Gathering Two Stateless Mobile Robots Using Very Inaccurate Compasses in Finite Time

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***Abstract*—This paper considers two asynchronous identical mobile robots in an environment devoid of any landmarks or common coordinate system. The robots, executing their own instance of the same algorithm, must cooperate to end up at the exact same location (not predetermined) within a finite time.**

The problem, known as gathering, is the simplest form of spontaneous agreement that can be reached between the robots. This simple problem is however notoriously impossible with two such robots. Surprisingly, the problem was shown to be solvable for three robots or more, adding a weak assumption to help break any potential symmetry in the system. Prior work has shown that the problem could be solved by letting each robot have access to some compass, provided that the divergence between the compasses is at most $45°$. The question remained open, however, as to whether the problem could still be solved with a larger divergence.

In this paper, we present a distributed algorithm that solves the gathering problem with two asynchronous robots, when their compasses can differ by any angle less than $180°$, which is obviously the largest divergence for which the compasses can still bring any useful information.

I. INTRODUCTION

During the past decade, increasingly more research has been focussing on the coordination and self-organization of mobile robot systems involving multiple simple robots working together, rather than a single highly-complex one. This view is motivated by a variety of reasons, including reduced manufacturing costs, increased fault resilience, improved overall maneuverability, or simply better polyvalence of the system. The challenge is however to ensure enough coordination so that the multiple robots appear as a single coherent system rather than as a set of independent entities.

The coordination of groups of robots has been studied from various perspectives (see [12] for a survey).

In Swarm intelligence (e.g., [9], [10], [11]), the approach consists in setting simple behaviors to individual robots and studying the global behavior that emerges from the interactions between these robots, for instance, inspired by the behavior of social insects. This approach is also called *behavior based.*

An other kind of approach to the problem of studying multi-robot systems, is that of Kawauchi, Inaba, and Fukuda [13] who have studied dynamically reconfigurable robotic system, consisting of several cells that can physically detach and combine autonomously according to the assigned task. In the same perspective is the work of Walter et al. [16] in metamorphic robot system.

Some of the studies addressed the problem from engineering stand point. For instance, the work of Belta et al [17], in planning and control of robots' motions.

Sometimes [14], coordination is also seen as an optimization problem and addressed by metaphors such as a free market economy. In networked robotics (e.g., [15]), the problem is sometimes expressed as a global control system that relies on tight real-time network guarantees. In distributed algorithms and self-stabilizing systems (e.g., [5], [8]), the aim is to provide a formal representation of multi-robots systems and coordination problems in which provably correct solutions can be developed and verified. All approaches bring light on different parts of the puzzle but much remains to be done before the pieces can actually be put together.

In this work, we consider the latter approach and focus on the interactions between the robots. We consider a system in which the robots are represented as points moving in a plane. Each robot executes its own instance of the same algorithm which consists of repeatedly (1) observing the environment, (2) computing a destination, and (3) moving toward it. Each action can take arbitrarily long, and a robot can also be idle for an arbitrary duration. The robots are not synchronized, they are identical (i.e., the algorithm cannot distinguish them), and they do not retain any information between activations. This last assumption is useful both for memory management and because an algorithm designed for such robots in inherently self-stabilizing.[1]

We focus on an agreement problem called gathering (also known as rendez-vous or point formation) in a system with no agreement on a global coordinate system and in the absence of any landmarks in the environment. In short, the problem requires that the robots, initially located at random locations, move in such a way that they eventually end up at the exact same location, not determined a priori. The algorithm must ensure that the final configuration is obtained within a finite number of steps, from any initial situation and in every possible execution. While being very simple to express, this problem has the advantage of retaining the inherent difficulty

[1] Self-stabilization is the property of a system which, starting in an arbitrary state, always converges toward a desired behavior [2].

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of agreement, namely the question of breaking symmetry between the robots. Among other things, being able to gather at a point means that the robots have the inherent ability to agree on a common origin.

In their SYM model [8], referred to as semi-synchronous model, Suzuki and Yamashita proposed an algorithm to solve the gathering problem deterministically in the case where robots have unlimited visibility. For a system with two robots, they have proven that it is impossible to achieve the gathering of two *stateless* mobile robots that have no common orientation in a finite time.

Prencipe [6] has shown that gathering stateless robots cannot be done deterministically without some additional assumptions. For instance, gathering is possible if robots share a common direction, as given by *perfect* compasses [3]. Similarly, if robots can detect multiplicity (i.e., count robots that share the same location) gathering is possible for *three or more* robots [1].

Our work is motivated by the pragmatic standpoint that (1) compasses are error-prone devices in reality, and (2) multiplicity detection allows for gathering in situations with more than two robots.

In our recent work, we have proposed a self-stabilizing algorithm with which two asynchronous robots can gather in finite time using inaccurate compasses with a divergence of as much as $45°$ [7]. [2] The question was still open whether it is also possible to solve the problem when the compasses diverge by a larger angle? We addressed the question, and the answer is yes.

In this paper, we propose a self-stabilizing algorithm with which a pair of asynchronous mobile robots can gather in finite time, provided that their compasses diverge by less than $180°$. This is the main contribution of this paper, and this closes the question since it is obviously impossible to do better than this.

The remainder of this paper is organized as follows. Section II describes the system model and the basic terminology. In Section III, we describe our gathering algorithm based on compass inconsistencies and give a tight bound. Finally, Section IV concludes the paper.

II. System Model and Definitions

A. System Model

In this paper, we consider the CORDA model of Prencipe [5], which is defined as follows. The system consists of a set of autonomous mobile robots $\mathcal{R} = \{r_1, \cdots, r_n\}$. A robot is modelled as a unit having computational capabilities, which can move freely in the two-dimensional plane. In addition, robots are equipped with sensor capabilities to observe the positions of other robots, and form a local view of the world. The robots are modelled and viewed as points in the Euclidean plane.[3] The local view of each robot includes a unit of length, an origin, and the directions and orientations of the two x and y coordinate axes as given by a compass.

[2]A similar result was obtained and presented by Imasu et al. [4] at a domestic workshop in Japan.

[3]We assume that there are no obstacles to obstruct vision. Moreover, robots do not obstruct the view of other robots and can "see through" other robots.

We further assume that the robots are *stateless*, meaning that they keep information neither on previous observations nor on past computations.

The cycle of a robot consists of four states: Wait-Look-Compute-Move.

- *Wait*. In this state, a robot is idle. A robot cannot stay permanently idle (see Assumption 2) below. At the beginning all robots are in Wait state.
- *Look*. Here, a robot *observes* the world by activating its sensors, which will return a snapshot of the positions of all other robots with respect to its local coordinate system.
- *Compute*. In this state, a robot *performs* a local computation according to its algorithm. The algorithm is the same for all robots, and the result of the *compute* state is a destination point.
- *Move*. The robot *moves* toward its computed destination. The robot moves toward the computed destination, but the distance it moves is unmeasured; neither infinite, nor infinitesimally small (see Assumption 1). Hence, the robot can only go towards its goal, but the move can end anywhere before the destination.

Finally, in the model, there are two limiting assumptions related to the cycle of a robot.

Assumption 1: It is assumed that the distance traveled by a robot r in a move is not infinite. Furthermore, it is not infinitesimally small: there exists a constant $\delta_r > 0$, such that, if the target point is closer than δ_r, r will reach it; otherwise, r will move towards it by at least δ_r.

Assumption 2: The time required by a robot r to complete a cycle (Wait-Look-Compute-Move) is not infinite. Furthermore, it is not infinitesimally small; there exists a constant $\epsilon_r > 0$, such that the cycle will require at least ϵ_r time.

B. Definitions

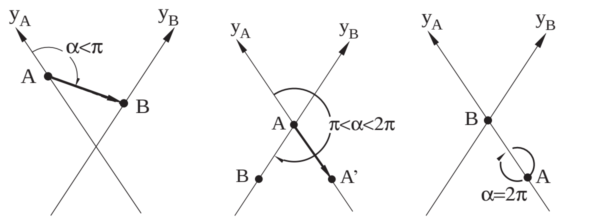
Definition 1 (Relative north): A relative north $\overrightarrow{N_A}(t)$ is a vector that indicates a north direction for some robot A at some time t.

Definition 2 (Inaccurate compasses): Informally, compasses of a pair of robots A and B are *inaccurate* by some angle θ iff., the absolute difference between the north of A and B, $\overrightarrow{N_A}$ and $\overrightarrow{N_B}$ is at most θ at any time t. In addition, the north directions of A and B are invariant over time. The special case when $\theta = 0$ represents perfect compasses.

Formally, compasses are *inaccurate* by some angle θ iff., the following two properties are satisfied:

1) Inaccuracy: $\forall A, B \in \mathcal{R}, \forall t, |\measuredangle \overrightarrow{N_A}(t)\overrightarrow{N_B}(t)| \leq \theta$,
2) Invariance: $\forall A, \forall t, t', \overrightarrow{N_A}(t) = \overrightarrow{N_A}(t')$.

In the sequel, we consider compasses that diverge by an angle strictly less than $180°$. Obviously, compasses that could differ by an angle of $180°$ would provide no information at all. Hence, the compasses considered in this paper represent the worse inaccuracy in compasses. Nevertheless, we will show that we can solve the gathering of two stateless robots.



(a) $0 < \alpha \leq \pi$, then A moves directly to B. :**Direct move**

(b) $\pi < \alpha < \pi + \theta$, then A moves by distance $\|\overline{AB}\|$ to A': **Side move south**.

(c) $\pi + \theta < \alpha \leq 2\pi$, then A does not move: **No move**.

Fig. 1. Principle of the algorithm.

TABLE I
COMBINATION OF MOVES FOR ROBOTS A AND B ALLOWED BY THE ALGORITHM WHEN $\theta < \pi$. THE TABLE IS SYMMETRICAL.

	Robot A		
Robot B	$0 < \alpha \leq \pi$ *(direct move)*	$\pi < \alpha < \pi + \theta$ *(side move south)*	$\pi + \theta < \alpha \leq 2\pi$ *(no move)*
$0 < \alpha \leq \pi$ *(direct move)*	◯	◯	◯
$\pi < \alpha < \pi + \theta$ *(side move south)*	◯	◯	◯
$\pi + \theta < \alpha \leq 2\pi$ *(no move)*	◯	◯	not applicable

III. GATHERING WITH VERY INACCURATE COMPASSES

In this section, we provide an algorithm for solving the gathering of two asynchronous stateless mobile robots when their compasses diverge by an angle $\theta < \pi$.

A. Algorithm Overview

Consider a local x-y coordinate system where, the positive y-axis points North and hence the positive x-axis points East. Let also the location of the robot be the origin of its local coordinate system.

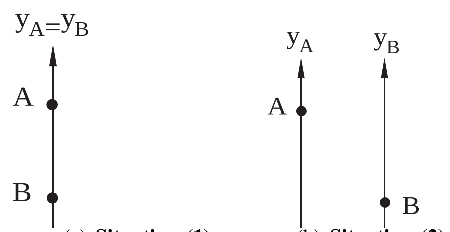
Let A be some robot, and let B be the position at which the other robot is located. We denote by α the angle between the y-axis of robot A, namely y_A and the segment $\overline{AB}$ in clockwise direction. That is, $\alpha = 0$ when B is on the positive y_A axis and $\alpha = \pi/2$ when B is on the positive x-axis of robot A. Finally, let θ be the difference in north direction indicated by the two local coordinate systems of robot A and B. In our algorithm, we assume that $0 \leq \theta < \pi$. Then, robot A decides its movement as follows:

- If $0 < \alpha \leq \pi$, then robot A moves directly on the segment $\overline{AB}$ to B. We refer to this move as ***direct move***.
- If $\pi < \alpha < \pi + \theta$, then robot A moves towards its south by the distance $\|\overline{AB}\|$. We will refer to this move as ***side move south***.
- If $\pi + \theta < \alpha < 2\pi$, then robot A does not move. We refer to this move as ***no move***.

The pseudo-code is given in Algorithm 1, and Table I summarizes the different moves of robot A and B.

Algorithm 1 Gathering two of asynchronous robots, when compass divergence $\theta < \pi$.

if (r sees only itself) **then** {*gathering terminated*}
  Do_nothing();
else
  B := position of the other robot B;
  y_A := y-axis of robot A;
  α := angle between y_A and $\overline{AB}$ in clockwise direction;
  if ($0 < \alpha \leq \pi$) **then** {direct move}
    robot A moves to robot B;
  else if ($\pi < \alpha < \pi + \theta$) **then** {side move south}
    robot A moves toward its south by distance $\|\overline{AB}\|$;
  else if ($\pi + \theta < \alpha \leq 2\pi$) **then** {no move}
    Do_nothing();
  end if
end if



(a) **Situation (1)**. (b) **Situation (2)**.

Fig. 2. Situations of A and B where $\theta = 0$.

B. Description of Situations

In this section, we define the different possible situations of robot A and B, when their compasses are inconsistent by $0 \leq \theta < \pi$. Without loss of generality, we consider that the north of robot B, denoted by y_B is always on the right hand side of the north of robot A, denoted by y_A. Thus, we define the following 10 situations[4]:

- **Situation (1):** $y_A = y_B$, and robots A and B are located on the same y-axis (refer to Fig. 2(a)).
- **Situation (2):** y_A of robot A and y_B of robot B are parallel. Besides, A and B are not located on the same y-axis (refer to Fig. 2(b)).

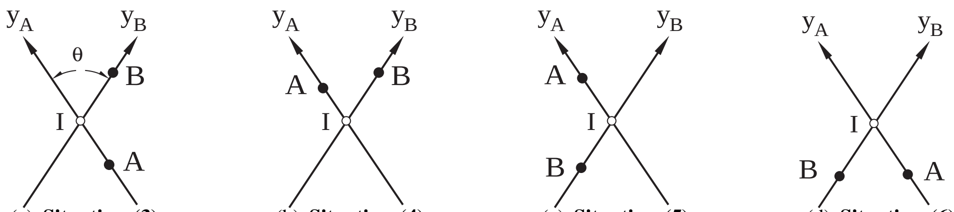
Situations (1) and (2) refer to cases when θ is equal to zero. In the following cases, we consider that θ is different than zero. Let I be the intersection of y_A and y_B. Then, four cases arise when both A and B are not at I.

- **Situation (3):** A is below I, and B is above I (see Fig. 3(a)).
- **Situation (4):** Both A and B are above I (see Fig. 3(b)).
- **Situation (5):** A is above I, and B is below I (see Fig. 3(c)).
- **Situation (6):** Both A and B are below I (see Fig. 3(d)).

Finally, we distinguish the following four cases (refer to Fig. 4) when either robot A or B is at I.

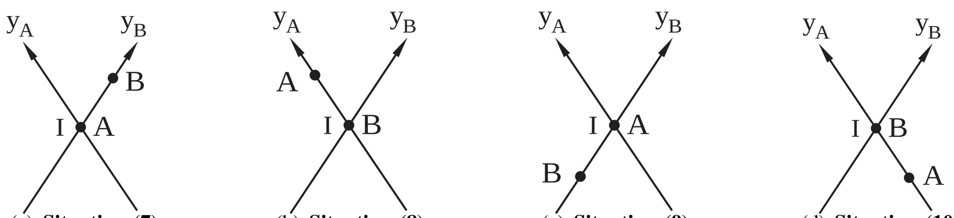
- **Situation (7):** A is at I and B is above I.
- **Situation (8):** A is above I and B is at I.

[4]If the north of robot B is on the left hand side of the north of robot A, then by symmetry we will have the same 10 situations.



(a) **Situation (3)**. (b) **Situation (4)**. (c) **Situation (5)**. (d) **Situation (6)**.

Fig. 3. Situations of A and B where $\theta \neq 0$ and both A and B are not at I.



(a) **Situation (7)**. (b) **Situation (8)**. (c) **Situation (9)**. (d) **Situation (10)**.

Fig. 4. Situations of A and B where $\theta \neq 0$ and either A or B is at I.

- **Situation (9):** A is at I, and B is below I.
- **Situation (10):** A is below I, and B is at I.

Due to lack of space, the proofs are omitted. Complete proofs of correctness are however available in a longer version of this paper [18].

IV. Conclusion

In this paper, we presented a self-stabilizing algorithm to gather, in a finite time, two stateless asynchronous mobile robots equipped with compasses that can differ by an angle strictly less than $180°$. Obviously, this is the largest divergence for which compasses can still bring useful information since two compasses that can differ by an angle of up to $180°$ provide no information at all.

The natural problem of generalizing our algorithm to an arbitrary finite number of robots remains open. We conjecture that a smaller angle of divergence of compasses is required. Another interesting issue to investigate is to consider the variance in the north directions indicated by compasses over time, and how it affects the solvability of the gathering problem. This remains an open question.

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