VCGG: A Varying Cone Distributed Topology-Control Algorithm for Wireless Ad Hoc Networks*

Jianxin Wang, Yuhong Luo, Jiawei Huang
School of Information Science
and Engineering,
Central South University
Changsha, China
{jxwang,yhluo,jwhuang}@mail.csu.edu.cn

Xi Zhang†
Networking and Information Systems Labs.
Dept. of Electrical and Computer Engineering
Texas A&M University
College Station, TX 77843, U.S.A.
xizhang@ece.tamu.edu

ABSTRACT

In order to increase node lifetime and system throughput, the topology of wireless Ad Hoc networks can be controlled by changing the transmission power at each node. In this paper, we propose an energy-efficient distributed topology-control algorithm, Varying Cone on Gabriel Graph (VCGG). By selecting logical neighbor nodes through deleting the farthest node, VCGG builds a degree-bounded, power spanner and planar sub-graph using the merits of a varying cone. The simulation results show that our proposed VCGG outperforms the existing SΘGG and SYaoGG algorithms in terms of power efficiency, the number of communication neighbors and interference reduction.

1. INTRODUCTION

Wireless Ad Hoc networks have received much attention in recent years due to their potential applications, such as the emergency disaster relief. Nevertheless, the limited battery power characteristic of wireless Ad Hoc networks limits the lifetime of wireless devices and the networks. A communication session is achieved either through single-hop transmission if the recipient is within the transmission range of the source node, or by relaying through intermediate nodes [1,2]. For the purpose of energy conservation, each node in a wireless Ad Hoc network can change the network topology by adjusting its transmission range and/or selecting specific nodes to forward its messages, controlling its neighbor set. The primary goal of topology control in wireless Ad Hoc networks is to maintain network connectivity, prolong network lifetime, and increase throughput, and thus achieve power-efficient routing [3,4].

*This work is supported in part by the National Natural Science Foundation of China No.60673164; the Provincial Natural Science Foundation of Hunan No.06JJ10009; the U.S. National Science Foundation CAREER Award under Grant ECS-0348694.

†The corresponding author: xizhang@ece.tamu.edu

QShine'08 July 28-31, 2008, Hong Kong, China

DOI 10.4108/ICST.QSHINE2008.

To simplify the problem, we make the following assumptions:

(1) Each node has an omni-directional antenna and a single transmission of a node can be received by any node within its vicinity which is a disk centered at this node.

(2) Each node knows its location information.

(3) The nodes are quasi-static for a period of time.

(4) In the power-attenuation model [5], the signal power falls as γ^{β}, where γ is the distance between the source and destination, and β $(2 \leq \beta \leq 4)$ is a real constant dependent on the wireless environment. The transmission range is large enough such that the emission power is the major component and the receiving power is negligible.

Based on these assumptions, the topology-control problem is formulated as follows:

A graph $UDG(V)$ (Unit Disk Graph), briefly UDG, is composed of a set V with n nodes. Each node is randomly located in the plane and has maximum transmission power of 1 unit, i.e., $G = (V, E)$, $E = \{(u, v) \mid ||uv|| \leqslant 1\}$, where $\|uv\|$ denotes the Euclidean distance between nodes u and v. Our goal in performing topology control is to find an undirected subgraph G' of G, such that G' consists of all the nodes in G but has fewer edges. G' satisfies the following properties:

(1) If G is a connected graph, G' is still a connected graph.

(2) t-spanner: G' has a stretch factor t if and only if for any link $u, v \in G$, $p_{G'}(u, v) \leq t \cdot p_G(u, v)$, where $p_G(u, v)$ is the total power consumption of the shortest path between u and v in G. The G' is called a t-spanner of the G and t is the power stretch factor. If we use length instead of power in $p_G(u, v)$, t is the length stretch factor.

(3) Degree Bounded: the degree of G' is small and bounded by a constant, i.e. $\{degree(v) \mid degree(v) \leq c, \forall v \in G'\}$, where c is a small constant. A small node degree reduces the MAC-level contention and interference, it may also help to mitigate the well known hidden and exposed terminal problems [6,7].

(4) Planar: there are no edges crossing each other. This

enables several localized routes, such as GPSR [8], GOAFR [9], to be performed on top of this structure to guarantee the packet delivery without a routing table.

The rest of the paper is organized as follows. Section 2 describes the related works. Section 3 proposes the VCGG algorithm and characterizes its properties. Section 4 evaluates our proposed VCGG algorithm and compare it with the other existing algorithms through simulations. The paper concludes with Section 5.

2. RELATED WORKS

Several geometrical structures have been studied recently both by computational geometry scientists and network engineers [10]. The *relative neighborhood graph*(RNG) [11] consists of all edges uv such that there is no point $w \in V$ with uw and wv satisfying $\|uw\| < \|uv\|$ and $\|wv\| < \|uv\|$. The *Gabriel graph* (GG) [12,13] contains an edge uv from G if and only if $disk(u, v)$ contains no other vertex $w \in V$ inside, where $disk(u, v)$ is the disk with edge uv as a diameter. The *Yao graph* ($\overrightarrow{YG_k}$) [14], in which at each node u, any k equally-separated rays originated at u define k cones. In each cone, we choose the shortest edge $\overrightarrow{uv}$, if there is one, and add a directed link. Ties are broken arbitrarily or by the smallest ID. The final directed graph is called the Yao graph.

Bose et al. proposed a centralized method, Cdel, with running time $O(n \log n)$ to build a degree-bounded planar spanner for a two dimensional point set [15]. It has been noted in [10, 16-20] that the distributed algorithms are more suitable for wireless Ad Hoc networks because a centralized algorithm needs global knowledge, which may introduce high communication overhead especially in large scale networks.

Wang and Li proposed the first efficient localized algorithm to build a degree-bounded planar spanner BPS for wireless ad hoc networks [18]. Its theoretical bound on the node degree, however, is a large constant and the communication cost of their method is high due to the collection of 2-hop information for every wireless node. In [20], a distributed algorithms is proposed to construct a degree-bounded planar structure of both RNG and GG.

Song et al. proposed two methods OrdYaoGG and SYaoGG to construct degree-bounded power spanner, by applying the ordered Yao structures on the Gabriel graph [19]. They achieved better performance with much lower communication cost, compared with the method in [18]. SYaoGG costs $3n$ messages for its construction, and guarantees that there are at most one neighbor node in each of the $k = 9$ equal-sized cones.

Li et al. proposed SΘGG to further reduce the medium contention than SYaoGG by selecting fewer communication neighbors and more widely θ-separated neighbors [10]. SΘGG does not rely on the absolute cone partition by adopting the θ-separated and has a smaller average node degree, interference and transmission range than SYaoGG.

Li et al. also proposed the Cone-based Topology Control (CBTC) algorithm to decrease the node transmission power and maximum node interference [17]. Basically, it is similar to the Yao structure for topology control. The traditional Yao structure has the constant k cones, but the number of cones that could be considered in CBTC is $2n$.

Table 1 shows that SΘGG has preferable metrics in the power spanner, degree bound and planar than prior algorithms. However, the simulation results in [10] show that both SΘGG and SYaoGG did not efficiently decrease the node transmission power and the maximum node interference compared to Gabriel graph.

In this paper, we propose an energy-efficient distributed topology-control algorithm, *Varying Cone on Gabriel Graph* (VCGG). Using a new mechanism for selecting logical neighboring nodes by first deleting the farthest node, the VCGG algorithm builds a degree bounded, power spanner and planar subgraph by using the merits of the varying cone.

Table 1: The Characteristics of Current Approaches

	t-spanner	Degree Bounded	Planar	Cost
RNG	$n-1$	$n-1$	$\surd$	$O(n)$
GG	1	$n-1$	$\surd$	$O(n)$
YG	$\frac{1}{1-(2\sin\frac{\pi}{k})^\beta}$	$k, n-1$		$O(n)$
Cdel*	$(1+\pi)C_{del}$, $C_{del} \le \frac{4\sqrt{3}\pi}{9}$	27	$\surd$	$O(n^2)$
BPS*	$max(\frac{\pi}{2}, \pi\sin\frac{\alpha}{2}+1)\cdot C_{del}(1+\varepsilon)$	$19+\lceil\frac{2\pi}{\alpha}\rceil$, $0<\alpha<\frac{\pi}{3}$	$\surd$	$O(n)$
SYaoGG	$\rho = \frac{(\sqrt{2})^\beta}{1-(2\sqrt{2}\sin\frac{\pi}{k})^\beta}$	k	$\surd$	$O(n)$
SΘGG	$\rho = \frac{(\sqrt{2})^\beta}{1-(2\sqrt{2}\sin\frac{\pi}{k})^\beta}$	$k-1$	$\surd$	$O(n)$

*Use length stretch factor, where n is the number of nodes and k is the number of equal-sized cones

3. VCGG ALGORITHM

3.1 The selection of the varying cone

The varying cone of the VCGG algorithm uses the concept of θ-Domination Region, in the definition given below we follows the one given in [10].

Definition 1 θ-Domination Region: For each neighbor node v of a node u, the θ-domination region of v is the 2θ-cone emanated from u, with the edge uv as its axis.

We use *firstly deleting the farthest node* (FDFN) to select logical neighbors on a Gabriel graph as follows:

(1) Each node u, assuming it has some BLACK logical neighbors in GG (a node is marked BLACK if it is processed and is marked WHITE if it is unprocessed), selects its closest BLACK neighbor, say w, and removes all links to all neighbors in the θ-domination region of w. It then repeats the above procedure until no processed logical neighbors in GG are left.

(2) Assume that node u also has some unprocessed logical neighbors and node v is the farthest node of them. The

node u firstly selects node w on condition that node w is the closest node among the remainder WHITE neighbors in the θ-domination region of the node v. The node u removes all links which connects node u with the remainder WHITE neighbors in the θ-domination region of w (of course including node v). It then repeats the above procedure until no WHITE neighbors in GG are left, or the distance of the remainder WHITE neighbors to node u is smaller than that of selected neighbors. If node u also has some WHITE neighbors, it deals with them using the same way as processing BLACK logic neighbors.

Lemma 1 The Subgraph constructed by the FDFN is t-spanner if the θ-domination region of node u satisfies the following two conditions:

(1) If node w selected by node u is BLACK, the θ-domination region of node w is less than $\pi/3$ for all WHITE nodes, say v ($d_w \le d_v$), where d_w is distance between node u and w), $\pi/4$ for all WHITE nodes ($d_w > d_v$) and $2\arcsin 2^{-\frac{1+\beta}{\beta}}$ for all BLACK nodes.

(2) If node w selected by node u is WHITE, the θ-domination region of node w is less $2\arcsin 2^{-\frac{1+\beta}{\beta}}$ than for all WHITE nodes, say v ($d_w \le d_v$) , and $\pi/6$ for all WHITE nodes ($d_w > d_v$).

PROOF. As shown in Figures 1-2, we suppose node w , v are the neighbors of node u and $\theta = \angle wuv$, node x, u are the neighbors of node w and $\gamma = \angle uwx$. Node u selects logical neighbors according to FDFN as follows:

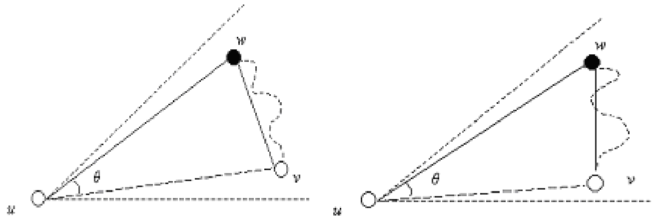
1) Node w is BLACK, node v is WHITE and $\|uw\| \le \|uv\|$ (See Figure 1(a)). Node u selects w as its logical neighbor and deletes the link uv. The power stretch factor is:

$$\begin{aligned}
&\|uv\|^\beta = \|uw\|^\beta + p(w,\dots,v) \le \|uw\|^\beta + t\cdot\|wv\|^\beta \\
&\le \|uv\|^\beta + t\cdot(2\sin\tfrac{\theta}{2}\cdot\|uv\|)^\beta \le t\cdot\|uv\|^\beta \\
&t \ge \tfrac{1}{1-(2\sin\frac{\theta}{2})^\beta}, \\
&t = \tfrac{1}{1-(2\sin\frac{\theta}{2})^\beta}, \quad \text{where} \quad \theta = \tfrac{\pi}{3} - \varepsilon(\varepsilon > 0).
\end{aligned}$$

2) Node w is BLACK, node v is WHITE and $\|uw\| > \|uv\|$(See Figure 1(b)), Node u selects w as its logical neighbor and deletes the link uv. Letting $\eta = \angle wvu$, we can get $\eta < \pi/2$ from Gabriel graph property. Also notice that $\|uw\| = \|uv\| \cdot \frac{\sin\eta}{\sin(\theta+\eta)}$ and $\|wv\| = \|uv\| \cdot \frac{\sin\theta}{\sin(\theta+\eta)}$, and then the power stretch factor is:

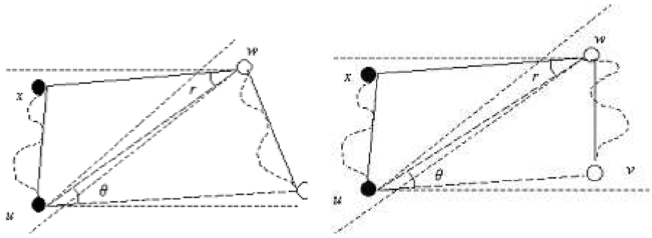
$$\begin{aligned}
&\|uv\|^\beta \le \|uw\|^\beta + t\cdot\|wv\|^\beta \\
&= \|uv\|^\beta\cdot(\tfrac{\sin\eta}{\sin(\theta+\eta)})^\beta + t\cdot\|uv\|^\beta\cdot(\tfrac{\sin\theta}{\sin(\theta+\eta)})^\beta \le t\cdot\|uv\|^\beta \\
&t \ge \tfrac{\sin^\beta\eta}{\sin^\beta(\theta+\eta)-\sin^\beta\theta} \\
&t \ge \tfrac{1}{\cos^\beta\theta-\sin^\beta\theta}, \quad \text{when} \quad \eta \to \tfrac{\pi}{2}, \\
&t = \tfrac{1}{\cos^\beta\theta-\sin^\beta\theta}, \quad \text{where} \quad \theta = \tfrac{\pi}{4} - \varepsilon\ (\varepsilon > 0).
\end{aligned}$$

3) Node w is WHITE and $\|uw\| \le \|uv\|$, node x is BLACK and $\|wx\| \le \|uw\|$ (See Figure 2(a)). Firstly, node u selects w as its logical neighbor and deletes the link uv, then node w selects x as its logical neighbor and deletes the link uw. The power stretch factor is:



(a) $\|uw\| \le \|uv\|$ (b) $\|uw\| > \|uv\|$

Figure 1: Link uw is kept in the final structure



(a) $\|uw\| \le \|uv\|$ (b) $\|uw\| > \|uv\|$

Figure 2: Link uv is removed when processing u and link uw is then removed by node w later

$$\begin{aligned}
&\|uv\|^\beta = p(u,\dots,x) + \|wx\|^\beta + p(w,\dots,v) \\
&\le t\cdot\|ux\|^\beta + \|wx\|^\beta + t\cdot\|wv\|^\beta \\
&\le t\cdot(2\cdot\sin\tfrac{\gamma}{2}\cdot\|uv\|)^\beta + \|uv\|^\beta + t\cdot(2\cdot\sin\tfrac{\theta}{2}\cdot\|uv\|)^\beta \\
&\le t\cdot\|uv\|^\beta \\
&t \ge \tfrac{1}{1-(2\sin\frac{\theta}{2})^\beta-(2\sin\frac{\gamma}{2})^\beta} \\
&t \ge \tfrac{1}{1-2(2\sin\frac{\theta}{2})^\beta},\ let\ \theta = \gamma \\
&t = \tfrac{1}{1-2(2\sin\frac{\theta}{2})^\beta}, \quad \text{where} \quad \theta = 2\arcsin 2^{-\frac{1+\beta}{\beta}} - \varepsilon\ (\varepsilon > 0).
\end{aligned}$$

4) Node w is WHITE and $\|uw\| > \|uv\|$, node x is BLACK and $\|wx\| \le \|uw\|$ (See Figure 2(b)). Firstly, node u selects w as its logical neighbor and deletes the link uv, then node w selects x as its logical neighbor and deletes the link uw. The power stretch factor is:

$$\begin{aligned}
&\|uv\|^\beta = p(u,\dots,x) + \|wx\|^\beta + p(w,\dots,v) \\
&\le t\cdot\|ux\|^\beta + \|wx\|^\beta + t\cdot\|wv\|^\beta \\
&\le t\cdot(2\cdot\sin\tfrac{\gamma}{2}\cdot\|uw\|)^\beta + \|uw\|^\beta + t\cdot\|wv\|^\beta \\
&\le t\cdot(2\cdot\sin\tfrac{\gamma}{2}\cdot\tfrac{\|uv\|}{\cos\theta})^\beta + (\tfrac{\|uv\|}{\cos\theta})^\beta + t\cdot(\tfrac{\sin\theta}{\cos\theta}\cdot\|uv\|)^\beta \\
&\le t\cdot\|uv\|^\beta \\
&t \ge \tfrac{1}{\cos^\beta\theta-\sin^\beta\theta-(2\sin\frac{\gamma}{2})^\beta} \\
&t = \tfrac{1}{\cos^\beta\theta-\sin^\beta\theta-(2\sin\frac{\gamma}{2})^\beta}, \\
&\text{where} \quad \gamma = 2\arcsin 2^{-\frac{1+\beta}{\beta}} - \varepsilon,\ \theta = \tfrac{\pi}{6} - \varepsilon(\varepsilon > 0). \quad \square
\end{aligned}$$

Definition 2 Cone α is the cone of degree α defined by two rays originated at u and aimed to two uninterrupted logical neighbors respectively, as shown in Figure 3.

Figure 3 shows that node u selects two uninterrupted logical neighbors w and v, cone α is the cone of angle $\angle wuv$. Suppose cone $\alpha > 2\theta$, if there is at least a node, say x, in the shadow of angle $\angle\alpha - 2\theta$, then node u needs to add node x as its logical neighbor due to maintaining power spanner of the

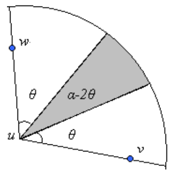


Figure 3: A α-cone between node w and v

subgraph. The cone α varies because θ-domination region changes with processing state of logical nodes and distance between logical nodes and node u. Only if cone $\alpha \leq 2\theta$ or there has no logical nodes in the shadow of angle $\angle\alpha - 2\theta$, the subgraph is a t-spanner.

3.2 The VCGG algorithm

The VCGG algorithm uses varying cone and FDFN mechanism to select logical neighbors as follows:

1. First, each node self-constructs the Gabriel graph locally according to the algorithm in [19]. Initially, all nodes mark themselves WHITE.

2. Once a WHITE node u has the smallest ID among all its WHITE neighbors, it uses the following strategy to select neighbors:

(1) Node u first sorts all its BLACK neighbors (if available) in $Nb(u)$ in the distance-increasing order, then sorts all its WHITE neighbors (if available) in $Nw(u)$ similarly.

(2) Node u scans the sorted list $Nb(u)$ from left to right. In each step, it keeps the current pointed neighbor w in the list, while deleting every conflicted node v in the remainder of the $Nb(u)$ list and in the whole of the $Nw(u)$ list. Here a node v conflicting with w means that node v is in the θ-dominating region of node w according to condition (1) of the θ-dominating region given in Lemma 1.

(3) If the sorted list $Nw(u)$ is not empty and the distance between the first node from right in the $Nw(u)$ list and node u is bigger than distance between every node in the $Nb(u)$ list and node u. Node u search a node x from left to right in the $Nw(u)$ list, which the first node from right in the $Nw(u)$ list is in the θ-dominating region ($\theta = 2\arcsin 2^{-\frac{1+\beta}{\beta}}$) of node x. If such node x exists, node u deletes every conflicted node v in the remainder of the $Nw(u)$ list and moves node x to the $Nb(u)$ list. Node v is in the θ-dominating region of node w according to limit (2) of the θ-dominating region in lemma 1, of course including the first node from right in the $Nw(u)$ list. Repeat step (3) until no such node x is exists.

(4) If the sorted list $Nw(u)$ is not empty, Node u scans the sorted list $Nw(u)$ from left to right. In each step, it keeps the current pointed neighbor w in the list, while deleting every conflicted node v in the remainder of the $Nw(u)$ list. Node v is in the θ-dominating region ($\theta = 2\arcsin 2^{-\frac{1+\beta}{\beta}}$) of node w. Node u moves all remainder nodes in the $Nw(u)$ list to the $Nb(u)$ list after scanned the $Nw(u)$ list.

(5) Node u then marks itself Black, and notifies each deleted neighbor v by a broadcasting message UpdateN.

3. Once a node v receives the message UpdateN from a neighbor u, it checks whether itself is in the nodes set for deleting: if so, it deletes the sending node u in its neighbor lists, otherwise, marks u as BLACK.

4. When all nodes are processed, all selected links $\{uv|v \in N(u); v \in GG\}$ form the final network topology, denoted by $VCGG$. Each node can shrink its transmission range as long as it sufficiently reaches its farthest neighbor.

3.3 Properties of the VCGG algorithm

Theorem 1 The VCGG is t-spanner.

PROOF. The VCGG algorithm uses varying cone and FDFN mechanism to select logical neighbors. It is obvious that the VCGG is t-spanner according to Lemma 1 and definition 2. $\square$

Corollary 1 The VCGG algorithm has better power stretch factor than SYaoGG and SΘGG when $\varepsilon = 2\arcsin 2^{-\frac{1+\beta}{\beta}} - \frac{2\pi}{k}$, where $k \geq 9$ and $2 \leq \beta \leq 4$ in the θ-domination region.

PROOF. The power stretch factor of SYaoGG or SΘGG is $t = \frac{(\sqrt{2})^\beta}{1-(2\sqrt{2}\sin\frac{\pi}{k})^\beta}$,$(k \geq 9, 2 \leq \beta \leq 4)$[10,19]. The power stretch factor of the VCGG is only relative to ε in the θ-domination region according to Lemma 1. Based on our derivations, if setting $\varepsilon = -\frac{2\pi}{k} + 2\arcsin 2^{-\frac{1+\beta}{\beta}}$, ($k \geq 9$ and $2 \leq \beta \leq 4$), then we need to consider the following four cases for different values of θ:

CASE 1. $\theta = \frac{\pi}{3} - \varepsilon$:

$$t = \frac{1}{1-(2\sin\frac{\theta}{2})^\beta} = \frac{1}{1-(2\sin(\frac{\pi}{k}+\frac{\pi}{6}-\arcsin 2^{-\frac{1+\beta}{\beta}}))^\beta}$$

We only need to prove the following inequality:

$$\frac{1}{1-(2\sin(\frac{\pi}{k}+\frac{\pi}{6}-\arcsin 2^{-\frac{1+\beta}{\beta}}))^\beta} \leq \frac{(\sqrt{2})^\beta}{1-(2\sqrt{2}\sin\frac{\pi}{k})^\beta},$$

which is equivalent to $f(\beta) \triangleq (\sqrt{2})^\beta(1-(2\sin(\frac{\pi}{k}+\frac{\pi}{6}-\arcsin 2^{-\frac{1+\beta}{\beta}}))^\beta) + (2\sqrt{2}\sin\frac{\pi}{k})^\beta \geq 1$.

Taking the first derivative of $f(\beta)$, we can get $f'(\beta) > 0$. Thus the function $f(\beta)$ is an increasing function. Therefore, we have $2(1-4\sin^2(\frac{\pi}{k}+\frac{\pi}{6}-\arcsin 2^{-1.5}))+8\sin^2\frac{\pi}{k} > 2(1-4\sin^2(\frac{\pi}{9}+\frac{\pi}{6}-20.7°)+8\sin^2\frac{\pi}{k} > 1.01 > 1$.

CASE 2. $\theta = \frac{\pi}{4} - \varepsilon$:

Using the similar way in **CASE 1**, we get $t = \frac{1}{\cos^\beta\theta - \sin^\beta\theta} = \frac{1}{\cos^\beta(\frac{\pi}{4}-2\arcsin 2^{-\frac{1+\beta}{\beta}}+\frac{2\pi}{k}) - \sin^\beta(\frac{\pi}{4}-2\arcsin 2^{-\frac{1+\beta}{\beta}}+\frac{2\pi}{k})} \leq \frac{(\sqrt{2})^\beta}{1-(2\sqrt{2}\sin\frac{\pi}{k})^\beta}$.

CASE 3. $\theta = 2\arcsin 2^{-\frac{1+\beta}{\beta}} - \varepsilon = \frac{2\pi}{k}$:

$t = \frac{1}{1-2(2\sin\frac{\theta}{2})^{\beta}} = \frac{1}{1-2(2\sin\frac{\theta}{k})^{\beta}} < \frac{(\sqrt{2})^{\beta}}{1-2(2\sin\frac{\theta}{k})^{\beta}}$.

CASE 4. $\theta = \frac{\pi}{6} - \varepsilon$ and $\gamma = 2\arcsin 2^{-\frac{1+\beta}{\beta}} - \varepsilon$:

Using the similar way in **CASE 1**, we get

$t = \frac{1}{\cos^{\beta}\theta - \sin^{\beta}\theta - (2\sin\frac{\gamma}{2})^{\beta}} = \frac{1}{\cos^{\beta}\theta - \sin^{\beta}\theta - (2\sin\frac{\pi}{k})^{\beta}}$

$\leq \frac{(\sqrt{2})^{\beta}}{1-(2\sqrt{2}\sin\frac{\pi}{k})^{\beta}}$.

In the above four cases, the power stretch factor of VCGG is smaller (better) than that of SYaoGG or SΘGG, when $k \geq 9$ and $2 \leq \beta \leq 4$. □

By introducing varying cone, VCGG can get more agile than k equally-separated cone in SYaoGG or θ-domination region in SΘGG. It decreases efficiently the node power, number of communication neighbors and node interference.

Theorem 2 Let $M = 2\pi/(\pi/6 + 2\arcsin 2^{-\frac{1+\beta}{\beta}} - 2\varepsilon)$. If value M is a decimal fraction, the degree of VCGG is bounded by $2\lfloor M \rfloor$. If value M is integer, the degree of VCGG is bounded by $2M - 1$.

PROOF. The proof of Lemma 1 shows that degree of VCGG may achieve the maximum value in case (4) when selecting WHITE nodes. The worst case is that the minimum θ-domination region is $\pi/6 - \varepsilon$ and $2\arcsin 2^{-\frac{1+\beta}{\beta}} - \varepsilon$, $(\varepsilon > 0)$ in each side of the logical neighbors respectively. So, it is at most $M = 2\pi/(\pi/6 + 2\arcsin 2^{-\frac{1+\beta}{\beta}} - 2\varepsilon)$ such regions in a circular area. It at most adds one node as logical neighbor between two such regions, like the shadow in Figure 3. Thus, if value M is a decimal fraction, the degree of VCGG is bounded by $2\lfloor M \rfloor$. If value M is integer, the degree of VCGG is bounded by $2M-1$ because there is at least a pair overlapped region of logical neighbors. □

Theorem 3 If the UDG is a connected graph, the VCGG is still a connected graph.

PROOF. We prove the connectivity by contradiction. As shown in Figures 1-2, we assume that link uv is the shortest link in UDG whose connectivity is broken by VCGG algorithm. Without loss of generality, we assume the link uv is removed while processing node u, because of the existence of another node w.

(1) As shown in Figure 1, it happens when node w is processed and node v is unprocessed. Hence, $\|wv\| < \|uv\|$ (otherwise $\angle uvw > \pi/2$ violates the Gabriel graph property). Since node w is a processed node and node u decides to keep link uw, the link uw will be kept in VCGG. According to assumption that u and v are not connected in VCGG, w and v are not connected either. That is to say, uv is not the shortest link whose connectivity is broken. It is a contradiction.

(2) As shown in Figure 2(a), $\|uw\| < \|uv\|$. Notice that $\angle vuw < \pi/4$, hence $\|wv\| < \|uv\|$. In other words, both link wv and uw are smaller than link uv. Since there are no paths $p(u,\ldots,v)$ according to the assumption, either the path $p(u,\ldots,w)$ or $p(v,\ldots,w)$ is broken. That is to say, either the connectivity of wv or uw is broken. Thus, uv is not the shortest link whose connectivity is broken, it is a contradiction.

(3) As shown in Figure 2(b), notice that $\angle vuw < \pi/4$, hence $\|wv\| < \|uv\|$. In other words, link wv is smaller than link uv. Notice that $\|wx\| < \|uw\|$ and $\angle xwu < \pi/4$, hence $\|xu\| < \|uw\|$. It does not break the connectivity of uw while node u selects node x as its logical neighbor and deletes the link uw according to the above (2). Since there are no paths $p(u,\ldots,v)$ according to the assumption, the path $p(v,\ldots,w)$ is broken. That is to say, the connectivity of wv is broken. Thus, uv is not the shortest link whose connectivity is broken, it is a contradiction.

Then we can get that if the UDG is a connected graph, the VCGG is still a connected graph. □

Theorem 4 The VCGG is a planar graph.

PROOF. The VCGG construction does not add any edges to the original graph. On the contrast, it only deletes edges. The planar property is inherited from GG graph. □

Theorem 5 Assuming that both the ID and the geometry position can be represented by $\log n$ bits each, the total number of messages of the VCGG is upper-bounded by $3n$.

PROOF. First, building GG in VCGG can be done using only n messages [19]: each message represented by $\log n$ bits contains the ID and geometry position of a node. Second, to build *VCGG*, initially, the number of edges in Gabriel Graph is $[n, 3n-6]$ since it is a planar graph. We will remove some edges from GG to bound the logical node degree. Clearly, there are at most $2n$ such removed edges since we keep at least $n-1$ edges from the connectivity of the final structure. Thus the total number of messages used to inform the deleted edges from GG is at most $2n$. So, the total number of messages of the VCGG algorithm is upper-bounded by $3n$. □

Corollary 2 The FDFN mechanism of selecting logical neighbors in VCGG algorithm can efficiently decrease the node power and node interference.

PROOF. As shown in Figure 4, node x, w, y, v are WHITE, and $\|uw\| < \|ux\| \leq \|uy\| < \|uv\|$, $\angle wuy < \theta$, $\angle xuy > \theta$, $\angle wuv > \theta$. The FDFN mechanism of selecting logical neighbors firstly selects node y as the logical neighbor of node u due to the farthest node v is in the θ-dominating region of node y. Node u deletes conflicted node w and v by removing link uw and uv. Then node u selects node x as its logical neighbor. The power of node u is $\|uy\|^{\beta}$. However, SΘGG algorithm selects logical neighbors in the distance-increasing order, the power of node u is $\|uv\|^{\beta}$. We know that $\|uy\| \geq \|uv\| \cdot \cos\frac{2\pi}{k}$ $(k \geq 9)$ according the Gabriel graph property, in the worst, the power of node u in the SΘGG algorithm is 1.3^{β} than that of in the VCGG algorithm. Clearly, the FDFN mechanism can efficiently decrease the node power and node interference, this fact is demonstrated by the simulation results given in Section 4. □

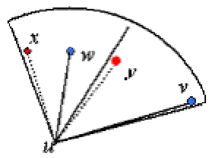


Figure 4: The method of node u selecting logical neighbors

Corollary 3 The varying cone in the VCGG algorithm makes selecting logical neighbors more agile and decreases the node power and number of communication neighbors efficiently.

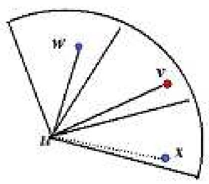


Figure 5: The bigger power and number of logical neighbors due to k-equally cones

Proof. As shown in Figure 5, node x and w are BLACK, node v is WHITE, and $\|uw\| \leq \|ux\| < \|uv\|$, $\angle wux < 2\pi/3, \angle xuv > \theta, \angle wuv > \theta$. Node u selects node w and x as its logical neighbors and deletes the link uv due to cone $\angle wux < 2\pi/3$ and node v in the cone $\angle wux$. The power of node u is $\|ux\|^{\beta}$. Node u selects node w, v and x as its logical neighbors in k equally-separated cone of SYaoGG or θ-domination region of SΘGG, the power of node $u$is $\|ux\|^{\beta}$. We know that $\|uv\| \leq \sqrt{2}\|ux\|$ $(k \geq 9)$ according the Gabriel graph property. The node power reduces by about $(\sqrt{2})^{\beta}$. □

4. PERFORMANCE EVALUATIONS

In the simulation, we compare the performance of VCGG with that of MST (Minimum Spanning Tree), GG, SΘGG and SYaoGG. We generate n random wireless nodes in a 16×16 unit squares. We set power attenuation constant $\beta = 2$, and the parameter $k = 9$ in SYaoGG and SΘGG, $\varepsilon = 2\arcsin 2^{-1.5} - 2\pi/9 \approx 1.4°$ in VCGG. The transmission range of each node is set to 4 units. We test the average power spanning ratios of all pairs of nodes, the average (and the maximum) (physical and logical) node degree of all nodes, and the node power by changing the node number n from 30 to 360. For each number $n = 30i$, $1 \leq i \leq 12$, we generate 500 vertex sets.

(1) Power Efficiency

Figure 6 summarizes the experimental results of power stretch factors of all these topologies as the average power spanning ratio.

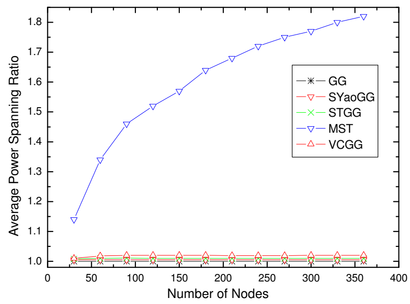
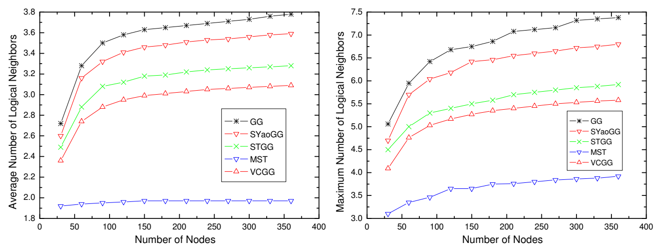


Figure 6: Average power spanning ratio of various structures

The average power panning ratio GG, SYaoGG or SΘGG is less than 1.021, that of VCGG is less than 1.03, and that of MST is maximum. The power stretch factor of VCGG can be controlled in a constant by limiting and controlling θ-domination region. The average power spanning ratio of VCGG has a little increase (<0.01) due to VCGG reducing node power efficiently and moving more links of neighbors. But length stretch factor of MST is at least $\Omega(\frac{\log n}{\log \log n})$, and its power stretch factor is $n-1$. So MST has bigger power spanning ratio than others, its power spanning ratio increases with the number of nodes.

(2) Number of communication neighbors



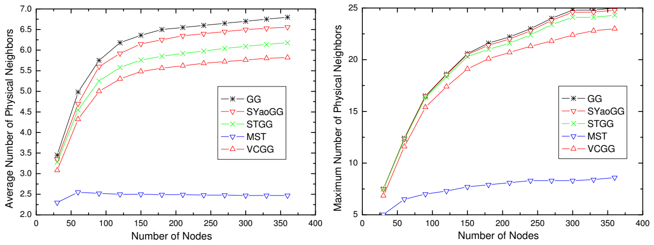
(a) Average communication neighbors (b) Max communication neighbors

Figure 7: The communication neighbors of various structures

Figures 7(a) and (b) show the average communication neighbors and maximum communication neighbors of various structures. VCGG makes use of merits of varying cone and FDFN mechanism. The varying cone makes selecting logical neighbors more agile than k equally-separated cone in SYaoGG or θ-domination region in SΘGG. The FDFN mechanism can efficiently decrease the node power. Thus, VCGG can efficiently decrease the node power and number of communication neighbors. The average number of communication neighbors of VCGG decreases about 0.2 than that of SΘGG, the maximum number of communication neighbors of VCGG decreases about 0.5 than that of SΘGG.

(3) Interference

The node interference is defined as the number of nodes within its transmission range. Figures 8(a) and (b) show

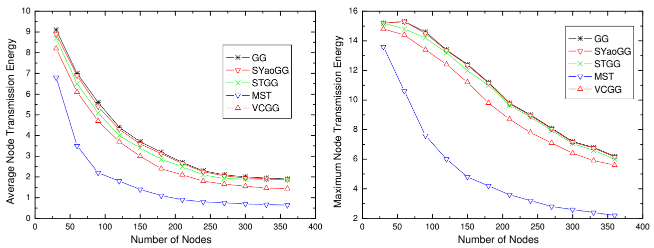


(a) Average node interference (b) Max node interference

Figure 8: The node interference of various structures

the average node interference and the maximum node interference of various structures. By using merits of varying cone and FDFN mechanism, VCGG can efficiently decrease the node power and node interference. The average node interference of VCGG decreases about 0.5 than that of SΘGG, and the maximum node interference of VCGG decreases about 1.3 than that of SΘGG.

(4) Node power



(a) Average node power (b) Max node power

Figure 9: The node power of various structures

Figures 9(a) and (b) show the average node power and the maximum node power of various structures. As shown in Figure 9, the VCGG algorithm can efficiently decrease the node power because of using merits of varying cone and FDFN mechanism. The average node power of VCGG decreases about 0.4 than that of SΘGG, the maximum node power of VCGG decreases about 1.0 than that of SΘGG. It can efficiently decrease energy consumption and node interference and extend the lifetime of nodes.

5. CONCLUSIONS

The topology of a wireless Ad Hoc network can be controlled by changing the transmission power at each node. The primary goal of topology control is to design power-efficient algorithms that maintain network connectivity and optimize performance metrics such as nodes lifetime and throughput. In this paper, we proposed a varying cone distributed topology-control algorithm on Gabriel graph. The VCGG algorithm builds a degree bounded, power spanner and planar subgraph by making use of merits of varying cone and a FDFN mechanism of selecting logic neighbor nodes. The varying cone makes selecting logical neighbors more agile than k equally-separated cone in SYaoGG or θ-domination region in SΘGG. The FDFN mechanism can efficiently decrease the node power. The simulation results show that our proposed VCGG algorithm performs better, in terms of power efficiency, number of communication neighbors and interference, than the existing SΘGG and SYaoGG algorithms.

6. REFERENCES

[1] J. Wieselthier, G. Nguyen, A. Ephremides. Distributed algorithms for energy-efficient broadcasting in ad hoc networks. In Proc. of IEEE Milcom, 2002.

[2] Y. Luo, J. Wang, J. Chen, S. Chen. A Distributed Algorithm based on Probability for Refining Energy-Efficiency of Multicast Trees in Ad Hoc Networks. In Proc. of IEEE LCN, 2005.

[3] R. Rajaraman. Topology control and routing in ad hoc networks: A survey. SIGACT News, 2002, 33:60-73.

[4] X. Li. Algorithmic, geometric and graphs issues in wireless networks. Wireless Communications and Mobile Computing, 2002, 3(2):119-140.

[5] T.S. Rappaport. Wireless Communications: Principles and Practices. Prentice Hall, 1996.

[6] L. Kleinrock, J. Silvester. Optimum transmission radii for packet radio networks or why six is a magic number. In Proc. of the IEEE National Telecommunications Conference, 1978.

[7] Yongho S, Jaewoo P, Yanghee C. Multi-Rate aware routing protocol for mobile ad hoc neworks. In: Proc. of IEEE VTC, 2003.

[8] B. Karp, H. Kung. GPSR: Greedy perimeter stateless routing for wireless networks. In: Proc. of ACM MobiCom, 2000.

[9] F. Kuhn, R. Wattenhofer, A. Zollinger. Worst-case optimal and average-case efficient geometric ad-hoc routing. In: Proc. of ACM MobiHoc, 2003.

[10] X. Li, W. Song, W. Wang. A Unified Energy Efficient Topology for Unicast and Broadcast. In: Proc. of ACM MobiCom, 2005.

[11] G. Toussaint. The relative neighborhood graph of a finite planar set. Pattern Recognition, 1980, 12(4):261-268.

[12] Wan PJ, Yi CW. On the longest edge of gabriel graphs in wireless ad hoc networks. IEEE Transaction on Parallel and Distributed Systems, 2007,18(1):111-125.

[13] K. Gabriel, R. Sokal.A new statistical approach to geographic variation analysis. Systematic Zoology, 1969,18(3):259-278.

[14] A. Yao. On constructing minimum spanning trees in k-dimensional spaces and related problems. SIAM Journal of Computing, 1982,11(4):721-736.

[15] P. Bose, J. Gudmundsson, M. Smid. Constructing plane spanners of bounded degree and low weight. In: Proc. of ESA, 2002.

[16] R. Wattenhofer, L. Li, P. Bahl, Y. Wang. Distributed topology control for wireless multihop ad-hoc networks. In: Proc. of IEEE INFOCOM, 2001.

[17] L. Li, J. Halpern, P. Bahl, Y. Wang, R. Wattenhofer. A Cone-Based Distributed Topology-Control Algorithm for Wireless Multi-Hop Networks. IEEE/ACM Transaction on Networking, 2005,13(1):147-159.

[18] G. Calinescu. Computing 2-hop neighborhoods in ad hoc wireless networks. In: Proc. of AD-HOC Networks and Wireless Conference, 2003.

[19] W. Song, Y. Wang, X. Li. Localized Algorithms for Energy Efficient Topology in Wireless Ad Hoc Networks. In: Proc. of ACM MobiHoc, 2004.

[20] A. A.-K. Jeng and R.-H. Jan. The r-neighborhoodgraph: An adjustable structure for topology control in wireless ad hoc networks. IEEE Transation Parallel and Distributed Systems, 2007,18(4):536-549.