and four special cases of transformation are considered. Experimental studies confirming the feasibility of the proposed methods are presented. The challenges in real time implementation of the proposed methods are discussed.

The remainder of this paper is organized as follows. Section II answers the question on whether an analytic or algorithmic method is suited for transformation. Section III states the problem of transformation, followed by four special cases of transformation. Section IV introduces the mathematical tool for transforming patterns and describes how it is applied on a pattern. Section V considers the challenges in implementing the method in real time and reports the merits and demerits of the method. Section VI concludes this paper and reports future work.

II. MICROSCOPIC VS. MACROSCOPIC

Before proceeding further it is necessary to distinguish between macroscopic and microscopic parameters of a swarm pattern.

Microscopic parameters are specific to robots constituting the swarm. In other words, microscopic parameters define the individual behaviours of swarm robotic agents. Microscopic parameters are reported in [6] [7]. It is noted that behaviour based pattern formation approaches tend to be microscopic in nature. Models that are microscopic might not be scalable.

On the other hand macroscopic properties of a group of robots are properties that affect the group behaviour of the system. They are abstract properties of a pattern, which when modified facilitates a change in the pattern. The control of a swarm of robot by varying abstract properties, namely variance and centroid is reported by Belta and Kumar [8]. Angular separation, linear distance, formation radius and rotation are macroscopic properties defined in a mathematical model for swarm systems proposed in [9].

There are atleast five main benefits of using macroscopic parameters. Firstly, implicit coordination, which refers to the coordination of a pattern comprising of mobile robots, need not be specified externally. Coordination is achieved as a result of varying the macroscopic properties. Secondly, Group behaviour definition, which refers to the collective behaviour of the group, is possible by controlling the macroscopic parameters. The individual behaviour of the units is affected by the variation in the macroscopic property. Thirdly, Adaptability, which refers to the ability of the group to adjust to change of internal or external circumstances, can be affected by macroscopic parameters. Fourthly, Stability, which refers to the factor by which the robot group maintains a pattern, can be controlled by using macroscopic parameter to dampen the propagation of errors. Fifthly, higher order parameters can control parameters of lower order.

It is evident that macroscopic parameters are advantageous when compared to microscopic parameters. To compare microscopic and macroscopic parameters, two pattern transformation methods are proposed in this paper. One method considers only macroscopic parameters while the second method considers both macroscopic and microscopic methods.

III. PATTERN TRANSFORMATION

Considering the fact that pattern transformation is minimally addressed and general methods for transformations are not investigated, it is necessary to define the problem of swarm pattern transformation.

Definition: Consider a pattern P with geometric relationships represented as G_P . The pattern P comprises of N robots such that their positions are given by $p_i(x_i, y_i)$ where $p_i \in \mathbb{R}^2$ and $i = 1, 2, \dots, N$. Pattern P transforms into the pattern Q with geometric constraints or relationships represented as G_Q . The pattern Q also comprises of N robots such that the position of the robotic agents is given by $q_i(x_i, y_i)$ where $q_i \in \mathbb{R}^2$ and $i = 1, 2, \dots, N$.

The function which enables the transformation of the pattern P to Q is given by $f(P) = Q$. In other words,

$$f(p_1(x_1, y_1), p_2(x_2, y_2), \dots, p_N(x_N, y_N)) = q_1(x_1, y_1), q_2(x_2, y_2), \dots, q_N(x_N, y_N).$$

The application of an inverse transformation function on the transformed pattern Q yields the pattern P , given

by $f^{-1}(Q) = P$. The transformation on the pattern also results in a transformation of the geometrical relationships from G_P to G_Q between the participating agents in the pattern.

Four cases of transformation based on the above definition are derived by imposing restrictions on the geometrical constraints.

Case 1: $G_Q = G_P$ after a transformation that involves repositioning all agents. This case is relevant when robotic agents in the pattern have repositioned, yet the geometrical pattern has not changed. Such a transformation is termed as Elementary transformation in this paper. This term also refers to those transformations very basic in nature. For instance, a swarm could be rotated with respect to its centroid or translated such that all robotic agents have repositioned themselves. Though the orientation of the pattern has changed, the configuration of the pattern remains unaltered. Mathematically, the case of elementary transformation would be such that $G_Q = G_P$ and $\forall i: p_i(x_i, y_i) \neq q_i(x_i, y_i)$.

Case 2: $G_Q = G_P$ after a transformation without repositioning all agents. This case considers the rotation or translation of the swarm with respect to some robotic agent whose position remains fixed. This case is also classified under Elementary transformation, yet repositioning of all agents has not occurred. Mathematically, this case of elementary transformation would be such that $G_Q = G_P$ and $\exists i: p_i(x_i, y_i) = q_i(x_i, y_i)$

Case 3: $G_Q \neq G_P$ after a transformation that involves repositioning all agents. This relates to the case when the geometrical constraints of the pattern have changed and a new pattern has emerged. It is termed a Geometric transformation. This concept is relevant when robotic agents in the pattern reposition to result in a geometry change. For instance, the shape of a swarm could be geometrically transformed from a polygon to a line. It is interesting to note that the scaling of a pattern would result in a geometric transformation, since the geometrical constraints are dissimilar in both cases. Mathematically, the case of geometrical transformation would be such that $G_Q \neq G_P$ and $\forall i: p_i(x_i, y_i) \neq q_i(x_i, y_i)$.

Case 4: $G_Q \neq G_P$ after transformation without repositioning all agents. This case considers the geometric transformation such that the position of one or more than one robotic agent remains fixed. It is classified under geometric transformation, yet repositioning of all agents has not occurred. Mathematically, the case of geometrical transformation would be such that $G_Q \neq G_P$ and $\exists i: p_i(x_i, y_i) = q_i(x_i, y_i)$.

Cases 1 and 2 relate to elementary transformation of the pattern. In these cases, the geometric constraint or relationship persists even after elementary transformation. Cases 3 and 4 consider geometric transformation. In these cases the geometrical relationships change after transformation.

The swarm model presented in [9] is chosen for the study of transformation methods in this paper. The model is chosen since it is derived mathematically and formulated by microscopic and macroscopic modeling. The macroscopic parameters of the model are extensively used in pattern formation and scaling of the pattern.

The following sections consider two methods for transformation, namely a macroscopic transformation method and a mathematical transformation method. Cases 1, 3 and 4 of transformation are considered in the transformation methods. To achieve geometrical transformation a series of operations are performed in both methods. Case 2 will be reported in a future paper.

IV. METHOD 1: MACROSCOPIC TRANSFORMATION

The first transformation method proposed in this section considers cases 1, 3 and 4 of transformation which is inclusive of elementary and geometric transformation applied on the swarm model.

Transformations of cases 1, 3 and 4 are achieved by varying the macroscopic parameter, namely the formation radius (along x and y axis) of the swarm model. It is interesting to note that a sequences of operations performed on the swarm model results in a transformation. The set of operations are:

1) *Rotation*: The initial step of rotation of the model is performed to achieve collision avoidance during the next step. A predefined angle offset is used to rotate the swarm. Though the robots are repositioned, the operation results in the same polygonal pattern with a different orientation from the former. Here, the concept of elementary transformation is introduced. Though all robots were repositioned in this operation, a geometric transformation is not evident since the shape of the pattern is retained. Though a geometric transformation is not evident, yet an elementary transformation of case 1 is achieved in this step.

2) *Macroscopic Parameter Operation*: Following a rotation operation, the macroscopic parameter is set to be modified. Deflating the model along the y-axis would result in a deformed polygonal pattern. The deflation of the model is performed by decrementing the magnitude of the formation radius along the y-axis. When deflation has reached its maximum value, the robotic agents have aligned themselves entirely along the x-axis. Maximum deflation is achieved when the formation radius value along any axis vanishes. An inflation operation along the other perpendicular axis simultaneously while deflating would result in a pattern with larger inter-linear distance between the agents (a measure for avoiding collisions). This variation is possible due to the notion of flexibility in rigid patterns.

3) *Further Rotation*: This step is performed to achieve equidistance between the participating agents. Though the

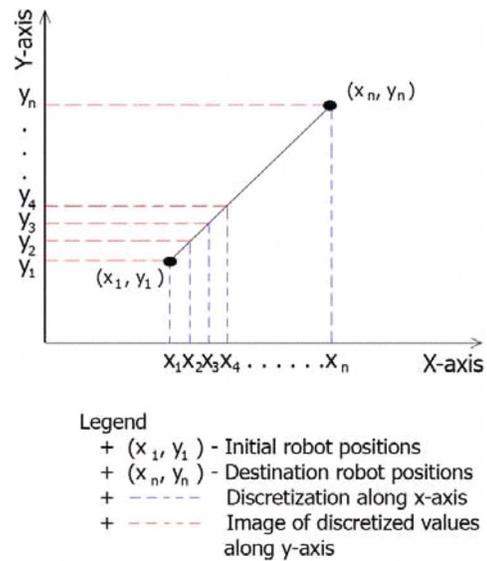


Fig. 1. Fundamental on discretizing an event. Domain values (on x-axis) are discretized yielding corresponding range values (on y-axis).

pattern has transformed its shape by this step, the participating agents are still governed by the rules of the swarm model. A corrective rotation measure would ensure that the agents are loosely equidistant.

V. METHOD 2: MATHEMATICAL TRANSFORMATION

The method proposed in this section considers case 3, which is achieved by using a mathematical transformation tool. Many mathematical tools are available for transformations which include stretching, rotating, reflecting and translating transformations. The linear fractional transformation is one such readily available mapping function that maps a set of points from one plane to another.

The transformation is given by $f(z) = \frac{az + b}{cz + d}$, where z , a , b , c and d are complex numbers satisfying $ad - bc \neq 0$. The linear fractional transformation is also known as a Moebius transformation [10].

The transformation functions are applied onto the swarm pattern which is polygonal in shape. Since the vertices of the polygonal pattern lie on the circle circumscribing the pattern [9], a circle to line and a line to circle transformations of the complex plane are used. However, the transformation function cannot be applied directly to the multi-robot pattern. This is due to the fact that the multi-robot pattern is defined on a global frame of reference while the mathematical function is applicable on the local frame of reference. Hence, the sequence of operations performed on the multi-robot pattern is:

1) *Transformation from global to local frame of reference*: The frame of reference of the multi-robot is temporarily transformed from the global to a local frame. The local frame of reference considered is such that the circumscribing circle is divided into four equal quadrants. Hence the centroid of the pattern lies on the origin position of the local frame.

2) *Discrete Transformation*: This step applies the mathematical transformation function on the multi-robot model. The transformation of a circle to a line is obtained from $w = i \frac{(1-z)}{(1+z)}$. Applying the equation on the Euclidean plane, the mapping function is deduced as $\left(\frac{2y}{1+x^2+y^2}, \frac{1-x^2-y^2}{1+x^2+y^2} \right)$. The transformation from a line to a circle is applied by considering a special case of the Moebius transformation. The transformation $w = \frac{1}{z}$ maps every straight line or circle onto a circle or straight line. It is also known as the inversion in the unit circle or reciprocal transformation. Applying the equation on the Euclidean plane, the mapping function is otherwise written as $\left(\frac{x}{x^2+y^2}, \frac{y}{x^2+y^2} \right)$. The destination coordinates obtained by the mathematical functions are the coordinates to which individual robot agents need to reposition while the pattern transforms. However, it is evident that these transformation functions are discrete in nature yielding only one set of destination coordinate rather than sub-goals or intermediate destination coordinates.

3) *Transformation from local to global frame of reference with magnification*: The destination coordinates are obtained on the local frame of reference. Hence, the local frame needs to be shifted to the global frame of reference. Since the mathematical functions considered in step (ii) are reducing functions (destination coordinates reduce the span of the pattern), a magnification ratio is used in the local frame to achieve gain in the destination coordinates.

4) *Path planning by discretization*: Since the achieved destination coordinate set is discrete, the major challenge in repositioning agents is to plan their path to the destination coordinates. In this paper, the technique adopted to reposition robots is along straight line trajectories without collisions. The straight line path between the agent and its estimated destination is discretized. Figure 1 illustrates the straight line discretization process. The domain values of the path are sliced to extrapolate the range values. This relates to the underlying principle of Discrete Event Simulations (DEVS). The potential of DEVS in path planning for robots is reported in [11].

VI. SIMULATION STUDIES

Simulation studies were pursued to validate and visualize the proposed mathematical approach for pattern transformation. The feasibility of the proposed approach was validated on the Processing [12] and Traer Physics [13] environment. Processing is an open source programming language and environment enabling visualizations for learning and prototyping. Traer Physics is a particle physics simulation engine for Processing.

The traer physics library has provisions for modeling a particle system, particles, springs and attractive or repulsive forces [13]. The particle system enables prototyping particles and forces. Particles represent objects having four properties, namely mass, position, velocity and age. Particles can be stationary or dynamic in an environment. Springs can connect two particles and prevent collisions. Springs are characterized by three properties, namely rest length, strength and damping. Attractions or repulsions pull particles together or apart and have two properties, namely strength and minimum distance. The simulations reported in this paper employ particle system, particles and attractive or repulsive forces.

The swarm pattern is designed as particles in an open environment with forces, namely macro and micro level forces of attraction and repulsion acting on the pattern. The macro level forces include repulsive forces, which act on the centroid of the swarm. The forces of repulsion are generated from obstacles (modeled as forces) in the environment. All robotic agents align themselves around the centroid with respect to the forces forming a virtual structure polygonal pattern. Obstacles in the path of the pattern are detected by the computation of the net force acting on the group of robots. Beyond a maximum threshold value of force, the pattern reacts appropriately by transforming its shape to avoid obstacles. The pattern regains its polygonal shape when the net force acting on the centroid decreases below a minimum threshold value, such as when the pattern has escaped from obstacles. The inter-agent bonding force and the forces of interaction with the centroid contributed to the micro level forces. The pattern generates a propulsive force to trace paths against repulsive forces.

The experimental setup comprised a tunnel through which the swarm had to displace. The walls of the tunnel generated repulsive forces and acted as the obstacle. The swarm initiated its motion from the left of the tunnel and aimed to reach a goal beyond the tunnel on the right side. While attempting to pass through the tunnel, the swarm transformed its shape to avoid the obstacle. Both transformation methods discussed in Section IV and V were implemented.

TABLE I
PRE DEFINED ROTATION VALUES & ESTIMATED COLLISIONS

ANGLE OFFSET	15°	30°	45°	60°
NO. OF ROBOTS				
3	-	-	-	1
4	-	-	2	-
5	-	1	-	-
6	-	3	-	3

The macroscopic method of Section IV which consists of a sequence of three operations was implemented. Firstly, the swarm model was rotated to avoid collisions while deflating. Table I illustrates the different rotation angles that were applied on the swarm. Higher value angles resulted in collisions for most patterns. Angles less than 15 degrees proved effective for collision avoidance.

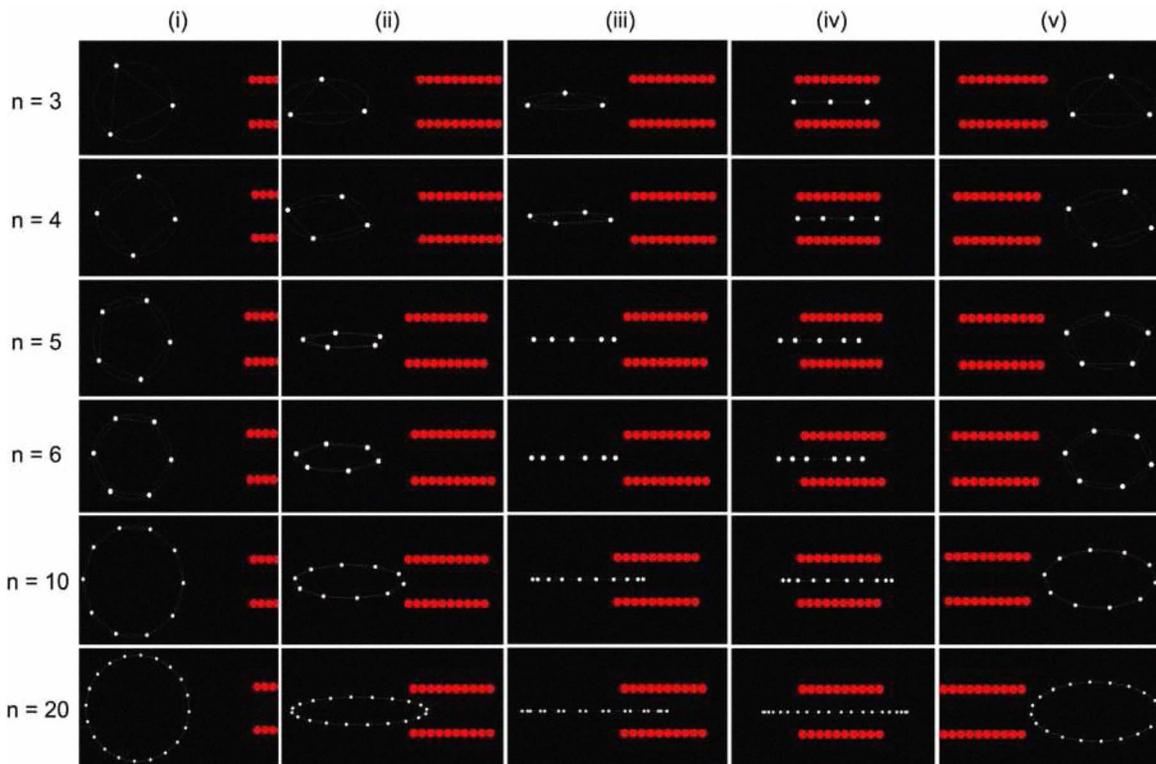


Fig. 2. Simulation results for transformation. (i) Rotated swarm model for various number of robots, (ii) Deflation of the model along the y – axis (For $n = 10$ and 20 , inflation along x – axis performed), (iii) Transformed pattern without corrective rotation measure, (iv) Transformed pattern after corrective rotation measure is applied (Except for $n = 3$ and 4 , since equidistance is more or less achieved), (v) Inverse transformation by inflation back to original pattern.

Secondly, the macroscopic parameters were varied. This variation resulted in deflation or inflation of the pattern (along x or y axis). Thirdly, a corrective rotation was applied to avoid agents from colliding against each other. Hence by transformation of the pattern, the swarm successfully displaced itself through the obstacle path. Figure 2 is a snapshot of the simulation studies for $n = 3$ to 6 , 10 and 20 robots transforming as per the first method

It is observed that a geometric transformation is obtained by performing a sequence of operations which consists of an elementary transformation. A regular pattern transforms to an irregular polygonal pattern while reconfiguring.

The mathematical method of Section V which consists of a sequence of four steps was implemented. Firstly, the swarm pattern was transformed from the global to a local frame such that the centroid of the pattern lies on the origin of local frame of reference. Hence, the pattern is equally spanned over the four quadrants in the local frame of reference, which was necessary for proper implementation of the transformation functions.

Secondly, the discrete transformation function was applied on the microscopic property, namely the position coordinates of the individual robots in the pattern. The transformation from a circle to line was employed in order for the pattern to pass through the tunnel in the environment. Beyond the obstacles, the transformation from a line to circle was employed. Both transformation operations yield a set of discrete destination coordinates for each robot.

Thirdly, transformation from the local to global frame of reference was performed. The destination coordinates obtained in the local frame of reference were such that the pattern radius is reduced. Hence a magnification of the coordinates in the local frame was performed and further mapped on to the global frame of reference.

Fourthly, path planning by discretization was executed. This step is essential to determine the sub goals or intermediate position coordinates. Repositioning the robots to sub-goals or intermediate coordinates is a computationally expensive process. Straight line trajectories from agents to calculated destination coordinates without collisions were considered in the work reported in paper. Figure 3 is a snapshot of the simulation studies for 17 robots that transform shape as per the second method.

It is observed that the circle to line transformation yielded a pattern in which robotic agents were loosely equidistant. The line to circle transformation employing the reciprocal transformation yielded a polygon irregular in nature. This was due to the nature of the reciprocal transformation which was anticipated.

It was observed that in both methods, the swarm successfully displaced itself through the obstacle path by transforming shape. The transformed patterns were loosely equidistant. Collision avoidance between repositioning agents is not implicitly guaranteed. Hence, atleast one operation in both methods ensured collision avoidance. The geometrical transformation of a circle to a line in both cases was achieved by transforming a regular polygonal pattern to an irregular pattern by repositioning agents. The observations

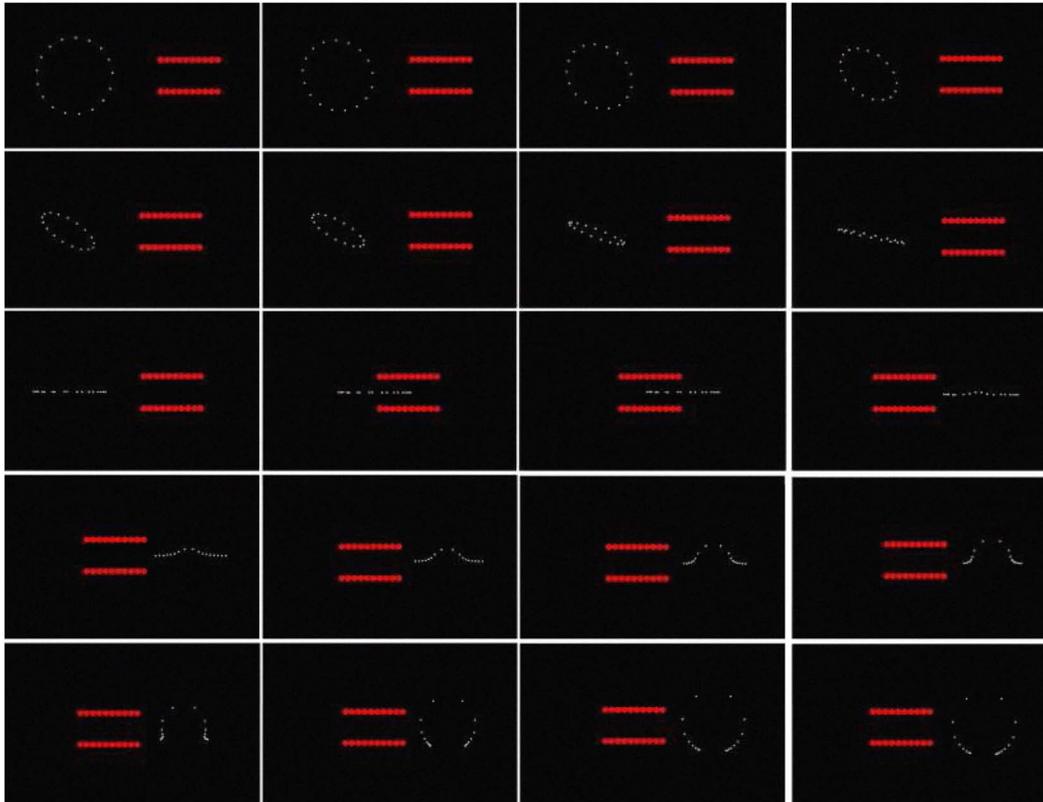


Fig. 3. Simulation studies for transformation in two stages. Firstly, a circle to a line transformation (First three rows excluding the fourth sequence in the third row). Secondly, a line to circle transformation (Fourth sequence of the third row and last two rows).

are consistent with the theoretical studies in Section IV and V and according to the authors expectation.

The time taken to transform a pattern in the two transformation methods for different number of robots in the configuration was measured. This experiment was carried out for different number of robots varying from 3 to 25 and keeping the initial formation radius of the swarm a constant. Figure 4 is a graph based on the results obtained from simulation and was plotted using MATLAB. It shows the time taken for transformation versus the number of robots in the pattern.

The average time taken to transform a pattern in the first transformation method was computed as 20.35 seconds. In the second transformation method the average time was noted as 21.40 seconds, slightly higher than the first method. It is understood from the graph that the mathematical transformation method employing both macroscopic and microscopic parameters is not advantageous for small number of robots. For smaller number of robots, the robots in the mathematical method traverse more distance within the pattern. As the number of robots increase, the distance traversed by a robot within the pattern decreases. The mathematical method performs better than the macroscopic method for higher number of robots. As the number of robots increase, the mathematical method tends to be effective since the time taken to transform decreases. However, the first method performs consistently for any number of robots.

In summary, the simulation studies confirm the feasibility of the proposed methods. The transformation cases discussed

in Section III are considered in the transformation method. A brief comparison between the method employing only macroscopic parameters and the method employing both microscopic and macroscopic parameters is presented.

VII. COMPARING THE METHODS

The transformation methods presented in this paper are feasible methods for reconfiguring patterns. However, it is noted that the mathematical method employing Moebius transformation is not strictly macroscopic in nature. The microscopic properties of the swarm units are taken into consideration. For example, path planning of individual robots is necessary to reposition the robots. The method is not advantageous for small number of robots in the pattern. Moreover the mathematical transformation method is a discrete transformation method. Hence, discretizing and quantizing the path to reposition are required. This is a computationally expensive process unsupported and unwarranted on minimal processing swarm units. Therefore global planning is required thereby increasing wireless communication overheads. A high bandwidth for communication and synchronized and consistent communication with a centralized unit are challenges in realizing the mathematical method in real time.

On the other hand, the macroscopic method considers the group behaviour of the swarm system. Hence, individual robots need not be addressed, eliminating microscopic parameter operations. For example, transformation in the first

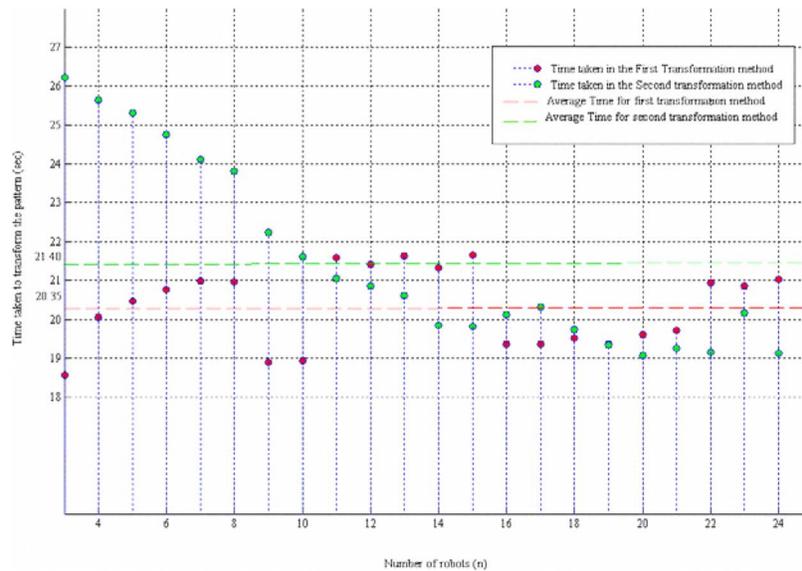


Fig. 4. Graph drawn based on number of robots in the pattern vs. time taken to transform the pattern for the two transformation methods.

method is obtained by a sequence of operations performed on the entire swarm pattern rather than considering individual robot path planning. The macroscopic method is observed to be consistent in the time taken for transformation, and is also a continuous method thereby reducing computations for individual robot planning. This method would hence offer better synchronization between the swarm units since local planning is sufficient. Hence wireless communication overheads are relatively less compared to the mathematical method.

By implementing a macroscopic method in a real time robot system, planning overheads for individual robots could be minimized. However, a mathematical transformation function is advantageous since it belongs to an analytical class of tools and mathematical analysis is possible. Therefore, this approach will also be explored in future work.

VIII. CONCLUSIONS

In this paper, the need for investigating feasible methods for transforming patterns is considered. Two transformation methods, namely a macroscopic method and a mathematical method are proposed. The first method is a macroscopic parameter method while the second considers both, macroscopic and microscopic parameters. A formal definition to transformation is presented with four special cases of transformation. Elementary and geometrical transformations are considered by repositioning agents. A swarm model reported in [9] is used to study the transformation methods. Transformation using both methods is achieved by a sequence of operations performed on the swarm pattern. The proposed methods are implemented on the Processing and Traer Physics environment. A comparison between the two methods considering transformation time from one pattern to another is presented. The simulation studies confirm the feasibility of the proposed methods.

Future work will include the real time implementation of the proposed transformation methods on a swarm robot

system. The challenges in mapping simulation studies to real time robot systems will be studied. Efforts will be made to explore continuous mathematical transformation methods which are expected to minimize individual robot path planning.

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