Spectral Sensing with Coupled Nanoscale Oscillators

Nikolai Nefedov[1,2]
[1]Nokia Research Center, Zurich, Switzerland
[2]Signal and Information Processing Lab (ISI) ETH Zurich, Switzerland
email:nikolai.nefedov@nokia.com

ABSTRACT

In this paper we propose a method to perform tunable spectral sensing using globally coupled oscillators. The suggested system may operate is analog (RF) domain without high speed ADC and heavy digital signal processing. Oscillator arrays may be made of imprecise elements such as nano-resonators. Provided a proper coupling, the system dynamics can be made stable despite the imprecision of components. Global coupling could be implemented using a common load and controlled by digital means to tune the bandwidth. This method may be used for spectral sensing in cognitive radio terminals.

H.4: Information Systems Applications: Miscellaneous

G.1.7: Mathematics of Computing: Numerical Analysis: ODE

G.3.14: Statistical Computing

1. INTRODUCTION

Development of high data rate communication systems requires implementation of complicated algorithms operating at high speed with low power consumption. Despite the progress in digital CMOS technology, high data rate digital processing poses a number of problems both for signal digitizing and its processing.

These problems motivate a search for new architectures capable to combine advances in scaling, new materials and information processing at nano-scale. Currently there is growing interest in development of signal processing systems using nano-scale devices such as molecular electronics, nano-wires, nanomechanical systems and etc [1]-[9]. For example, scaling MEMS to nano-scale gave rise for low power nano-electromechanical systems (NEMS) with operational frequencies above GHz (frequency range typical for modern communication systems) [5],[6].

Recent advances in nanotechnology and scaling allow to build systems with a large number of nano-resonators (such as CNT-based NEMS [8]) integrated within CMOS chip. On the other hand, scaling poses a problem to maintain accuracy of elements. It is especially pronounced in fabrication of nano-scale devices where top-down design is not possible, and technology process and impurities may result in unpredictable components' accuracy.

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A practical way to cope with inaccuracy may be seen in building hybrid systems combining low precision analog/nano components with digital calibration/control. Besides, in network structures one may use a proper coupling, such that the system dynamics may be made stable despite the imprecision of components [10]. In particular, collective behavior of coupled NEMS networks with assistance of coupling and calibration provided by digital CMOS may allow to implement low power information processing algorithms in analog domain with digital calibration/control. For example, it makes feasible spectral processing in RF domain which is of the special importance for high data rate wireless communications systems. The suggested method may be used for spectral sensing in cognitive radio terminals [11], where wide radio spectrum bands are to repeatedly scanned in real-time with low power consumption.

The paper is organized as follows. Sec.2 provides a background on coupled oscillators. Proposed method of spectral sensing based on inhibitory globally coupled arrays and its possible NEMS implementation are outlined in Sec.3 and Sec.4, respectively, with conclusions followed in Sec.5.

2. COUPLED OSCILLATORS

2.1 Background

Coupled osillators have been under intensive studies in different fields of science for decades (e.g., see [10][12][13][14] and references within). In this section we outline the basic concepts and introduce notation used in the following.

Let's consider an oscillator in a steady state limit cycle described by a single variable, the phase of the limit cycle θ. This results in the simple motion equation, $\frac{d\theta}{dt} = \omega$, where ω is the frequency of the oscillator with period $T = 2\pi/\omega$. The solution for this equation is $\theta(t) = \omega t + \theta(0)(\mathrm{mod}\, 2\pi)$, where $\theta(0)$ is the initial phase at time $t = 0$. Geometrically $\theta(t)$ is presented as a point moving on a circle. A system of N coupled oscillators resides on N circles and forms N-dimensional torus T^N with vector coordinates $\boldsymbol{\theta} = (\theta_1, \theta_2, ...\theta_N)$, $0 < \theta_n(t) < 2\pi$, $n = 1, ...N$.

We consider weak coupling among the oscillators, such that oscillators maintain their limit-cycle behavior perturbed by coupling. It allows us to ignore coupling affect on oscillator's amplitudes and describe system only with phase relationships as

$$\frac{d\theta_n}{dt} = \omega_n + Q_n(\theta_1, \theta_2, ...\theta_N)$$

where is ω_n is natural (uncoupled) frequency of n-th oscillator, Q_n presents a coupling effect from all other oscillators on the n-th oscillator. The coupling function Q_n is 2π-periodic in each of its arguments (evolution of θ_n depends on a position of each of the N oscillators in their cycles, no dependance on a number of cycles passed). We further assume that coupling may be separated into contributions due to the interaction between any two pairs of oscillators (i.e., the total coupling effect is the sum of single-link effects)

$$Q_n(\theta_1, \theta_2, ...\theta_N) = \sum_{m=1}^{N} q_{nm}(\theta_m, \theta_n)$$

For weak coupling $q_{nm}(\theta_m, \theta_n) = q_{nm}(\theta_m - \theta_n)$ and $q_{nm}(0)$=0. The latter corresponds to no interactions if two identical oscillators are in phase with each other. Such coupling is known as diffusive coupling, motion equations are

$$\frac{d\theta_n}{dt} = \omega_n + \sum_{m=1}^{N} q_{nm}(\theta_m - \theta_n). \quad (1)$$

Since g_{nm} is to be 2π-periodic, we may present it in Fourier series. Taking only the first Fourier term, (1) may be written

$$\frac{d\theta_n}{dt} = \omega_n + \sum_{m=1}^{N} a_{nm} \sin(\theta_m - \theta_n) \quad (2)$$

Cases when $a_{nm} > 0$ and $a_{nm} < 0$ are called *excitatory* and *inhibitory* coupling, respectively. In the simplest example of $N = 2$ coupled oscillators, their motion is described by

$$\frac{d\theta_1}{dt} = \omega_1 + a_{12} \sin(\theta_2 - \theta_1)$$
$$\frac{d\theta_2}{dt} = \omega_2 + a_{21} \sin(\theta_1 - \theta_2)$$

By defining a phase lag $\varphi(t) = \theta_1(t) - \theta_2(t)$ the motion equations are simplified to

$$\frac{d\varphi}{dt} = (\omega_1 - \omega_2) + (a_{12} + a_{21}) \sin\varphi$$

which may be solved exactly. A special solution called 1:1 phase-locking motion (phase lag $\varphi = const$) is

$$\varphi = \arcsin\frac{\omega_1 - \omega_2}{a_{12} + a_{21}}.$$

This equation may have either no, one or two solutions depending on whether $\left|\frac{\omega_1 - \omega_2}{a_{12} + a_{21}}\right| > 1, = 1,$ or < 1. In the former case, no phase-locking solution, the system is said to drift. Note that the coupling $a_{nm} > 0$ may be increased until the systems goes from drift into phase-locking.

2.2 Globally coupled oscillators

A special case of (2) with $a_{nm} = a/N$ ($a = const > 0$) corresponds to uniform global coupling and is well known as globally-coupled Kuramoto model [12][13]

$$\frac{d\theta_n}{dt} = \omega_n + \frac{a}{N} \sum_{m=1}^{N} \sin(\theta_m - \theta_n) \quad (3)$$

This equation may have a range of solutions from periodic to chaotic. For example, periodic solutions include: (i) phase sync state $\theta_n(t) = \theta_0(t)$ for all n; (ii) "phase-lock" state where all oscillators have the same waveform but are shifted by a fixed phase $\theta_n(t) = \theta_0(t + nT/N)$, where T is oscillation period (creating a rotating wave); (iii) partial sync there both states may co-exist. Kuramoto showed that the system (3) may be analytically tractable and in the limit $N \to \infty$ there is a critical value of coupling strength k_c, such that for $a/N = k > k_c$ both frequency and phase sync appear in the system [12].

Let's define a complex mean field for N oscillators with equal unit amplitude as

$$R(t) = \frac{1}{N} \sum_{n=1}^{N} e^{i\theta_n(t)} = re^{i\psi(t)}$$

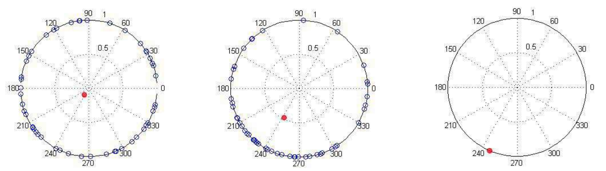
Global coupling may be seen as the total mean field effect acting on a selected oscillator, then (3) may be rewritten as

$$\frac{d\theta_n}{dt} = \omega_n + kr \sin(\psi - \theta_n) \quad (4)$$

where k is the strength of all-to-all coupling and r is mean-field amplitude. If identical oscillators are all in phase-sync, then oscillations added in phase create just one oscillation with max mean field amplitude ($r = 1$), whereas for random phases oscillators show a chaotic behavior with minimum mean field amplitude ($r = 0$). For this reason the mean-field amplitude r is also referred as the order factor.

2.3 Oscillators with attractive global coupling

First let's consider oscillators with equal natural frequencies $\omega_n = \omega_0$. On the phase plot this common frequency appears as the collective angle motion of all oscillators. In the following we use moving coordinates where $\omega_0 = 0$. Figures below present time evolution of randomly initialized oscillator phases obtained by numerically solving system of equations (4) for $N = 50$. Starting from uniform random phase distribution (Fig.1a) the positive coupling $k > 0$ in globally coupled network results first in phase clustering (Fig.1b) followed by phase synchronization. As expected, the order factor (shown as red-filled circle) is growing as phases of oscillators are grouped and approaches max when phase synchronization is reached (Fig.1c).



a) t=0 b) t=10 c) t=40

Fig.1. Evolution of phases of $N = 50$ identical ($\omega_n = \omega_0$) oscillators with global coupling $k = 0.3$ in time t .

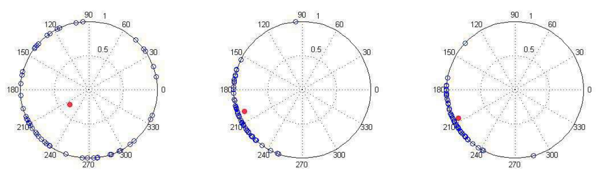


Fig.2. Evolution of phases of $N = 50$ oscillators with random normal distributed frequencies ($\sigma_\omega^2 = 0.02$) with different global coupling: a) $t = 20$, $k = 0.3$ no sync; b) $t = 20$, $k = 0.5$ part sync,c) $t = 40$, $k = 0.6$, sync.

In practice the oscillators' frequencies are not identical, their natural frequencies may be modeled as random values taken from some distribution. As an example, below we consider Gaussian frequency distribution with zero mean and variance σ_ω^2=0.02. System dynamics for non-identical oscillators is more complicated. Provided that coupling strength is large enough compared to frequency variations, the system evolves from quasi-chaotic (Fig.1a; Fig.3a) to partial synchronization (Fig.2b; Fig.3b), where oscillators with close frequencies are frequency locked, resulting in growing mean-field (red-filled point at Fig.2) which in turn attracts further staying apart (in frequency) oscillators into the frequency lock. As illustration, evolution of different oscillators' frequencies in time at different coupling strengths are depicted at Fig.3. Note that oscillators with natural frequencies $|\omega_n - \omega_0| > kr$ can not be attracted to the frequency lock, it results in partial frequency sync and lower steady-state order factor r. But even in case when all oscillators are frequency-locked, it results at best in phase-mode locking (constant phase difference), but not in phase synchronization (where phase difference is zero). Phase-mode locking among oscillators result in fluctuating order factor with variance proportional to $N^{-1/2}$.

Effect of frequency synchronization in globally attractively-coupled system allows to built fixed-frequency oscillators from resonators with randomly spread frequencies. Possible applications of these systems to appear elsewhere [16].

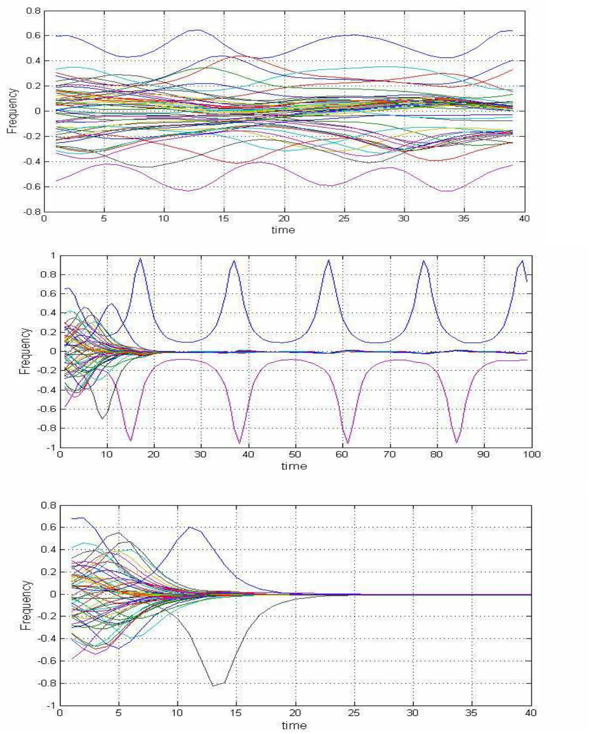


Fig.3. Evolution of random ($\sigma_\omega^2 = 0.02$) oscillators' frequencies in time: a) k=0.3 no sync; b) k=0.5 part.sync; c) k=0.6 sync.

3. INHIBITORY COUPLED OSCILLATORS

As mentioned above, it is well known that oscillator arrays with different natural frequencies may be driven into a collective behavior without external force provided strong enough positive global coupling. The convergence of these kinds of systems into a sync (measured by the order factor) depends mainly on coupling strength that masks external excitations and limits signal processing possibilities. To make coupled oscillators to be more sensitive for external excitations we suggest to translate external signals into coupling strength. However, before this mapping we need to keep the system out of sync when no external signal is present. It can be done by utilizing *inhibitory* coupling preventing the system to fall into a collective behavior. Then, once the external excitation is above the inhibitory coupling, it forces the system (proportionally to the strength of the external excitation) to move into more ordered behavior. As we outline below, it seems feasible to build systems sensitive to a strength and the spectral content of the excitation.

In case of inhibitory global coupling it may be shown that for identical oscillators ($\omega_n = \omega_0$) without external forcing the mean field reaches zero from any initial condition (cf. Fig.4a) at arbitrary small negative coupling strength $k < 0$ (Fig.4b). The stationary regime corresponds to all oscillators having identical amplitude, but different phases. Note that the phase distribution is not uniform nor unique: the stationary regime may have multiple phase distributions among individual oscillators subject to the only constraint $\sum_n e^{j\theta_n} = 0$.

For non-equal oscillators' frequencies and without external force the mean field oscillates at small values of inhibitory coupling. The transition to the synchronized regime depends on number of oscillators in the array. For $N > 3$ the oscillators do not reach phase-sync at any value of coupling, but $r \to 0$ as the strength of inhibitory coupling increases. If natural frequencies, e.g., ω_{n-1} and ω_n, are sufficiently close, then there is phase-locking between these oscillators resulting in a stable independent of time periodic motion $\varphi_n = \theta_n - \theta_{n-1}$. For large values of $|\theta_n - \theta_{n-1}|$ there is a phase drift with the faster oscillation processing over the slow one.

3.1 Forced oscillations

Models for phase coupled oscillators (3) may be generalized to include external forcing

$$\frac{d\theta_n}{dt} = \omega_n + Q_n(\theta_1, \theta_2, ...\theta_N) + Q_F((\theta_1^{ext}, \theta_2^{ext}, ...\theta_M^{ext}) \quad (5)$$

where Q_F is 2π-periodic function in all of its arguments. Repeating derivations of Sec.2 we may write

$$\frac{d\theta_n}{dt} = \omega_n + kr\sin(\psi - \theta_n) + q_n^{ext}\sin(\theta_n^{ext}) \quad (6)$$

Last term in (6) may be replaced by $q_n^{ext}\sin(\theta_n - \omega_{ext}t)$, where ω_{ext} is the frequency of the external force. Introducing new variable $\varphi_n = \theta_n - \omega_{ext}t$ we obtain again the equation (6) for φ_n but with other natural frequencies $\omega_n - \omega_{ext}$. Hence, periodic external forcing may be seen as a modification of natural frequencies in the oscillator array.

If external periodic forcing $\omega_{ext} = \omega_0$ applied, oscillators with identical frequencies synchronize with identical phases to a non-zero mean field (the convergence is faster as $|k|$ increases).

For negatively-coupled oscillators with non-equal frequencies and external periodic forcing $\omega_{ext} = \omega_0$, there will be fluctuations of mean field at any value of coupling. Oscillator phases and frequencies evolve from chaotic to partially sync as amplitude of external force is increasing.

Dependence of (averaged) order factor from amplitude A of external periodic driving force $\omega_{ext} = \omega_0$ at different coupling strength is shown at Fig.5. As one can see, at given coupling the order factor r depends monotonically from amplitude of the external force. This property may be used for energy sensing of external signals addressed below. Fig.6 shows averaged order factor for global inhibitory-coupled oscillators (coupling $k = -1$) with different amplitude-frequency characteristics of forcing. Dependence of order factor of coupled oscillators from frequency of forcing signal with fixed amplitude $A = 1$ at different coupling is depicted at Fig.7.

As one may see, by adjusting coupling strength one may tune the frequency bandwidth. It allows to utilize amplitude-frequency selectivity and its dependance on tuning and the amplitude of external excitation for spectral sensing in radio systems (e.g., cognitive radio). Below we outline a possible implementation of this kind of sensing with nanoscale resonators.

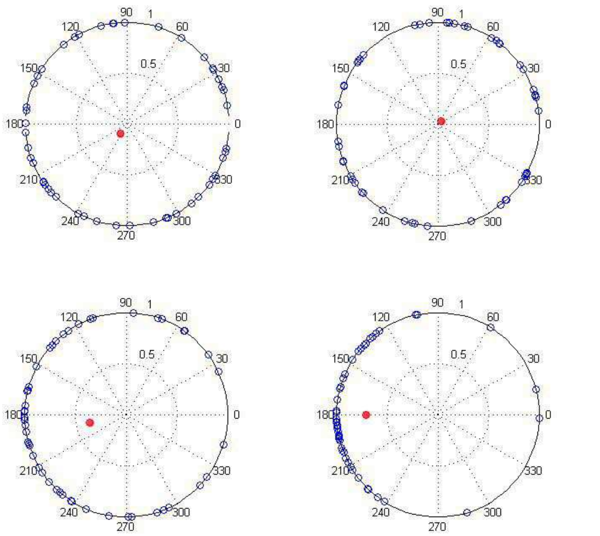


Fig.4. Evolution of phases of $N = 50$ oscillators with random normal distributed frequencies ($\sigma_\omega^2 = 0.02$) with inhibitory global coupling $k = -1$ and different amplitude of external forcing A: (a) initial condition; (b) A=0, (c) A=0.5, (d) A=1 at t=100.

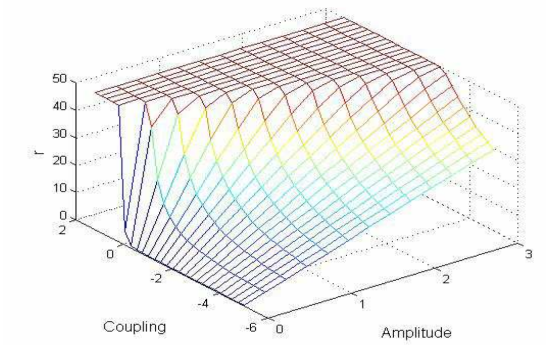


Fig.5. Order factor of coupled oscillators as function of coupling and amplitude of forcing signal $\omega_{ext} = \omega_0$.

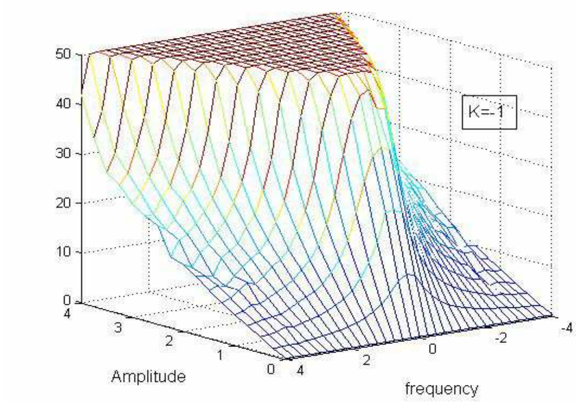


Fig.6. Order factor of coupled oscillators ($k = -1$) as function of amplitude and frequency of forcing signal.

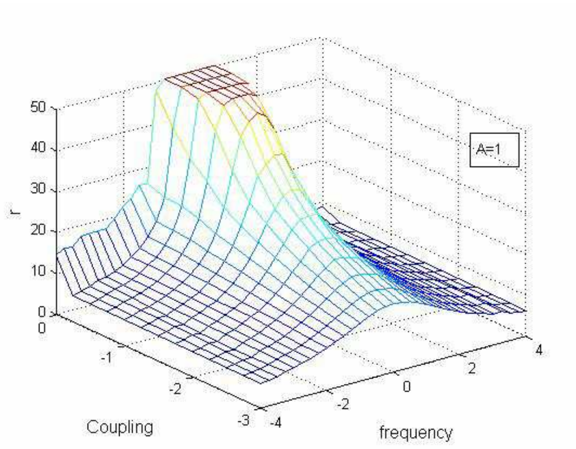


Fig.7. Order factor of coupled oscillators as function of coupling and frequency of forcing signal, A=1.

4. NANOSCALE RESONATORS

Electromechanical systems (EMS) convert electrical signals into mechanical motion by exciting a resonant mode of a mechanical element. Then mechanical response, namely the displacement of the element, is transduced back into electrical signals.

Resonant nanostructures have been addressed in a number of papers. For example, parametric resonance in an electrostatically driven nanowire is studied in [3], laser-driven limit cycle oscillators in NEMS (nano-EMS) resonators in different disc shapes and wires are reported in [7], mechanically coupled NEMS with resonant frequencies up to 18 MHz are presented in [15].

Besides silicon-based NEMS, carbon nanotubes (CNT) are under intensive studies because of their superior mechanical properties, small cross-sections and possibility for defect-free self-assembling [2]. Additionally, the CNT can act as a transistor, may be able to sense its own motion and can be made CMOS compatible. Recently it is shown that CNT can be used as nano-switches [4] and as GHz oscillators [5][6]. As an example we outline oscillator arrays formed by CNT-based NEMS.

Let's consider a suspended CNT clamped on both sides to metal pads (source and drain) and capacitively coupled to a gate as reported in [8], tunable CNT-based NEMS is depicted at Fig.8. Similar to MEMS, we may add positive feedback loop that converts drain output current into voltage (I/V block at Fig.8) which then is fed back to the gate to excite CNT resonance modes. Provided a proper positive feedback this structure may be used as limit-cycle oscillator (rotator) shown as the dashed-dot box at Fig.8. The applied DC gate voltage changes CNT strain and hence controls the eigenmodes which may be excited by external AC source ω_{ext}.

One-dimension motion of a nanotube can be described by Duffing equation [2]

$$m\ddot{x} + b\dot{x} + k_1 x + k_3 x^3 = dE_{cap}/dx$$

where $E_{cap} = C(x)V^2/2$ is capacitance energy, $C(x)$ is the displacement dependent capacitance, b and k_i are dumping and spring coefficients. In general a linearly (weakly) coupled system may be described as

$$m\ddot{x}_n + f(x_n)\dot{x}_n + h(x_n) = \sum_m (b_{nm}\dot{x}_m + k_{nm}x_m)$$

It may be shown that for small displacements and weak interactions globally coupled Duffing oscillators may be described by (4) [9]. Under these assumptions we may consider a system of *inhibitory* globally coupled oscillators shown at Fig.9. Here current outputs from oscillators $I(\omega_n)$ are combined and coupled by connecting to a common load (similar to globally coupled Josephson junctions) followed by the feedback via current/voltage conversion (I/V block at Fig.9). The amount and sign of the feedback may be controlled by operational amplifiers (OA) with tunable amplification. The same OA is used to block external signal leakage into the common load. In the absence of external signal the global negative feedback is (digitally) calibrated to prevent system convergence into frequency lock and keep the order parameter close to zero[1]. In the presence of external signals $V(\omega_{ext})$ the output of the oscillator array depends on amplitude and frequency content of excitation (cf.Fig.5-Fig.7). Global coupling strength k may be digitally controlled to tune the bandwidth of spectral sensing (Fig.7). Note that the absolute value of order factor is increasing with the number oscillators allowing significant signal amplification, which is important for nano-scale resonators such as NEMS.

CNT strain and its eigenmodes may be tuned by changing DC voltage V_g (Fig.8). Tuning voltage gate V_g may be used to adjust the whole set of frequencies $\Omega = \{\omega_1, ..., \omega_N\}$ in a given oscillator array (Fig.9).

Several oscillator arrays with different frequency sets Ω_n tuned by DC gate voltage V_g (to adjust frequency) and coupling k (to adjust bandwidth) may be used to make coarse spectral sensing as shown at Fig.10. In particular, when voltage $V(r_{\Delta\Omega_n})$ corresponding to order factor(s) $r_{\Delta\Omega_n}$ of some frequency band(s) is below a threshold E_0 (set by the control block), it indicates potential spectrum holes. Then control block sends information on relevant frequency(s) band $\Delta\Omega_n$ to the frequency synthesizer which sets relevant local frequency for the mixer, at the same time the input RF signal is connected to the mixer. After downconverting the baseband signal in a selected (relatively narrow) frequency band is digitized by ADC and then analyzed in the feature detector. If needed the feature detector may refine frequency and bandwidth of the osillator array via feedback control.

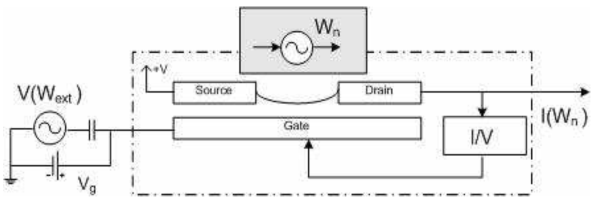


Fig.8. Tunable CNT-based NEMS oscillator (cf. [8]).

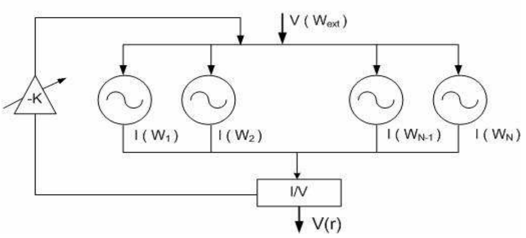


Fig.9. Oscillator array with global coupling.

[1] Recall that there is also positive feedback within each oscillator to maintain limit cycle rotations.

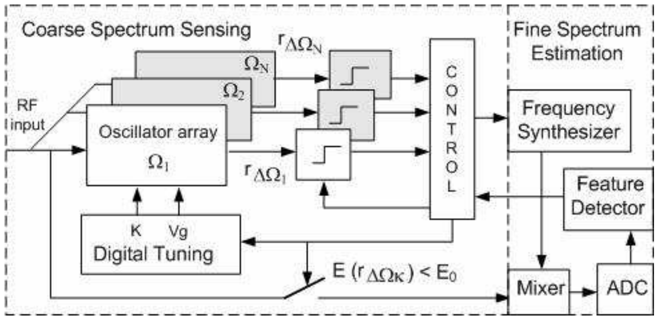


Fig.10. Tunable coupled oscillator arrays for spectral sensing.

5. CONCLUSIONS

In this paper we present the method of coarse spectral sensing in analog domain using globally coupled limit-cycle oscillators which may be made of imprecise nanoscale components such as CNT-based NEMS. Provided proper coupling, the system collective behavior (measured by order factor) becomes sensitive to amplitude and frequency content of excitation, digital control of coupling allows to tune the spectral sensing position and bandwidth. This method may be used in cognitive radio terminals.

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