Joint Congestion Control, Contention Control and Resource Allocation in Wireless Networks *

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ABSTRACT

Traditional congestion control protocols assume that each link provides a fixed capacity, while it is not always the case in wireless networks which have shared and variable medium. In this paper, we incorporate variable link capacity as a function of resource allocated, and random-access interference model dependent on physical location, in addition to congestion control, into the network utility maximization framework. Despite non-convexity and non-separability of the primal formulation, we transform the problem and apply a two-level dual based decomposition for solving it. We then propose practical algorithm and prove their convergence to the globally optimum. By collaboratively optimization of transmission rate at the transport layer, link persistence probability at the media-access control layer, and allocated resource at the physical layer, our algorithm can improve the system performance which is further demonstrated by numerical results.

1. INTRODUCTION

In order to meet the increasing demand for high performance wireless networks, limited network resource need to be collaboratively optimized to improve system efficiency. In wireline networks, network utility maximization (NUM) has recently emerged as a powerful framework for investigating network resource allocation problems and Internet congestion control protocols, e.g., [1], [2], [3]. However, these result can not be directly applied to wireless network for two major reasons.

First, the wireless channel is a shared and interference-limited medium. Therefore, the transmission flows may compete even if they are not sharing a wireless link, which may result in transmission failure as long as they locate sufficiently close and the interference is sufficiently large. To avoid degradation in efficiency, medium access control (MAC) protocols need to be joint designed with congestion control. Various algorithm either scheduling-based or random-access-based have been developed to tackle these issues such as [4], [5] [6], [7], [8], [9].

Second, unlike their wireline counterparts with fixed link capacities, wireless networks have "elastic" link capacities dependent on the resource allocated. Intuitively, a bottleneck link can be alleviated by allocating more resource thus increase link capacity instead of reducing the allowed transmission rates from all the sources using this link. For this reason, there is a balance between "supply" of limited resource and "demand" of link capacities. Extension of the basic NUM problem in this direction have been recently investigated in [10], [11], [12], [13], [14].

To fully utilize the property of wireless networks, joint consideration of congestion control, contention control and resource allocation is needed. In this paper, we study this problem using the NUM framework. Based on random-access MAC and system-wide resource allocation model, we extend the basic NUM and obtain a rigorous and systematic design. By applying duality theory, the non-separable problem is decomposed into three subproblems: congestion control subproblem, contention control subproblem and resource allocation subproblem, coordinated by a price updating master problem. With this decomposition, a subgradient algorithm for collaborative optimization is proposed and performance improvement have been demonstrated via numerical results.

The rest of this paper is organized as follows. Section II, provide the system model considered in this paper. Section III present decomposition method and the joint congestion control, contention control and resource allocation (JCCRA) algorithm. Section IV illustrate how our algorithm can improve system performance through numerical results. Finally, we conclude the paper in Section V.

2. SYSTEM MODEL

Consider a wireless network represented by a directed graph $G = (N, L)$ (*e.g.* Fig. 1), where N is the set of nodes and L is the set of links. We define $L_{out}(n)$ as the set of outgoing

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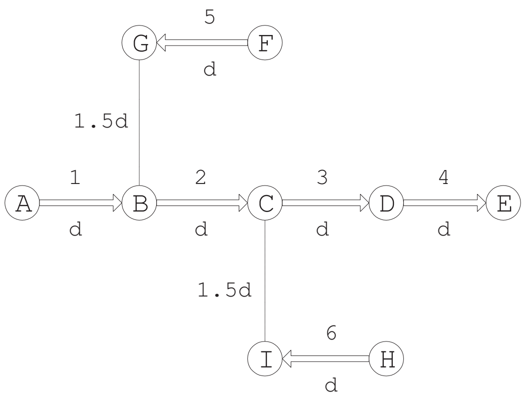

Figure 1: Network Topology

links from node n, $L_{in}(n)$ as the set of incoming links to node n. Let $L^I(n)$ denote the set of links whose transmissions get interfered from the transmission of node n and $N^I(l)$ denote the set of nodes whose transmissions cause interference to the receiver of link l, excluding the transmitter node of link l.

At each time slot, each node n transmits data with a fixed probability q_n. When it determines to transmit data, it choose one of its outgoing links $l \in L_{out}(n)$ with a probability of p_l/q_n, where p_l is the link persistence probability and $\sum_{l\in L_{out}(n)} p_l = q_n, \forall n \in N$.

Assume each link has a general concave utility function $U_l(x_l)$, an increasing nonlinear function of its average data rate x_l. In our model, x_l is obtained as:

$$x_l = c_l(r_l)p_l \prod_{k\in N^I(l)} (1 - q_k), \quad \forall l \tag{1}$$

where $c_l(r_l)$ is flexible capacity of link l dependent on allocated resource r_l, such as power, bandwidth, antenna, *etc.*, with a constraint on the total budget R_T:

$$\sum_{l\in L} r_l \le R_T \tag{2}$$

The objective of the problem is to obtain optimal link rates $\mathbf{x}$, persistence probability $\mathbf{p}, \mathbf{q}$ and resource allocation $\mathbf{r}$ so as to maximize the aggregate utility of all links in the network. This problem can be formulated as the following nonlinear programming:

$$\begin{aligned}
\max \quad & \sum\nolimits_{l\in L} U_l(x_l) \\
\text{s.t.} \quad & x_l \le c_l(r_l)p_l \prod\nolimits_{k\in N^I(l)} (1 - q_k), \quad \forall l \\
& \sum\nolimits_{l\in L_{out}(n)} p_l = q_n, \quad \forall n \\
& 0 \le q_n \le 1, \quad \forall n \\
& 0 \le p_l \le 1, \quad \forall l \\
& \sum\nolimits_{l\in L} r_l \le R_T
\end{aligned} \tag{3}$$

The problem formulated in (3) incorporates congestion control at the transport layer, contention control at the MAC layer and resource allocation at the physical layer. The three layers are coupled through the first constrain, which reveals that both link capacity and persistence probability can affect link rate. The transport layer link rate, MAC layer persistence probability and physical layer resource should be jointly optimized to maximize the aggregate link utility. Due to the first constraint, (3) is in general non-convex and non-separable, thus difficult to obtain global optimality in a distributed way. Under certain conditions, however, it can be transformed into a convex and separable optimization problem, as will be discussed in the next section.

3. CROSS-LAYER DESIGN VIA DUAL DECOMPOSITION

In this section, we first reformulate the problem in (3), then decompose the problem into multilevel subproblems controlled by a master problem through price and derive a dual-based algorithm for solving it.

3.1 Reformulation and Dual problems

Since the problem in (3) is non-separable, we transform it by taking log of both sides of the first constraints. This reformulation casts the problem into

$$\begin{aligned}
\max \quad & \sum\nolimits_{l\in L} U_l'(x_l') \\
\text{s.t.} \quad & c_l'(r_l) + \log p_l \\
& + \sum\nolimits_{k\in N^I(l)} \log(1 - q_k) - x_l' \ge 0, \quad \forall l \\
& \sum\nolimits_{l\in L_{out}(n)} p_l = q_n, \quad \forall n \\
& 0 \le q_n \le 1, \quad \forall n \\
& 0 \le p_l \le 1, \quad \forall l \\
& \sum\nolimits_{l\in L} r_l \le R_T
\end{aligned} \tag{4}$$

where $x_l' = \log(x_l'), U_l'(x_l') = U_l(e^{x_l}), c_l'(r_l) = \log c_l(r_l)$. Assume concavity of $c_l'(r_l)$, the constraint set in (4) is convex due to the convexity of $-\log$ function, but in order to obtain convex programming, we still need to check the concavity of $U_l'(x_l')$. Define

$$h_l(x_l) = \frac{d^2U_l(x_l)}{dx_l^2} x_l + \frac{dU_l(x_l)}{dx_l} \tag{5}$$

it can be demonstrated that if $g_l(x_l) < 0$, $U_l'(x_l')$ is a strictly concave function of x_l' [8]. Throughout this paper, we will assume this condition is satisfied , thus the optimization in (4) is convex, which enable a dual decomposition approach.

The Lagrangian function associated with problem (4) can

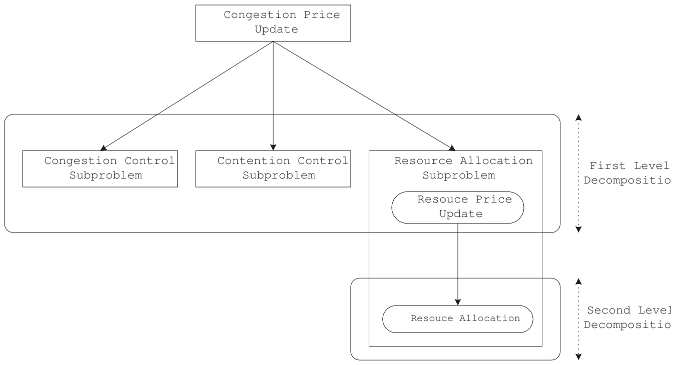

Figure 2: Illustration of the two level dual decomposition

be written as follows:

$$
\begin{aligned}
L(\mathbf{x}', \mathbf{p}, \mathbf{q}, \mathbf{r}, \boldsymbol{\lambda}) &= \sum_{l\in L} U_l'(x_l') \\
&\quad + \sum_{l\in L} \lambda_l \left(c_l'(r_l) + \log p_l + \textstyle\sum_{k\in N^I(l)} \log(1-q_k) - x_l' \right) \\
&= \sum_{l\in L} \{U_l'(x_l') - \lambda_l x_l'\} + \sum_{l\in L} \lambda_l \log p_l \\
&\quad + \sum_{l\in L} \sum_{k\in N^I(l)} \log(1-q_k) + \sum_{l\in L} \lambda_l c_l'(r_l) \\
&= \sum_{n\in N} \lambda_l \sum_{l\in L_{out}(n)} \{U_l'(x_l') - \lambda_l x_l'\} + \sum_{n\in N} \sum_{L\in L_{out}(n)} \lambda_l \log p_l \\
&\quad + \sum_{n\in N} \sum_{m\in L^I(n)} \lambda_m \log(1-q^k) + \sum_{l\in L} \lambda_l c_l'(r_l)
\end{aligned} \tag{6}
$$

where λ_l is the Lagrange multiplier on link l with an interpretation of "contention price". The Lagrangian dual function is

$$g(\boldsymbol{\lambda}) = \max_{\substack{\sum_{l\in L_{out}(n)} p_l = q_n, \quad \forall n \\ \mathbf{0}\preceq\mathbf{q}\preceq\mathbf{1} \\ \mathbf{0}\preceq\mathbf{p}\preceq\mathbf{1} \\ \mathbf{r}^T\mathbf{1}\le R_T}} L(\mathbf{x}', \mathbf{p}, \mathbf{q}, \mathbf{r}, \boldsymbol{\lambda}) \tag{7}$$

and the dual problem is

$$\min_{\boldsymbol{\lambda}\succeq\mathbf{0}} g(\boldsymbol{\lambda}) \tag{8}$$

Due to the convexity of (4), there is no duality gap between the primal and dual problem [15].

3.2 Decomposition and JCCRA Algorithms

Using decomposition theory [16], we can derive a two-level dual decomposition and obtain three subproblems: congestion control subproblem, contention control subproblem and resource allocation subproblem (fig. 2).

The congestion control subproblem is

$$\max_{\mathbf{x}'} \sum_{l\in L_{out}(n)} U_l'(x_l') - \lambda_l x_l' \tag{9}$$

Since the problem is naturally decoupled, it can be conducted in parallel at each node n

$$\max_{x_l'} U_l'(x_l') - \lambda_l x_l', \quad \forall l \in L_{out}(n) \tag{10}$$

The contention control subproblem is

$$\max_{\substack{\sum_{l\in L_{out}(n)} p_l = q_n \\ 0\le p_l\le 1, \forall l\in L_{out}(n) \\ 0\le q_n\le 1}} \sum_{L\in L_{out}(n)} \lambda_l \log p_l + \sum_{m\in L^I(n)} \lambda_m \log(1-q_k) \tag{11}$$

Define λ_n and k_n as

$$\lambda_n = \textstyle\sum_{l\in L_{out}(n)} \lambda_l, \quad \forall n \tag{12}$$

$$k_n = \textstyle\sum_{l\in L_{out}(n)} \lambda_l + \sum_{k\in L^I(n)} \lambda_k, \quad \forall n \tag{13}$$

problem (11) has closed-form solution [8]

$$
\begin{aligned}
p_l(\boldsymbol{\lambda}) &= \begin{cases} \frac{\lambda_l}{k_n} & k_n \neq 0 \\ \frac{1}{|L_{out}(n)|+|L^I(n)|} & k_n = 0 \end{cases}, \quad \forall n, \forall l \in L_{out}(n) \\
q_n(\boldsymbol{\lambda}) &= \begin{cases} \frac{\lambda_n}{k_n} & k_n \neq 0 \\ \frac{\lambda_n}{|L_{out}(n)|+|L^I(n)|} & k_n = 0 \end{cases}, \quad \forall n
\end{aligned} \tag{14}
$$

The resource allocation subproblem is

$$\max_{\substack{\sum_{l\in L} r_l \le R_T \\ r_l\ge 0}} \lambda_l c_l'(r_l) \tag{15}$$

which can be solved via a second-level dual decomposition. The Lagrangian associated with problem (15) is

$$
\begin{aligned}
L(\mathbf{r}, \mu) &= \sum_{l\in L} \lambda_l c_l'(r_l) + \mu(R_T - \sum_{l\in L} r_l) \\
&= \sum_{l\in L} (\lambda_l c_l'(r_l) - \mu r_l) - \mu R_T
\end{aligned} \tag{16}
$$

where dual variable μ can be interpreted as "resource price". The Lagrange dual function is

$$g(\mu) = \max_{\mathbf{r}\succeq\mathbf{0}} L(\mathbf{r}, \mu) \tag{17}$$

Therefore, optimal resource allocation yielding the following second-level subproblems

$$\max_{r_l\ge 0} \lambda_l c_l'(r_l) - \mu r_l, \quad \forall l \in L \tag{18}$$

and a secondary master dual problem updating μ

$$\mu(t+1) = \left[\mu(t) - \alpha(t) \left(R_T - \sum_{l\in L} -r_l^\star(\mu(t)) \right) \right]^+ \tag{19}$$

where t is the iteration index, α is a positive stepsize, $r^\star$ is optimal solution for problem (18) and $[.]^+$ denotes the projection onto the nonnegative orthant.

We are now ready to solve the dual problem in (8) using a subgradient projection algorithm [18]. At each node n for $\forall l \in L_{out}(n)$, the dual variable λ, which stands for price charged at each link, can be updated as

$$
\begin{aligned}
\lambda_l(t+1) = \Bigg[\lambda_l(t) - \alpha(t) \Bigg(& c'^\star_l(t) + \log p_l^\star(t) \\
& + \textstyle\sum_{k\in N^I(l)} \log(1-q_k^\star(t)) - x'^\star_l(t) \Bigg) \Bigg]^+
\end{aligned} \tag{20}
$$

Table 1: JCCRA Algorithm for Wireless Networks

1. *Initialization*: For each node n

$t = 0, q_n(1) = \frac{\lambda_n}{|L_{out}(n)|+|L^I(n)|}$, and $\forall l \in L_{out}(n)$

$p_l(1) = \frac{1}{|L_{out}(n)|+|L^I(n)|}, \lambda_l(1) = 1, c'_l(1) = 1,$

2. *Iteration*: $t = t + 1$

(1) Inform $\lambda_l(t)$ to all nodes in $N^I(l)$,$\forall l \in L_{out}(n)$

(2) At each node n, obtain $q_n(t+1), x'_l(t+1), p_l(t+1)$, $\forall l \in L_{out}(n)$ using (14) and (10)

(3) Calculates link capacity $c_l(t+1)$ by solving second-level subproblem (18), (19)

(4) Updates congestion price $\lambda_l(t+1)$ according to (20)

3. *Termination*: Repeat 2 until convergence

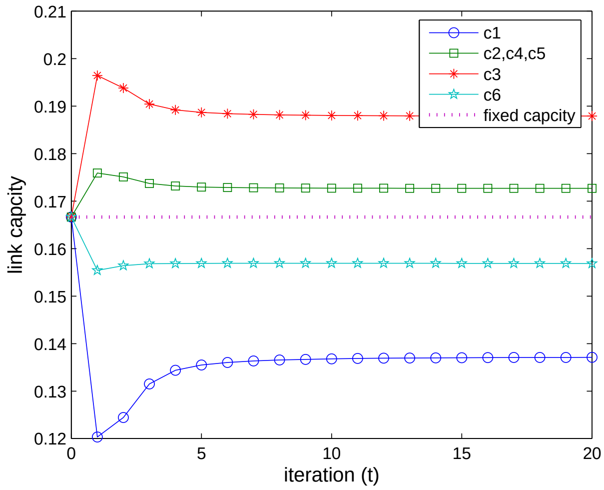

Figure 3: The evolution of link capacity

where $x'^{\star}$, $p^{\star}$ and $q^{\star}$, and $c'^{\star}$ are solutions to problems (11), (14) and (15), respectively.

We summarize the joint congestion control, contention control, and resource allocation (JCCRA) algorithm in table 1. The convergence and optimality of JCCRA algorithm can be established by theorem 1.

THEOREM 1. *JCCRA algorithm converge to the optimal dual solution* $\boldsymbol{\lambda}^{\star}$ *and at* $\boldsymbol{\lambda}^{\star}$, $\mathbf{x}'^{\star}, \mathbf{p}^{\star}, \mathbf{q}^{\star}, \mathbf{r}^{\star}$ *are optimal to problem (4), if following condition is satisfied*

$$\sum_{l \in L_{out}(n)} \lambda_l^{\star} + \sum_{k \in L^I(n)} \lambda_k^{\star} \neq 0, \quad \forall n \tag{21}$$

PROOF. The condition guarantees problem (4) to be strictly concave, therefore, the optimal solution is unique.

By Danskin's theorem [18]

$$\frac{\partial g(\boldsymbol{\lambda})}{\partial \lambda_l} = c'_l + \log p_l + \sum_{k \in N^I(l)} \log(1 - q_k) - x'_l \tag{22}$$

Therefore, (20) is a subgradient algorithm for problem (8). Properly choosing the stepsize $\alpha(t)$ (*e.g.*, [17], [18], [19]), the algorithm guarantees to converge to the optimal dual solution $\boldsymbol{\lambda}^{\star}$.

Due to the convexity of problem (4) and the uniqueness of solutions, $\mathbf{x}'^{\star}, \mathbf{p}^{\star}, \mathbf{q}^{\star}, \mathbf{r}^{\star}$ are optimal to the problem [15]. □

4. NUMERICAL RESULTS

In this section, we provide some numerical results for the JCCRA algorithm. The network topology in Fig. 1 is considered, which has six directed links with variable capacity. We assume if and only if the distance between the transmitter of one link and the receiver of the other link is less than $2d$, transmission of the first link will cause interference strong enough to influence reception of the second link.

Consider utility function $U_l(x_l) = (1-\theta)^{-1} x_l^{1-\theta}$ [20] with $\theta = 2$, thus $U'_l(x'_l)$ is strictly concave utility. The total resource is normalized to $R_T = 1$ and let $c_l(r_l) = r_l$ which guarantees the concavity of $c'_l(r_l)$.

We compare the JCCRA algorithm with algorithm 1 proposed in [8]. Algorithm 1 assume fixed capacity at each link, thus we set $c_l = 1/6$ in accordance with the JCCRA ensemble. In the following simulations, diminishing stepsize $\alpha(t) = 1/t$ is used [18].

First of all, the most important feature of the JCCRA algorithm is the adaptivity of link capacity to alleviate congestion. Fig. 3 presents results from this aspect. We can observe that more congested links (*e.g.*, c_3) are assigned with more resources thus have larger throughput, while the less congested ones (*e.g.*, c_1) have less capacity. Both algorithms are with the same overall capacity due to the total resource constraint. Dynamic resource allocation enables the system to improve performance as we will see in the following.

Table 2 shows the rate and utility comparison of the two algorithms. As expected, in terms of total rate and utility, there can be performance enhancement using the JCCRA algorithm. We can observe that utility of more congested links are greatly increased, while the less crowded ones are slightly decreased. Therefore, this tradeoff finally improve the system performance.

Finally, we show the evolution of link persistence probability in Fig. 4. In general, the probability does not vary too much for two algorithms. For the JCCRA algorithm, less congested links are assigned with large persistence probability in compensate for the reduction of capacity, which can be regarded as another reason for the performance improvement.

Table 2: Rate and utility comparison of the JCCRA alg. and alg. 1

	x_1	x_2	x_3	x_4	x_5	x_6	Total
JCCRA	0.0656	0.0512	0.0467	0.0512	0.0512	0.0569	0.3227
Alg. 1	0.0662	0.0482	0.0431	0.0482	0.0482	0.0552	0.3092
	U_1	U_2	U_3	U_4	U_5	U_6	Total
JCCRA	-15.26	-19.54	-21.41	-19.54	-19.54	-17.58	-112.86
Alg. 1	-15.10	-20.74	-23.18	-20.74	-20.74	-18.12	-118.61

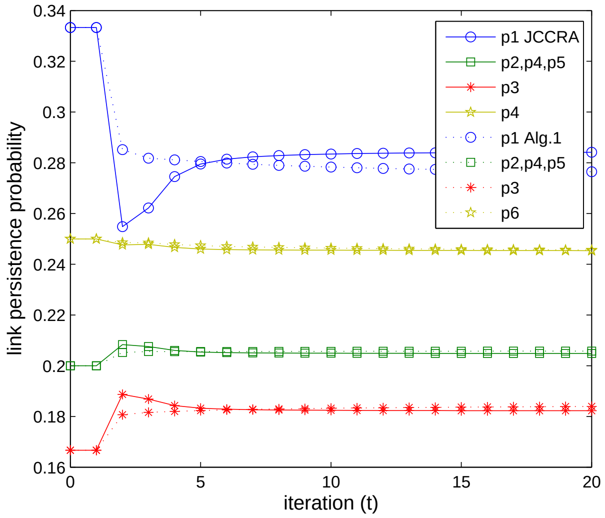

Figure 4: The evolution of link persistence probability

5. CONCLUSION

With the aim of fully utilizing the property of wireless networks, we jointly consider congestion control, contention control and resource allocation. Based on random-access MAC and system-wide resource allocation model, we extend the basic NUM and obtain a rigorous and systematic design. We use a two-level dual based decomposition for solving it, then propose practical algorithm and prove their convergence to the globally optimum. Performance improvement have been demonstrated via numerical results.

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