Joint Rate and Power Allocation in Spectrum Sharing Networks with Balanced QoS Provisioning and Power Saving

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ABSTRACT

In this paper, we study the joint rate and multi-channel power allocations in spectrum sharing networks (SSNs) with balanced QoS provisioning and power saving. We formulate this cross layer problem as a non-cooperative game G_{JRPA} in which each user aims to achieve its target data rate as exactly as possible and minimize its power consumption at the same time. We investigate the properties of Nash equilibrium (N.E.) for G_{JRPA}, including its existence, and properties of QoS provisioning as well as power saving. With the decomposition technique, we propose a distributed algorithm to find the N.E. for G_{JRPA} from any feasible initialization and provide a sufficient condition for its convergence. Simulation results are presented to show the validity of our proposed algorithm. Performance tradeoff between QoS provisioning and power saving are investigated via tuning the weighting factors. Besides, advantages of our G_{JRPA} over conventional iterative water filling (IWF) are also analyzed.

Categories and Subject Descriptors

C. 2.1 [**Network Architecture and Design**]: Wireless Communications

Keywords

Spectrum Sharing, Qos Provisioning, Game Theory

1. INTRODUCTION

The fast development of wireless communication systems with heterogeneous Quality of Service (QoS) requirements and different mobile device capabilities entails a great demand for flexible and efficient access mechanism to the valuable spectrum resources. Dynamic spectrum access (DSA), which allows different systems to share a common part of spectrum under certain restrictions, has been considered a promising technique to fullfill this demand for next generation ubiquitous wireless networks. At present three categories of DSA has been classified [1], including: (a) *Exclusive use model*; (b) *Hierarchical access model*; (c) *Open sharing model*. The first two access models, i.e., *Exclusive use model* and *Hierarchical access model* are based on interference-free restrictions, which means that strict orthogonality is guaranteed in radio resources (time slot and/or spectrum channel) used by different systems. While in the *Open sharing model* unlicensed systems[1] are free to use some common frequency band with their mutual interference limited. For example, Wi-Fi and Bluetooth horizontally share the unlicensed ISM band in adjacent area without causing harmful interference to each other. Despite gaining advantages of flexible and easy implementation, the open spectrum sharing faces the risks of inefficiency and unfairness in spectrum utilization, which is mainly due to the selfish nature of individual system. These drawbacks become more serious when mobile users with heterogeneous QoS requirements, different power capacities and asymmetric channel conditions coexist.

Current research reveals that blindly maximizing either the aggregate system utility [2] or each user's own utility [3] always results in unfairness among users and inefficiency in spectrum utilization of the whole system. [5] studies this unfairness issue with repeated game by assuming a long term interaction among users. [9] considers the heterogeneous QoS requirements in SSNs and proposes a non-cooperative game with piece-wise utility function. However, this simple and intuitive design of utility function has a drawback of non-continuity, therefore influencing the existence of N.E..

Besides QoS guarantee, power saving is also an important issue in SSNs because most of the mobile devices are power-limited. Motivated by these considerations, we study the joint rate and power allocations in SSNs with both QoS provisioning and power saving. We first formulate this problem as a non-cooperative game G_{JRPA} in which each user selfishly aims to achieve its target QoS requirement with as low power allocation as possible. A cost function with multi-objective expression is adopted for each user to indicate its consideration on both QoS provisioning and power

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[1]In [1] primary (licensed) system is referred to the system having exclusive ownership of some frequency band authorized by regulatory. Secondary (unlicensed) system is referred to the system having no pre-authorized frequency to use.
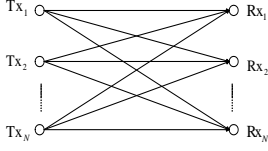
Figure 1: Network Model. N users share a common set of K channels.

saving. Then we investigate the properties of N.E. for our G_{JRPA}, including its existence, and properties of QoS provisioning as well as power saving. Furthermore, we analyze our proposed game model based on decomposition technique [6] and design a distributed algorithm to find the N.E for it. Different from [7] focusing on source coding in application layer, our game model addresses the link layer rate allocation and PHY layer multi-channel power allocation problem. Moreover, we characterize our G_{JRPA} as potential game [14] and provide the properties of N.E. from this particular game model's perspective.

We organize this paper as follows. In Section 2, we present the SSNs scenario and the game model with joint rate and power allocations. We investigate the properties of N.E. for our formulated game in Section 3. Decomposition structure is introduced in Section 4 and detailed study of PHY layer power allocation game is presented. We propose a distributed algorithm to find the N.E. in Section 5. Simulation results are presented in Section 6 and we conclude this paper in Section 7.

2. NETWORK MODEL AND GAME THE-ORETIC FORMULATION

We consider a multi-channel SSNs as shown in Figure 1, where a set of users $\mathcal{N} = \{1, 2, ..., N\}$ share a common part of spectrum B which is equally divided into a set of $\mathcal{K} = \{1, 2, ..., K\}$ channels. Each user $i \in \mathcal{N}$ consists of a transmitter TX_i and an intended receiver RX_i. We assume that each user i has a target QoS represented by target data rate R_i^{tar} at link layer. Each user i selfishly aims to achieve its R_i^{tar} as exactly as possible and minimize its power allocation at the same time. In this paper we focus on the cases that the target rate profile $(R_1^{tar}, R_2^{tar}, ..., R_N^{tar})$ of all users are acceptable to the network. Issues about determining the feasible region of target rate profile and designing an admission control mechanism with which users requiring unacceptable target rates can be identified are our future work.

Due to the co-channel interference among different users, their achievable data rates are coupled together and conflicting with each other. Therefore, we formulate this multiuser rate and power allocation problem as a non-cooperative game with the formal description given as follows:

$$G_{JRPA} = \{\mathcal{N}, \{\chi_i\}_{i \in \mathcal{N}}, \{\Psi_i\}_{i \in \mathcal{N}}\} \qquad (1)$$

where $\mathcal{N}$ denotes the set of users coexisting in SSNs, χ_i and Ψ_i represent user i's strategy space and cost function respectively. Since achieving the target rate R_i^{tar} and minimizing the power allocation are two conflicting objectives, we design a cost function Ψ_i for each user i with a multi-objective

expression [10] as follows:

$$\Psi_i(r_i, \mathbf{p}_i) = \alpha_i \left(\frac{r_i - R_i^{tar}}{R_i^{tar}}\right)^2 + \beta_i \left(\sum_{k \in \mathcal{K}} p_i^k\right) \qquad (2)$$

where the two-tuple expression $(r_i, \mathbf{p_i})$ includes user i's rate allocation r_i, and power allocation vector $\mathbf{p}_i = (p_i^1, p_i^2, ..., p_i^K)$ over the K channels. α_i and β_i represent user i's weighting factors on QoS provisioning and power saving. Large value of the ratio $\frac{\alpha_i}{\beta_i}$ means user i puts more emphasis on QoS provisioning.

In G_{JRPA}, due to the co-channel interference, each user i's strategy space χ_i is coupled with all the other users. Specifically, the strategy space of user i can be expressed as:

$$\chi_i(\mathbf{p}_{-i}) = \{(r_i, \mathbf{p}_i) \in \Omega_i^R \times \Omega_i^P | r_i \le \sum_{k \in \mathcal{K}} \Phi_i^k(p_i^k, \mathbf{p}_{-i}^k)\} \quad (3)$$

$\Omega_i^R = \{r_i \in \mathbb{R} | R_i^{min} \le r_i \le R_i^{max}\}$ is user i's rate allocation range, where R_i^{min} and R_i^{max} denote the practical lower and upper bounds of user i's data rate respectively. $\Omega_i^P = \{\mathbf{p}_i \in \mathbb{R}^K | \sum_{k \in \mathcal{K}} p_i^k \le P_i^{max}; pmin_i^k \le p_i^k \le pmax_i^k, \forall k \in \mathcal{K}\}$ is user i's power allocation range, where P_i^{max} denotes user i's power capacity. $pmin_i^k$ and $pmax_i^k$ denote the practical lower and upper bounds of user i's power allocation on channel k.[2] The constraint

$$r_i \le \sum_{k \in \mathcal{K}} \Phi_i^k(p_i^k, \mathbf{p}_{-i}^k) \qquad (4)$$

guarantees that user i's rate allocation r_i in link layer can't exceed the maximum achievable data rate according to its power allocation $\mathbf{p}_i$ in PHY layer, given all the other users' power allocations $\mathbf{p}_{-i} = (\mathbf{p}_1, \mathbf{p}_2, ..., \mathbf{p}_{i-1}, \mathbf{p}_{i+1}, ..., \mathbf{p}_N)$ fixed. Specifically, on each channel $k \in \mathcal{K}$, Φ_i^k has the information-theoretic capacity formula as:

$$\Phi_i^k(\mathbf{p}_i, \mathbf{p}_{-i}) = \frac{B}{K} \log(1 + \frac{p_i^k g_{ii}^k}{\eta_i^k}) \qquad (5)$$

where

$$\eta_i^k = \sum_{j \ne i, j \in \mathcal{N}} p_j^k g_{ji}^k + n_i^k$$

is the power of interference plus background noise measured by user i's receiver on channel k. g_{ji}^k is the channel gain from user j's transmitter to user i's receiver over channel k[3]. n_i^k is the power of background noise of user i over channel k. Therefore, based on (3) the strategy profile space χ for G_{JRPA} can be expressed as:

$$\chi = \{(r_1, \mathbf{p}_1, r_2, \mathbf{p}_2, ..., r_N, \mathbf{p}_N) | (r_i, \mathbf{p}_i) \in \Omega_i^R \times \Omega_i^P, \forall i \in \mathcal{N};$$
$$r_i \le \sum_{k \in \mathcal{K}} \Phi_i^k(p_i^k, \mathbf{p}_{-i}^k), \forall i \in \mathcal{N}\}$$

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[2] Practically, R_i^{min} can be set as user i's minimum tolerable data rate and R_i^{max} can be determined by user i's maximum achievable data rate without considering the interference from other users. We assume in this paper that $R_i^{min} < R_i^{tar} < R_i^{max}, \forall i \in \mathcal{N}$ hold. $pmin_i^k, pmax_i^k$ can be considered as user i's spectrum mask constraints on channel k, which are specific to cognitive radio system [13].

[3] We consider a relatively static network in this work, i.e., the set of channel gains $g_{ji}^k, \forall i, j \in \mathcal{N}, \forall k \in \mathcal{K}$ keeps unchanged during the time interval of interest.

Given $\mathbf{p}_{-i}$, each user $i \in \mathcal{N}$ faces a joint rate and power allocations problem as follows:

$$\min_{(r_i, \mathbf{p}_i) \in \chi(\mathbf{p}_{-i})} \Psi_i(r_i, \mathbf{p}_i) \qquad (6)$$

According to the definition of Nash Equilibrium (N.E.) [19], the equilibrium rate and power allocation profile $(\mathbf{r}^*, \mathbf{p}^*)$ for G_{JRPA} should satisfy the following equilibrium condition:

$$(r_i^*, \mathbf{p}_i^*) = \arg \min_{(r_i, \mathbf{p}_i) \in \chi(\mathbf{p}_{-i}^*)} \Psi(r_i, \mathbf{p}_i), \forall i \in \mathcal{N} \qquad (7)$$

which guarantees that no single user has the incentive to deviate from N.E. unilaterally.

3. PROPERTIES OF N.E.

In this section we first investigate the existence and uniqueness of N.E. for G_{JRPA}. Then we study the properties of QoS provisioning[4] and power saving of N.E..

3.1 Existence and Uniqueness

In G_{JRPA} each user i simultaneously achieves its equilibrium rate allocation r_i^* and power allocation $\mathbf{p}_i^*$ after arriving at N.E. characterized by (7). By viewing G_{JRPA} as a potential game [14], we describe the existence of N.E. in the following lemma.

Lemma 1: There always exists a pure strategy N.E. for G_{JRPA} on the strategy profile space χ if $\alpha_i > 0, \beta_i \geq 0, \forall i \in \mathcal{N}$.

PROOF. The G_{JRPA} can be considered as a potential game with exact potential function $\mathscr{F}$ given as:

$$\mathscr{F}(r_1, \mathbf{p}_1, ..., r_N, \mathbf{p}_N) = \sum_{i \in \mathcal{N}} \Psi_i(r_i, \mathbf{p}_i)$$

For $\forall i \in \mathcal{N}$ and $\forall (r_i^a, \mathbf{p}_i^a, r_{-i}, \mathbf{p}_{-i}), (r_i^b, \mathbf{p}_i^b, r_{-i}, \mathbf{p}_{-i}) \in \chi$, the potential function $\mathscr{F}$ has the property that

$\Psi_i(r_i^a, \mathbf{p}_i^a) - \Psi_i(r_i^b, \mathbf{p}_i^b) =$
$\mathscr{F}(r_1, \mathbf{p}_1, ..., r_{i-1}, \mathbf{p}_{i-1}, r_i^a, \mathbf{p}_i^a, r_{i+1}, \mathbf{p}_{i+1}, ..., r_N, \mathbf{p}_N) -$
$\mathscr{F}(r_1, \mathbf{p}_1, ..., r_{i-1}, \mathbf{p}_{i-1}, r_i^b, \mathbf{p}_i^b, r_{i+1}, \mathbf{p}_{i+1}, ..., r_N, \mathbf{p}_N)$

where $(r_i^a, \mathbf{p}_i^a)$ and $(r_i^b, \mathbf{p}_i^b)$ are two feasible rate-power allocation tuples of user i given all the other users' rate-power allocations fixed at $(r_{-i}, \mathbf{p}_{-i})$. Therefore, $\mathscr{F}$ exactly measures the difference in global cost due to the unilateral deviation from N.E. of each single user.

Let $\mathcal{P}_{min}$ denote the set of minima of $\mathscr{F}$ on χ. $\mathcal{P}_{min}$ always holds non-empty because χ is a compact set and $\mathscr{F}$ is continuous, bounded on χ. Furthermore, $\mathscr{F}$ is continuously differentiable in the interior of χ and convex on χ. Therefore, according to Theorem 3 and Corollary 5 [14], there exists at least one pure strategy N.E. in G_{JRPA} on χ. $\square$

Furthermore, still based on Theorem 3 [14], the uniqueness of N.E. for G_{JRPA} follows if the strategy profile space χ is a compact, convex set. However, in fact the convexity of χ cannot be guaranteed and many studies have been carried out either to convexify the capacity formula $\sum_{k \in \mathcal{K}} \Phi_i^k(p_i^k, \mathbf{p}_{-i}^k)$ by assuming the high SINR conditions [6][3] or to convexify the capacity region [8] by assuming a sufficient number of orthogonal channels and the feasibility of time sharing among users. In this work, we only report via simulation

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[4]In this paper, QoS provisioning is referred to achieving the target rate as exactly as possible.

that the uniqueness of N.E. for G_{JRPA} is always observed using our proposed algorithm if certain channel condition is met, which we will describe in Section 5.

3.2 QoS Provisioning and Power Saving

An important property of G_{JRPA} is that after arriving at the N.E. each user i can get a satisfactory balance between QoS provisioning and power saving according to its value of $\frac{\alpha_i}{\beta_i}$. Specifically, no user will get redundant data rate beyond its target rate and no power is wasted to achieve unwanted data rate, i.e., the rate allocation constraint (4) is strictly binding at the N.E.. We describe these two properties in the following lemma.

Lemma 2: The following two properties always hold at the N.E. of G_{JRPA}:
(a) $r_i^* = \sum_{k \in \mathcal{K}} \Phi_i^k(p_i^{k*}, \mathbf{p}_{-i}^{k*}), \forall i \in \mathcal{N}$.
(b) $R_i^{min} \leq r_i^* \leq R_i^{tar}, \forall i \in \mathcal{N}$.

PROOF. Lemma 2 can be proved by showing that contradiction will occur if (a) or (b) is violated at N.E..

With respect to (a), assume there exists a N.E. $(r_i^e, \mathbf{p}_i^e, r_{-i}^e, \mathbf{p}_{-i}^e)$ where for a particular user i', its equilibrium rate allocation $r_{i'}^e < \sum_{k \in \mathcal{K}} \Phi_{i'}^k(p_{i'}^{ke}, \mathbf{p}_{-i'}^k)$. Then according to the equilibrium condition (7), user i' always has the incentive to reduce power allocation on some channels such that its cost $\Psi_{i'}(r_{i'}, \mathbf{p}_{-i'})$ is reduced while keeping constraint (4) still satisfied, i.e., user i' will deviate from this assumed N.E. unilaterally. Therefore, $(r_i^e, \mathbf{p}_i^e, r_{-i}^e, \mathbf{p}_{-i}^e)$ cannot be the N.E. for G_{JRPA}.

With respect to (b), assume similarly that there exists a N.E. $(r_i^e, \mathbf{p}_i^e, r_{-i}^e, \mathbf{p}_{-i}^e)$ where for a particular user i', its equilibrium rate allocation $r_{i'}^e > R_{i'}^{tar}$. Then there always exists a corresponding rate $r_{i'}^* = 2R_{i'}^{tar} - r_{i'}^e < R_{i'}^{tar}$, which satisfies $(\frac{r_{i'}^* - R_{i'}^{tar}}{R_{i'}^{tar}})^2 = (\frac{r_{i'}^e - R_{i'}^{tar}}{R_{i'}^{tar}})^2$. Meanwhile, given $\mathbf{p}_{-i'}^e$, the optimal power allocation $\mathbf{p}_{i'}^*$ for (6) has the property that $\sum_{k \in \mathcal{K}} p_{i'}^{k*} < \sum_{k \in \mathcal{K}} p_{i'}^{ke}$ when rate $r_{i'}^*$ is adopted by user i'. Therefore, the cost $\Psi_{i'}(r_{i'}^*, \mathbf{p}_{i'}^*) < \Psi_{i'}(r_{i'}^e, \mathbf{p}_{i'}^e)$, i.e., $(r_i^e, \mathbf{p}_i^e)$ does not satisfy the equilibrium condition (7), meaning that $(r_i^e, \mathbf{p}_i^e, r_{-i}^e, \mathbf{p}_{-i}^e)$ cannot be the N.E. for G_{JRPA}. $\square$

4. DUAL DECOMPOSITION AND LAYERED STRUCTURE

We introduce partial dual decomposition into G_{JRPA} in this section, through which we show that our joint resource allocation game can be decomposed vertically into a set of rate allocations with QoS provisioning for each individual user and a weighted multi-channel power allocation game with linear pricing among all users.

Our G_{JRPA} differs from conventional models in that each user i has rate-power two-tuple strategy $(r_i, \mathbf{p}_i)$ simultaneously, and $r_i, \mathbf{p}_i$ are coupled by constraint (4). We use partial dual decomposition to relax this constraint and treat the corresponding dual variable z_i as a coordinator between rate allocation and power allocation for each user i. Specifically, the Lagrangian function for user i can be written as:

$$L_i(r_i, \mathbf{p}_i, z_i) = \alpha_i \left(\frac{r_i - R_i^{tar}}{R_i^{tar}}\right)^2 + \beta_i \left(\sum_{k \in \mathcal{K}} p_i^k\right) + z_i \left(r_i - \sum_{k \in \mathcal{K}} \Phi_i^k(p_i^k, \mathbf{p}_{-i}^k)\right)$$

Therefore, given dual variable z_i, each user i's optimization

problem (6) can be vertically separated into two subproblems as follows:

(a) Rate allocation with QoS provisioning in link layer

$$r_i^*(z_i) = \arg \min_{r_i \in \Omega_i^R} \alpha_i (\frac{r_i - R_i^{tar}}{R_i^{tar}})^2 + z_i r_i \qquad (8)$$

(b) Power allocation with linear pricing in PHY layer

$$\mathbf{p}_i^*(z_i, \mathbf{p}_{-i}) = \arg \min_{\mathbf{p}_i \in \Omega_i^P} \beta_i (\sum_{k \in \mathcal{K}} p_i^k) - z_i \sum_{k \in \mathcal{K}} \Phi_i^k(p_i^k, \mathbf{p}_{-i}^k) \quad (9)$$

Subproblems (a) and (b) are connected by the dual problem as follows:

$$\max_{z_i \geq 0} D_i(z_i) = \max_{z_i \geq 0} L_i(r_i^*(z_i), \mathbf{p}_i^*(z_i, \mathbf{p}_{-i}), z_i) \qquad (10)$$

For any given $\mathbf{p}_{-i}$, according to Slater's conditions [16], strong duality exists between each user i's primal convex problem (6) and dual problem (10). Therefore, the optimum solution for (6) can always be found by solving (10) without any performance loss.

We note in (9) that the optimal power allocation $\mathbf{p}_i^*(z_i, \mathbf{p}_{-i})$ of each user i also depends on other users' power allocations. Given a dual vector $\mathbf{z} = (z_1, z_2, ..., z_N)$, the PHY layer multi-channel power allocations can be expressed as a non-cooperative game with linear pricing mechanism among all users as follows:

$$G_{WPA}(\mathbf{z}) = \{\mathcal{N}, \{\Omega_i^P\}_{i \in \mathcal{N}}, \{\Theta_i(z_i)\}_{i \in \mathcal{N}}\} \qquad (11)$$

In $G_{WPA}(\mathbf{z})$, each user i has an independent strategy space only determined by its own power allocation range Ω_i^P and its cost function can be written as:

$$\Theta_i(z_i, \mathbf{p}_i, \mathbf{p}_{-i}) = \beta_i \sum_{k \in \mathcal{K}} p_i^k - z_i \sum_{k \in \mathcal{K}} \Phi_i^k(p_i^k, \mathbf{p}_{-i}^k) \qquad (12)$$

where the achievable data rate $\sum_{k \in \mathcal{K}} \Phi_i^k(p_i^k, \mathbf{p}_{-i}^k)$ is weighted by the dual variable z_i to indicate its QoS weighting factor and a linear pricing mechanism $\beta_i \sum_{k \in \mathcal{K}} p_i^k$ is incorporated to address its power saving consideration. In [3] the authors investigate the information rate maximization game G_{IRM} as follows:

$$G_{IRM} = \{\mathcal{N}, \{\Omega_i^P\}_{i \in \mathcal{N}}, \{-\sum_{k \in \mathcal{K}} \Phi_i^k(p_i^k, \mathbf{p}_{-i}^k)\}_{i \in \mathcal{N}}\} \qquad (13)$$

Our weighted power allocation game G_{WPA} generalizes G_{IRM} by incorporating both the QoS weighting factor from link layer and power saving factor from PHY layer into each user's cost function. Figure 2 shows the decomposition structure for our G_{JRPA}. Since G_{WPA} plays an important role in our decomposition structure and differs from many other studies, we investigate the properties of N.E. for G_{WPA} in detail and describe them in the following lemma.

Lemma 3: (a) There always exists a pure strategy N.E. for $G_{WPA}(\mathbf{z})$ on the strategy profile space $\Omega^P = \Omega_1^P \times \Omega_2^P \times ... \times \Omega_N^P$ for any given dual vector $\mathbf{z} = (z_1, z_2, ..., z_N) > \mathbf{0}$ and pricing vector $\beta = (\beta_1, \beta_2, ..., \beta_N) \geq \mathbf{0}$. (b) Furthermore, if the channel gains satisfy that

$$\rho(\mathcal{G}^k) < 1, \forall k \in \mathcal{K} \qquad (14)$$

where $\rho(.)$ denotes the spectrum norm and for each $k \in \mathcal{K}$, the N by N matrix $\mathcal{G}^k$ is defined as:

$$[\mathcal{G}^k]_{ij} = \begin{cases} \frac{g_{ji}^k}{g_{ii}^k}, \text{ if } i \neq j \\ 0, \text{ otherwise} \end{cases} \qquad (15)$$
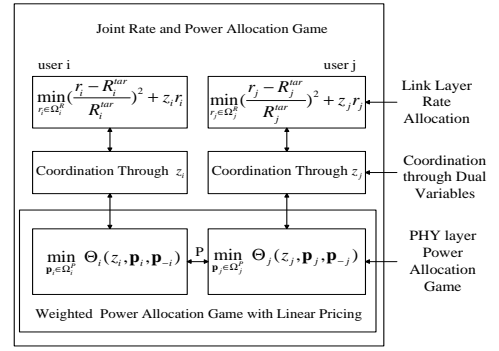

Figure 2: Decomposition Structure of G_{JRPA}

then there exists a unique pure strategy N.E. on Ω^P for $G_{WPA}(\mathbf{z})$.

PROOF. We only describe the key points in proof of Lemma 3 due to the limited space. Detailed procedures can be referred to [11]. With respect to the proof of existence, we show that: (a) for each user $i \in \mathcal{N}$, its strategy space Ω_i^P is a compact and convex set, and (b) for each user $i \in \mathcal{N}$, its cost function $\Theta_i(z_i, \mathbf{p}_i, \mathbf{p}_{-i})$ is continuous on strategy profile space Ω^P and strictly convex on $\mathbf{p}_i$. Therefore, according to Theorem 4.3 [18], there exists a pure strategy N.E. for $G_{WPA}(\mathbf{z})$ over Ω^P. With respect to the proof of uniqueness, we provide a sufficient condition for the uniqueness of N.E. for $G_{WPA}(\mathbf{z})$ by extending Theorem 2 in [3]. Lemma 3 (b) indicates that the equilibrium power allocation profile of $G_{WPA}(\mathbf{z})$ is a well-defined function of dual vector $\mathbf{z}$ as long as $\mathbf{z} > 0$, $\beta \geq \mathbf{0}$ and the channel condition (14) is met. This desirable property is exploited in our proposed algorithm to find the N.E. for G_{JRPA} in Section 5. $\square$

5. DISTRIBUTED RATE AND POWER ALLOCATION ALGORITHM

We propose a distributed algorithm to find the N.E. for G_{JRPA} in this section. Our algorithm is based on the decomposition structure of rate allocation (8), weighted power allocation game $G_{WPA}(\mathbf{z})$ (11) and the coordinating dual problem (10). To facilitate the proposed algorithm, we assume synchronization among all users and design two different time scales for rate and power updating. Specifically, let ΔT denote the long updating interval for both link layer rate allocation and dual updating. Meanwhile, let $\Delta t = \Delta T/M$ denote the short updating interval for PHY layer power allocation, where M is a sufficient large number such that for any given dual vector $\mathbf{z}$, the equilibrium power allocation for $G_{WPA}(\mathbf{z})$ can be reached before the end of ΔT. Figure 3 shows the procedures of rate, power allocation and dual updating in a single ΔT.

We propose our distributed rate and power allocation in algorithm (**DRPA**) below.

We remark our proposed algorithm as follows:

Remark 1: Algorithm (**DRPA**) shows that after receiving the dual variable z_i and pricing factor β_i, each user i requires channel gain g_{ii}^k and the power of interference plus background noise η_i^k to implement the Jacobian iteration for (17). η_i^k can be measured by each user i over channel k and g_{ii}^k can be obtained by channel estimation. Meanwhile, each user i allocates its rate r_i (16) and updates its dual
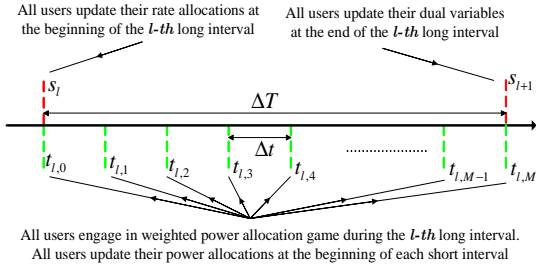
All users update their rate allocations at the beginning of the *l-th* long interval

All users update their dual variables at the end of the *l-th* long interval

All users engage in weighted power allocation game during the *l-th* long interval. All users update their power allocations at the beginning of each short interval

Figure 3: Rate, Power and Dual Updating in ΔT

Distributed Rate and Power Allocation (DRPA)
Initialization Step:
Each user $i \in \mathcal{N}$ initializes $r_i(s_0) \in \Omega_i^R$, $z_i(s_0) > 0$ and $\mathbf{p}_i(t_{0,0}) \in \Omega_i^P$, where we set $t_{0,0} = s_0$
Iteration Process:
(a) **Best Response for Rate Allocation:**
Given $z_i(s_l)$, each user $i \in \mathcal{N}$ updates r_i at the beginning of the lth long updating index $s_l = s_0 + l * \Delta T, l = 0, 1, 2, ...,$ as follows:

$$r_i(s_l) = [R_i^{tar} - \frac{(R_i^{tar})^2}{2\alpha_i} z_i(s_l)]_{R_i^{min}}^{R_i^{max}} \qquad (16)$$

(b) **Jacobian Iteration for Power Allocation:**
Given dual vector $\mathbf{z}(s_l)$, all users participate in $G_{WPA}(\mathbf{z}(s_l))$ during the lth long updating interval. Specifically, during the lth ΔT each user $i \in \mathcal{N}$ updates $\mathbf{p}_i$ at the beginning of the mth short updating index $t_{l,m} = s_l + m * \Delta t, m = 0, 1, ..., M$ as follows: (Initialize $p_i^k(t_{l,0}) = p_i^k(t_{l-1,M}), \forall k \in \mathcal{K}$)

$$p_i^k(t_{l,m}) = [\frac{z_i(s_l)}{\beta_i + \lambda_i(t_{l,m})} - \frac{\eta_i^k(t_{l,m-1})}{g_{ii}^k}]_{pmin_i^k}^{pmax_i^k}, \forall k \in \mathcal{K} \quad (17)$$

and $\lambda_i(t_{l,m})$ is the optimal dual variable for user i's power capacity constraint at the index $t_{l,m}$, which satisfies that:

$$\lambda_i(t_{l,m}) \geq 0, \quad \lambda_i(t_{l,m})(P_i^{max} - \sum_{k \in \mathcal{K}} p_i^k(t_{l,m})) = 0, \forall i \in \mathcal{N}$$

(c) **Subgradient Updating for Dual Vector:**
Each user $i \in \mathcal{N}$ updates z_i at the end of the lth long updating index as follows:

$$z_i(s_{l+1}) = [z_i(s_l) + \zeta_i(r_i(s_l) - \sum_{k \in \mathcal{K}} \Phi_i^k(p_i^k(t_{l,M}), \mathbf{p}_{-i}^k(t_{l,M})))]^+ \ (18)$$

(d) **Repeat Steps (a), (b) and (c) until convergence**
End of Algorithm

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variable z_i (18) using only local information. Therefore, the proposed distributed power allocation algorithm (**DRPA**) relies mostly on local measurements and requires few message exchange.
Remark 2: The convergence of algorithm (**DRPA**) can be described as follows. First of all, we provide a convergence lemma for the distributed power allocation (17) as follows:
Lemma 4: The Jacobian iteration for power allocation (17) during each long updating interval ΔT always converges to the equilibrium power allocation profile for $G_{WPA}(\mathbf{z})$ with any given dual vector $\mathbf{z} > \mathbf{0}$ and pricing vector $\beta \geq \mathbf{0}$ if the channel condition (14) is satisfied.

PROOF. Our proof is based on [12] and detail procedures can be referred to [11]. Lemma 4 states that the sufficient condition for the unique N.E. in $G_{WPA}(\mathbf{z})$ also serves as the convergence condition for (17). Similar results have been provided in Theorem 1 [4] for conventional IWF, where, however, the pricing mechanism for power allocation is not considered. □

Second, the sub-gradient updating (18) is guaranteed to converge to the optimal dual value if the step-size ζ is chosen according to the diminishing rule [17]. Furthermore, according to the convexity of (6) and Slater's condition, there exists no dual gap between the primal optimum of (6) and dual optimum of (10). Therefore, the whole algorithm (**DRPA**) can always converge to the N.E. for G_{JRPA}.
Remark 3: As described above, the value of M should be chosen large enough so that the Jacobian iteration for power allocation (17) can converge to the equilibrium point for $G_{WPA}(\mathbf{z})$ before the end of each ΔT. However, our simulation results show that convergence of power allocation for $G_{WPA}(\mathbf{z})$ during each ΔT is not strictly required and about $M = 2$ to 5 is generally enough for the whole algorithm (**DRPA**) to converge.

6. SIMULATION RESULTS

Simulation results are presented in this section to show the validity of our proposed distributed rate and power allocation algorithm. We build a network scenario where $N = 6$ users share $K = 10$ channels with bandwidth $B/K = 1$ per channel. Users are located at different places and the channel gains $g_{ij}^k, \forall i, j \in \mathcal{N}, \forall k \in \mathcal{K}$ are generated according to the path-loss model[5]. Same background noise power $n_i^k = 0.1, \forall i \in \mathcal{N}, \forall k \in \mathcal{K}$ is assumed and we only consider the channel realization with the condition (14) satisfied.

Figures 4, 5 and 6 show the convergence of our algorithm for a random channel realization. The weighting factors are set as $\frac{\alpha_i}{\beta_i} = 10, \forall i \in \mathcal{N}$. Figure 4 shows the convergence of total power allocation for each user. Figure 5 shows the convergence of rate allocation. The solid lines denote rate allocations of users 1, 3, 5. The dash-dot lines marked by circle, triangle and asterisk denote the achievable data rates according to users 1, 3, 5's power allocations in PHY layer. It is shown in Figure 5 that after convergence, the equilibrium multi-channel power allocations in PHY layer provide exactly the equilibrium rate allocations of users 1 ,3, 5 in link layer without any power waste. Meanwhile, users 1, 3,

[5] Specifically, $g_{ij}^k = d_{ij}^{-\gamma} \xi_{ij}^k$, where $d_{ij}^{-\gamma}$ is the path loss between TX_i and RX_j with exponent γ (we set $\gamma = 2$) and ξ_{ij}^k is a normalized random factor modeling the frequency selective fading across different channels.
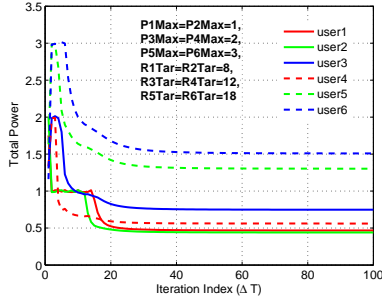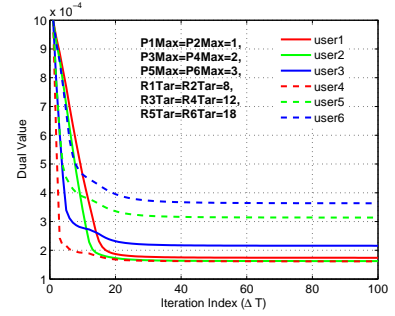
Figure 4: Convergence of Power Allocation

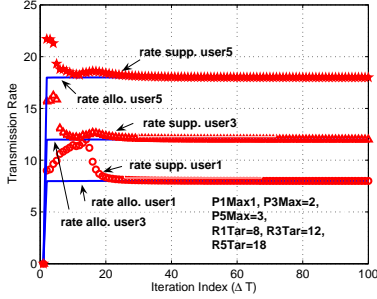**Figure 6: Convergence of Dual Updating**

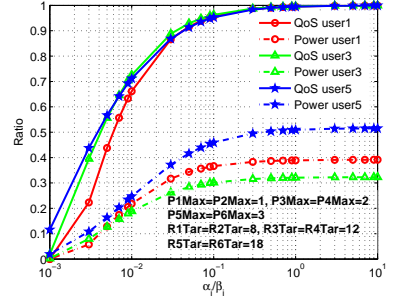**Figure 5: Convergence of Rate Allocation**

Figure 7: Tradeoff between QoS provisioning and Power Saving

5 almost get their target data rates without any redundance. Therefore, no power is wasted by any user to get unwanted rate. Figure 6 shows the convergence of coordinating dual variable for each user.

Figure 7 shows the tradeoff between QoS provisioning and power saving for each user i through tuning the value of $\frac{\alpha_i}{\beta_i}$. Specifically, the pair of solid and dash-dot lines marked with circle denote the ratios r_1^*/R_1^{tar} and $\sum_{k \in \mathcal{K}} p_1^{*k}/P_1^{max}$ for user 1 with $\frac{\alpha_1}{\beta_1}$ changing from 0.001 to 10 while fixing $\frac{\alpha_i}{\beta_i} = 10$ for $i = 2, 3, 4, 5, 6$. It is shown in Figure 7 that with large value of $\frac{\alpha_1}{\beta_1}$, (i.e., putting more emphasis on QoS guarantee), user 1 can achieve larger data rate but consuming more power. However, with small value of $\frac{\alpha_1}{\beta_1}$ (i.e., putting more emphasis on power saving), user 1 can save more power but with the degradation in data rate. The pair of solid and dash-dot lines marked with triangle show the similar performance tradeoff for user 3. So does the pair of lines marked with pentagram for user 5. Meanwhile, it is also shown in Figure 7 that each user i almost achieves above 95% of its target rate if it puts enough emphasis on QoS provisioning (e.g., $\frac{\alpha_i}{\beta_i} \geq 1$).

Table I shows the comparison of equilibrium results between our proposed G_{JRPA} and G_{IRM} [4] where each user blindly aims to maximize its transmission rate for one random channel realization. In Table I, $P_1^{max} = P_2^{max} = 1, P_3^{max} = P_4^{max} = 2, P_5^{max} = P_6^{max} = 3$ and $R_1^{tar} = R_2^{tar} = 6, R_3^{tar} = R_4^{tar} = 10, R_5^{tar} = R_6^{tar} = 14$. Meanwhile, we set $\frac{\alpha_i}{\beta_i} = 10, i = 1, 2, ..., 6$. It is shown in Table I that G_{IRM} generates the result that each user may attain redundant data rate larger than required at the cost of exhausting its power capacity. In comparison, at the N.E. of our G_{JRPA}, all users almost achieve their target rates with only negligible gaps

and none of them spends more power than needed to get any redundant rate, therefore achieving the power saving objective at the same time. Meanwhile, thanks to the QoS guiding and power saving properties, G_{JRPA} gains the advantage of lowering down the aggregate interference level in SSNs, which is especially useful in cases where the QoS requirements (i.e., target data rates) are inversely proportional to the power capacities. Table II demonstrates this advantage of G_{JRPA} using the same set of channel gains and power capacities as the case in Table I, but changing the target rates to $R_1^{tar} = R_2^{tar} = 12, R_3^{tar} = R_4^{tar} = 10, R_5^{tar} = R_6^{tar} = 6$. As shown in Table II, G_{IRM} generates the same equilibrium state as in Table I due to its blindness in QoS and power saving, which results in that users 1, 2 cannot get their target rates both due to their own power limitations as well as too much interference caused by other users. In comparison, at the N.E. of our G_{JRPA}, all users almost can get their target rates even without using up all of their power capacities. As explained before, the reason is that users 3, 4, 5, 6 become QoS guided and avoid too much power allocation to get unwanted data rates, which therefore reduces the aggregate interference level in SSNs and makes the target rates of users 1, 2 acceptable.

7. CONCLUSIONS

In this paper, we study the joint rate and multi-channel power allocations in SSNs with balanced QoS provisioning and power saving. We formulate this cross layer problem as a non-cooperative game G_{JRPA} in which each user aims to achieve its target QoS as exactly as possible and minimize its power allocation at the same time. We investigate the properties of N.E. for G_{JRPA}, including its existence, and

Table 1: Comparison of Rate Achieved and Power Consumed between G_{JRPA} and G_{IRM} (Proportional QoS Requirement-Power Capacity)

	G_{JRPA} Rate (Power)	G_{IRM} Rate (Power)
user1	5.9965 (0.3019)	8.1846 (1.0000)
user2	5.9969 (0.2643)	9.3892 (1.0000)
user3	9.9946 (0.6496)	12.4636 (2.0000)
user4	9.9940 (0.7167)	11.7067 (2.0000)
user5	13.9935 (0.9567)	17.5530 (3.0000)
user6	13.9932 (1.0144)	16.8805 (3.0000)

Table 2: Comparison of Rate Achieved and Power Consumed between G_{JRPA} and G_{IRM} (Inverse Proportional QoS Requirement-Power Capacity)

	G_{JRPA} Rate (Power)	G_{IRM} Rate (Power)
user1	11.9952 (0.6599)	8.1846 (1.0000)
user2	11.9957 (0.5944)	9.3892 (1.0000)
user3	9.9954 (0.5681)	12.4636 (2.0000)
user4	9.9955 (0.5577)	11.7067 (2.0000)
user5	5.9970 (0.2503)	17.5530(3.0000)
user6	5.9973 (0.2335)	16.8805 (3.0000)

properties of QoS provisioning as well as power saving. In addition, we propose a distributed algorithm to find the N.E. for G_{JRPA} and provide a sufficient condition for its convergence. Simulation results are presented to show the validity of our proposed algorithm and examples are given to illustrate the advantage of our G_{JRPA}. We make an assumption in this paper that the target rate profile $(R_1^{tar}, R_2^{tar}, ..., R_N^{tar})$ is acceptable to the whole SSNs. However, practical cases in which target rates of some users are not acceptable can occur. Our future work is to analyze these cases and design a distributed admission control mechanism so that users with unacceptable target rates can be identified.

8. REFERENCES

[1] Q. Zhao, A. Swami 'A Survey of Dynamic Spectrum Access: Signal Processing and Networking Perspectives,' in *Proc. IEEE ICASSP2007*

[2] J. Huang, R. Berry and M. L. Honig, 'Distributed Interference Compensation for Wireless Networks,' *IEEE JSAC*, 24(5):1074–1084, May 2006

[3] G. Scutari, D. P. Palomar and S. Barbarossa, 'Optimal Linear Precoding Strategies for Non-cooperative System Based on Game Theory - Part I: Nash Equilibrium,' *IEEE Trans. on Signal Prcessing*, 56(3):1230–1249, Mar. 2008

[4] G. Scutari, D. P. Palomar and S. Barbarossa, 'Optimal Linear Precoding Strategies for Non-cooperative System Based on Game Theory - Part II: Algorithms,' *IEEE Trans. on Signal Prcessing*, 56(3):1250–1267, Mar. 2008

[5] R. H. Etkin, A. K. Parekh and D. N. C. Tse 'Spectrum Sharing for Unlicensed Bands,' *IEEE JSAC*, (25)3:517–528, Apr. 2007

[6] M. Chiang, 'Balancing Transport and Physical Layers in Wireless Multihop Networks: Joint Optimal Congestion Control and Power Control,' *IEEE JSAC*, 23(1):104–116, Jan. 2006

[7] J. Yuan, and W. Yu, 'Distributed Cross-Layer Optimization of Wireless Sensor Network: A Game Theoretic Approach,' in *Proc. IEEE GLOBECOM2006*

[8] R. Cendrillon, and W. Yu, 'Optimal Multiuser Spectrum Balancing for Digital Subscriber Lines,' *IEEE Trans. on Communications*, 54(5):922–933, May 2006

[9] T. Jin, C. Chigan, and Z. Tian, 'Game-Theoretic Distributed Spectrum Sharing for Wireless Cognitive Radio Networks,' in *Proc. IEEE GLOBECOM2006*

[10] J. H. Zhu, C. Chen, B. Bensaou, and K. L. Huang, 'Tradeoff between Lifetime and Rate Allocation in Wireless Sensor Networks: A Cross Layer Approach,' in *Proc. IEEE INFOCOM2007*

[11] Y. Wu and D. H. K. Tsang 'Efficient Spectrum Sharing with Balanced QoS Provisiong and Power Saving' *Technical Reprot*, Mar. 2008, HKUST

[12] K. W. Shum, K. K. Leung, and C. W. Sung 'Convergence of Iterative Waterfilling Algorithm for Gaussian Interference Channels,' *IEEE JSAC*, 25(2):1091–1100, Aug. 2007

[13] F. Wang, M. Krunz, and S. Cui, 'Price-Based Spectrum Management in Cognitive Radio Networks,' *IEEE Journal on Selected Topics in Signal Processing*, 2(1):74–87, Feb. 2008

[14] G. Scutari, S. Barbarossa and D. P. Palomar, 'Potential Games: A Framework for Vector Power Control Problems with Coupled Constraints,' in *Proc. IEEE ICASSP2006*

[15] Y. Yuan, P. Bahl, R. Chandra, T. Moscibroda, and Y. Wu, 'Allocating dynamic time-spectrum blocks in cognitive radio networks,' in *Proc. ACM MobiHoc2007*

[16] S. Boyd, L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004

[17] S. Boyd, L. Xiao and A. Mutapcic, *Subgradient Method*. Lecture Notes of EE392o, Standford University, Autumn, 2003

[18] T. Basar, and G. T. Olsder, *Dynamic Noncooperative Game Theory*. 2nd ed. SIAM series classics in applied mathematics, Philadephia, PA: SIAM, 1995

[19] M. J. Osborne and A. Rubinstein, *A Course in Game Theory* MIT Press, 1994