Atomic Hierarchical Routing Games in Communication Networks

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***Abstract*—Theoretical studies on routing games in networks have so far dealt with reciprocal congestion effects between routing entities. But, with the advent of technologies like Cognitive Radio, we have networks which support differentiation of flows. In a two priority level model a user can be high priority or low priority and there is a cost for such a classification. The point of departure of this model from the traditional routing scenarios is the absence of reciprocity in the congestion effects: The low priority flow faces congestion from both high priority as well as low priority flow while the high priority flow is immune to the congestion effects from the low priority flow. This hierarchy is naturally present in contexts where there are primary (licensed) users and secondary (unlicensed) users who can sense their environment because there are equipped with a cognitive radio [9]. We study such kind of routing scenarios for the cases of atomic users. We establish the existence and the uniqueness of Nash equilibrium and further we show the existence of a potential function for linear congestion costs and a certain priority classification pricing scheme. Natural applications of this model to Cognitive Radio are also pointed out.**

I. Introduction

Non-cooperative routing has long been studied both in the framework of road-traffic as well as in the framework of wireless networks. Such frameworks allow to model the flow configuration that results in networks in which routing decisions are made in a non-cooperative and distributed manner between the users. In the case of a finite (not very large) number of agents, the resulting flow configuration corresponds to the so called Nash equilibrium defined as a situation in which no agent has an incentive to deviate unilaterally. In other words, the Nash equilibrium does not state what can or cannot happen when more than one decision maker changes their strategy (route) simultaneously. When the decision makers in a Nash game are discrete and finite in number, a Nash equilibrium can be achieved without the costs of all used routes being equal, contrary to Wardrop's equilibrium principle [11]. In some cases, Wardrop's principle has been shown to represent a limiting case of the Nash equilibrium principle, as the number of users becomes very large [4], [5], [13]. In the context of routing in networks, the Nash equilibrium has been introduced by Orda et al [10] as well as [2] in order to predict the traffic pattern that occurs when each of several non-cooperative sources of traffic (say a service provider) has traffic to send and it should split (or route) its traffic among several available paths in a network. Each service provider wishes to minimize its own cost, which is however influenced by the routing decisions of other service providers. A related model was studied thirty years ago in [14].

None of these mentioned works consider the cases where the congestion effects between the routed entities are not reciprocal. To build a theoretical framework for studying such problems and to study the issues involved is the main objective of this paper. We consider a different kind of *Hierarchical* Routing game in which the routed traffic can be of two types: High priority and Low priority. The high priority traffic is unaffected by the routing profile of the Low priority traffic. On the other hand the low priority traffic faces congestion effects from both high priority and low priority traffic. The non-atomic version of the game was studied in [**?**]. In this paper we consider the atomic version of the game. An atomic hierarchical game is played between controllers or service providers who can divide their total traffic flow into these two types, classifying it as high priority being more expensive than classifying it as low priority. After this division they have to make the decision of how to route their traffic on the network. Each service provider seeks to minimize its own cost which is influenced by: 1. His classification decision and 2. His routing decision and the routing decisions of all the other service providers.

A. Our Contributions

We first define the atomic hierarchical routing game for which the existence of Nash equilibrium is established under assumptions on the link costs. We then derive a pricing scheme for the low priority classification in a hierarchical game on parallel links with linear link costs which allows for an existence of potential under certain conditions. These conditions are naturally satisfied by symmetric games for which we are thus able to prove the uniqueness of Nash equilibrium and we are also able to present certain distributed algorithms based on evolutionary dynamics which converge to this equilibrium. Natural applications of these models to cognitive radio are pointed out. Our results are finally illustrated by an example of multi-channel hierarchical rate allocation under unslotted ALOHA protocol.

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II. Atomic routing game over parallel links

Consider a network topology given by $G = (E, \{s, d\})$ where E is a set of $|E| = K$ parallel links connecting source node s to destination node d. The links are assumed to be directed from the source to the destination. We consider a model where the low priority flow is compressed when it is routed while the high priority flow is unaffected. For example if the flow is in the form of a stream of packets, then the packet size is compressed by a factor $\gamma \in (0, 1)$ if it is routed as low priority. Consider N classes of users where class i has a total flow demand r^i to ship from a source s to a destination d. Let $x_l^{i,H}$ be the high priority flow and $x_l^{i,L}$ be the low priority flow that player i ships over link $l \in E$, $l = 1, \cdots, K$. With some abuse of notation we write

$$x_l^H = \sum_{i=1}^{N} x_l^{i,H},\; x_l^L = \sum_{i=1}^{N} x_l^{i,L}$$

$$x_l = x_l^H + x_l^L$$

Also,

$$x^{i,H} = \sum_{l=1}^{K} x_l^{i,H},\; x^{i,L} = \sum_{l=1}^{K} x_l^{i,L}$$

We define the following vectors

$$\bar{x}^{i,H} = (x_1^{i,H}, \cdots, x_K^{i,H})$$

$$\bar{x}^{i,L} = (x_1^{i,L}, \cdots, x_K^{i,L})$$

$$\bar{x}_l^{H} = (x_l^{1,H}, \cdots, x_l^{N,H})$$

$$\bar{x}_l^{L} = (x_l^{1,L}, \cdots, x_l^{N,L})$$

Introduce for each user i the cost density function $G_l^H(x_l^H)$ associated with classifying the traffic as High priority on link l and the cost density function $G_l^L(x_l^H)$ for classifying it as low priority on link l both dependent on the total high priority flow on the ink with $G_l^H(a) > G_l^L(a) \forall a \geq 0$. For now, we consider constant functions of the form $G_l^H(x_l^H) = C^H$ and $G_l^L(x_l^H) = C^L$ where C^L and C^H are costs per unit flow.

For each user i introduce for each link l a cost function $J_l^{i,H}(x_l^{i,H}, \bar{x}_l^H)$ for the high priority traffic routed by him on that link which is dependent only on the high priority traffic flow profile on that link. Further introduce the cost function $J_l^{i,L}(\gamma x_l^{i,L}, \bar{x}_l^H, \gamma \bar{x}_l^L)$ for the low profile traffic which depends on the entire flow profile on that link. Note that this function depends on the low priority flow after compression. We only consider cost functions of the form $J_l^{i,H}(x_l^{i,H}, \bar{x}_l^H) = x_l^{i,H} T_l(x_l^H)$ and $J_l^{i,L}(\gamma x_l^{i,L}, \bar{x}_l^H, \gamma \bar{x}_l^L) = x_l^{i,L} T_l(x_l^H + x_l^L)$ which depend on the total flows on the link rather than the profile of individual flows. The link congestion cost density functions $T_l(.)$ are convex, continuously differentiable and monotone increasing in their argument. So the optimization problem faced by player i is to find a flow profile $\bar{x}^i = (x^{i,H}, x^{i,L})$ which solves the following problem:

$$\min \sum_{l=1}^{K} \Big[x_l^{i,H} T_l(x_l^H) + \gamma x_l^{i,L} T_l(x_l^H + \gamma x_l^L) \Big] + G_l^H(x_l^H) x^{i,H} + G_l^L(x_l^H) x^{i,L}$$

or if we consider constant pricing functions,

$$\min \sum_{l=1}^{K} \Big[x_l^{i,H} T_l(x_l^H) + \gamma x_l^{i,L} T_l(x_l^H + \gamma x_l^L) \Big] + C^H x^{i,H} + C^L x^{i,L}$$

subject to

$$x^{i,H} + x^{i,L} = r^i \;\forall\, i = 1, \cdots, N$$

$$x_l^{i,H}, x_l^{i,L} \geq 0 \;\forall\, i = 1, \cdots, N\;, l \in E$$

Under the given assumptions on the cost functions, given that the classification costs are constant, the existence of Nash equilibrium follows from arguments similar to those in [10]. Consider the case where the classification costs are not constant. Then the Kuhn-Tucker conditions for this optimization problem are given by:

$$x_l^{i,H}(T_l(x_l^H) + x_l^{i,H} T_l'(x_l^H) + \gamma x_l^{i,L} T_l'(x_l^H + \gamma x_l^L) + G_l'^H(x_l^H) x_l^{i,H} + G_l^H(x_l^H) - \lambda^i) = 0,\, l \in E \quad (1)$$

$$T_l(x_l^H) + x_l^{i,H} T_l'(x_l^H) + \gamma x_l^{i,L} T_l'(x_l^H + \gamma x_l^L) + G_l'^H(x_l^H) x_l^{i,H} + G_l^H(x_l^H) - \lambda^i \geq 0,\, l \in E \quad (2)$$

$$x_l^{i,L}(\gamma T_l(x_l^H + \gamma x_l^L) + \gamma x_l^{i,L} T_l'(x_l^H + \gamma x_l^L) + G_l^L(x_l^H) - \lambda^i) = 0,\, l \in E \quad (3)$$

$$\gamma T_l(x_l^H + \gamma x_l^L) + \gamma x_l^{i,L} T_l'(x_l^H + \gamma x_l^L) + G_l^L(x_l^H) - \lambda^i \geq 0,\, l \in E \quad (4)$$

A. Existence of a potential for linear cost density functions under a certain pricing scheme

We now consider congestion cost per unit flow functions $T_l(.)$ to be linear in their argument i.e. $T_l(a) = k_l a\,, \forall a > 0$ for some positive constant k_l for each link $l \in E$. The Kuhn Tucker conditions then reduce to the following:

$$x_l^{i,H}(k_l x_l^H + k_l x_l^{i,H} + \gamma k_l x_l^{i,L} + G_l'^H(x_l^H) x_l^{i,H} + G_l^H(x_l^H) - \lambda^i) = 0,\, l \in E \quad (5)$$

$$k_l x_l^H + k_l x_l^{i,H} + \gamma k_l x_l^{i,L} + G_l'^H(x_l^H) x_l^{i,H} + G_l^H(x_l^H) - \lambda^i \geq 0,\, l \in E \quad (6)$$

$$x_l^{i,L}(\gamma k_l(x_l^H + \gamma x_l^L) + \gamma^2 k_l x_l^{i,L} + G_l^L(x_l^H) - \lambda^i) = 0,\, l \in E \quad (7)$$

$$\gamma k_l(x_l^H + \gamma x_l^L) + \gamma^2 k_l x_l^{i,L} + G_l^L(x_l^H) - \lambda^i \geq 0,\, l \in E \quad (8)$$

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We now make the following assumption:

A1. There exists an equilibrium such that for each link l, $x_l^{i,H} > 0$ for some i if and only if $x_l^{i,H} > 0$ for all i. Similarly $x_l^{i,L} > 0$ for some i if and only if $x_l^{i,L} > 0$ for all i

Under this condition, we can now sum the above variational inequalities over all players to obtain the following set of inequalities defined on total flows on the links:

$$x_l^H((N+1)k_l x_l^H + \gamma k_l x_l^L + G_l'^H(x_l^H)x_l^H + NG_l^H(x_l^H) - \lambda) = 0,\, l \in E \tag{9}$$

$$(N+1)k_l x_l^H + \gamma k_l x_l^L + G_l'^H(x_l^H)x_l^H + NG_l^H(x_l^H) - \lambda \geq 0,\, l \in E \tag{10}$$

$$x_l^L(\gamma N k_l(x_l^H + \gamma x_l^L) + \gamma^2 k_l x_l^L + NG_l^L(x_l^H) - \lambda) = 0,\, l \in E \tag{11}$$

$$\gamma N k_l(x_l^H + \gamma x_l^L) + \gamma^2 k_l x_l^{i,L} + NG_l^L(x_l^H) - \lambda \geq 0,\, l \in E \tag{12}$$

Here N is the number of players and $\lambda = \sum_i \lambda_i$. These are the variational inequalities that define the Wardrop equilibrium with link costs for a High priority user given by

$$c_l^H = (N+1)k_l x_l^H + \gamma k_l x_l^L + G_l'^H(x_l^H)x_l^H + NG_l^H(x_l^H)$$

and the link costs for Low priority user given by

$$c_l^L = \gamma N k_l(x_l^H + \gamma x_l^L) + \gamma^2 k_l x_l^L + NG_l^L(x_l^H)$$

Note that the decision for a user involves both: the choice of link and the choice of class, high priority or low priority. But for the existence of a potential function the following externality symmetry condition (see [12]) must be satisfied:

$$\frac{\partial(c_l^H)}{\partial x_l^L} = \frac{\partial(c_l^L)}{\partial x_l^H}$$

which leads to:

$$\gamma k_l = \gamma N k_l + NG_l'^L(x_l^H)$$

$$G_l^L = \frac{\gamma k_l(1-N)}{N}x_l^H + K^L$$

Where K^L is an arbitrary constant of integration (we assume it to be the same for all the links). Let us consider the classification cost density function for High priority flow to be of the form $G_l^H(x_l^H) = C^H$ where C^H is a constant. Now this is a potential population game with link costs given by:

$$c_l^H = (N+1)k_l x_l^H + \gamma k_l x_l^L + NC^H \tag{13}$$

$$c_l^L = \gamma^2(N+1)k_l x_l^L + \gamma k_l x_l^H + NK^L \tag{14}$$

and the potential function given by:

$$\Psi(\bar{x}) = \sum_{l \in E} \frac{(N+1)k_l(x_l^H)^2}{2} + \frac{(N+1)k_l\gamma^2(x_l^L)^2}{2} + NC^H x_l^H + K^L N x_l^L + \gamma k_l x_l^H x_l^L \tag{15}$$

Note that this function is strictly convex. From [12] its follows that the total link flows at Wardrop equilibrium can be computed as a solution to the global minimization problem of the convex function $\Psi(\bar{x})$ over its domain. The feasible domain is given by the following set of constraints:

$$\sum_{l \in E}(x_l^H + x_l^L) = \sum_i r^i = r$$

$$x_l^H, x_l^L \geq 0 \;\forall\, l \in E$$

Remark 2.1: Select K^L and C^H such that the following conditions are valid: $\min_{k_l}[\frac{\gamma k_l(1-N)}{N}r] + K^L > 0$ and $C^H > K^L$. Then since $N > 1$ we have the following characteristics of the function $G_l^L(x_l^H)$:

- $G_l^L(x_l^H)$ is linearly decreasing with increasing amount of High priority flow in the network.
- The first condition ensures that $G_l^L(x_l^H) > 0$ over the feasible domain. The second condition ensures that the high priority classification cost per unit is greater than the low priority classification cost per unit over the entire domain.

B. Characterization of equilibrium for symmetric players with linear congestion costs

We now consider the Hierarchical routing game with symmetric classes. We say that classes are symmetric if they have the same demands and link cost functions. Let us denote the demand $r^i = \bar{r}$ for each class i. It is clear that if the classes are symmetric, then assumption **A1** is valid and that the total flow on a link and at a particular priority at equilibrium is shared equally by all classes. We thus have the following theorem:

Theorem 2.1: Consider a hierarchical routing game between symmetric classes with linear congestion cost density functions as described above. Consider a classification cost density function for low priority users to of the form

$$G_l^L(x_l^H) = \frac{\gamma k_l(1-N)}{N}x_l^H + K^L$$

and that for the high priority users is constant C^H. Then,

- The unique Wardrop equilibrium flow profiles for the link costs given by c_l^H and c_l^L are obtained as a solution to the global minimization problem of minimizing the strictly convex function $\Psi(\bar{x})$ over the feasible domain.
- Under **A1**, there exists a Nash equilibrium to the original problem such that the total link flows are the unique Wardrop equilibrium with the transformed costs c_l^H and c_l^L
- If the users are symmetric, **A1** holds, and the Wardrop equilibrium leads to a unique Nash equilibrium for our original problem where, each class sends over the link and under a particular priority, $\frac{1}{N}$ of the total prescribed flow at Wardrop equilibrium.

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C. Distributed Computation of Nash equilibrium: Population dynamics

The existence of a potential function in a convex game ushers in a lot of nice properties. First of all, as mentioned before, the equilibrium can be computed as the global optimum of a single convex function: The potential function and a hoard of different algorithms for convex optimization can be used to compute the equilibrium. But there are another class of algorithms based on population dynamics which have been shown to converge to the Nash equilibrium for our case of potential games, see [12]. These population dynamics based algorithms can easily be implemented in a distributed fashion.

Let c_l^H and c_l^L for each $l \in E$ be as given by equations (13) and (14). Also let $P \in \{H, L\}$ Introduce the following distributed evolutionary dynamics:

- Replicator Dynamics:

$$\dot{x}_l^{i,P} = -x_l^{i,P}\left(c_l^P - \frac{1}{r^i}\sum_{l \in E}(x_l^{i,H}c_l^H + x_l^{i,L}c_l^L)\right)$$

- Brown – Von Neumann (BNN) Dynamics: Define

$$\zeta_{l'}^{i,P} = \max\left[\frac{1}{r^i}\sum_{l' \in E}(x_l^{i,H}c_l^H + x_l^{i,L}c_l^L) - c_{l'}^P, 0\right]$$

Then

$$\dot{x}_l^{i,P} = \left(r^i\zeta_l^{i,P} - x_l^{i,P}\sum_{l' \in E}(\zeta_{l'}^{i,H} + \zeta_{l'}^{i,L})\right)$$

A symmetric flow configuration is the one in which the individual flows of each user on any link are the same. Note that the dynamics are defined in terms of the individual path flows $x_l^{i,H}$ and $x_l^{i,L}$ rather than the total flows x_l^H and x_l^L of the potential game, and thus it is necessary to start from from a symmetric point so that the final configuration is symmetric, which according to theorem (2.1) is a necessary condition for it to be a Nash equilibrium. Using [12] that proves the convergence of BNN for potential population games, we see that BNN converges to the original Nash equilibrium for the symmetric case. Similar result holds for the replicator dynamics provided we start at an interior point, see [12], [15]. We thus have the following theorem.

Theorem 2.2: Assume that the classes are symmetric. Then **A1** holds and

- The BNN dynamics converges to the Nash equilibrium provided we start from a symmetric point.
- The Replicator dynamics converges to Nash equilibrium provided we start from an interior symmetric point.

III. APPLICATIONS

A hierarchical routing game arises naturally in some context of practical interests. For example, this hierarchy is naturally present in contexts where there are primary (licensed) users and secondary (unlicensed) users who can sense their environment because there are equipped with a cognitive radio [9]. In cognitive radio networks it requires that secondary users dynamically and opportunistically utilize unused licensed spectrum without interfering with the primary users. Another situation where this model finds a natural application is the two power ALOHA system. We shall consider this scenario as an illustration of the applicability of our model in the next section.

A. Hierarchical non-cooperative rate control over multiple ALOHA channels

Consider an infinite population of mobile terminals. We use the model of unslotted aloha where the global arrival of new packets from all the mobiles follows a poisson process with rate λ. Consider M channels over which the packets can be routed under the ALOHA protocol. Also two levels of power can be chosen by each mobile: High and Low. The time required to send the packet is $\frac{1}{2}$ time unit. So if there is a transmission in the period $[t-\frac{1}{2}, t+\frac{1}{2}]$, then there is a collision. If a High power packet collides with another High power packet then both packets are lost, similarly if a low power packet collides with a low power packet. If a high power packet collides with a low priority packet, then the high power packet transmission is successful while the low power packet is lost. This is called the capture phenomenon. All mobiles are identical in the sense that the contribution to the global rate λ of the arrival rate from each mobile, though negligible, is the same for all mobiles. Let η be the rate contribution of a single mobile where η is very small compared to λ. The population of mobiles is divided into N groups each being controlled by a controller i where $i = 1, \cdots, N$. All these controllers are symmetric and each controls the same population size of mobiles and hence the same rate. They face the decision of splitting this rate amongst different channels as well as power levels. The rate being controlled by each controller is given by $\frac{\lambda}{N}$. Let $\lambda_l^{i,H}$ and $\lambda_l^{i,L}$ be the high and low power rate of packets respectively from controller i on channel l. Let λ_l^H and λ_l^L be the total high and low power rates respectively on a channel l.

$$\sum_l \lambda_l^H + \lambda_l^L = \lambda$$

The probability of successful transmission for a packet choosing high power on a link l is

$$P^H(\lambda_l^H) = e^{-\lambda_l^H}$$

and that for a packet choosing low power is

$$P^L(\lambda_l^H + \lambda_l^L) = e^{-(\lambda_l^H + \lambda_l^L)}$$

The channel utility density per unit rate is assumed to be an increasing function of the packet transmission probability. We choose the logarithmic function, and then the channel cost density (negative of the utility density) is rendered linear in the rates on that channel:

$$c_l^H = -k_l \ln(e^{-\lambda_l^H}) = k_l(\lambda_l^H)$$

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and

$$c_l^L = -k_l \ln(e^{-(\lambda_l^H+\lambda_l^L)}) = k_l(\lambda_l^H + \lambda_l^L)$$

where c_l^H is the channel cost density for a controller routing on channel l using high power, c_l^L is the channel cost density for a controller routing on channel l using low power and k_l is a constant for each channel l. The classification cost density for transmitting a unit rate with high power is given by C^H and it is a constant. According to the pricing scheme derived in the previous section, the classification cost density for transmitting a unit rate as low power on a channel l is given by $G_l^L(\lambda_l^H) = \frac{k_l(1-N)}{N}\lambda_l^H + K^L$ such that K^L satisfies the properties mentioned before. The total cost density functions (channel and classification) are then given by :

$$\bar{c}_l^H = k_l(\lambda_l^H) + C^H$$

and

$$\bar{c}_l^L = k_l(\lambda_l^H + \lambda_l^L) + \frac{k_l(1-N)}{N}\lambda_l^H + K^L$$

The equilibrium total channel flows at high or low power can then be found as the ones which minimize the potential function:

$$\Psi = \sum_l \frac{(N+1)k_l(\lambda_l^H)^2}{2} + \frac{(N+1)k_l(\lambda_l^L)^2}{2}$$
$$+NC^H\lambda_l^H + NK^L\lambda_l^L + k_l\lambda_l^H\lambda_l^L \qquad (16)$$

1) Example: Consider a hierarchical ALOHA game between 4 symmetric controllers. There are three channels $1, 2$ and 3 with $k_1 = 1$, $k_2 = 2$ and $k_3 = 4$. Choose $\lambda = 1$, $K^L = 3.5$ and $C^H = 4$.

We minimize the potential function above to get the following values of total high power and low power rates on each channel at Nash equilibrium: $\bar{\lambda}^H = (\bar{\lambda}_1^H, \bar{\lambda}_2^H, \bar{\lambda}_3^H) = (0.0358, 0.0178, 0.0089)$ and $\bar{\lambda}^L = (\bar{\lambda}_1^L, \bar{\lambda}_2^L, \bar{\lambda}_3^L) = (0.5357, 0.2679, 0.1340)$

and thus the Nash equilibrium rates for each controller i are given by: $\bar{\lambda}^{i,H} = (\bar{\lambda}_1^{i,H}, \bar{\lambda}_2^{i,H}, \bar{\lambda}_3^{i,H}) = (0.00895, 0.00445, 0.002225)$ and $\bar{\lambda}^{i,L} = (\bar{\lambda}_1^{i,L}, \bar{\lambda}_2^{i,L}, \bar{\lambda}_3^{i,L}) = (0.133925, 0.066975, 0.0335)$

The cost at Nash equilibrium to each player is 1.0110 and the total cost incurred by all the players is 4.0440. We also compute the globally optimal solution. At this solution, the total high power and low power rates on each channel are: $\hat{\lambda}^H = (\hat{\lambda}_1^H, \hat{\lambda}_2^H, \hat{\lambda}_3^H) = (0.1429, 0.0713, 0.0357)$ and $\hat{\lambda}^L = (\hat{\lambda}_1^L, \hat{\lambda}_2^L, \hat{\lambda}_3^L) = (0.4287, 0.2144, 0.1070)$

The total globally optimal cost is 4.0089 which is lesser than the total cost at Nash equilibrium. Thus even though the Nash equilibrium is computed as a global optima of the potential function, minimizing this potential does not minimize the total cost incurred in the system.

IV. Conclusions

We introduced a new class of atomic routing games which allows for a hierarchy amongst the routed entities with non-reciprocal congestion effects, applications of which arise frequently in practical scenarios especially with the advent of cognitive radio technology. Through our results we were able to highlight the issues that arise in modeling routing scenarios in the absence of reciprocal congestion effects between classes of users. Finally a feasible pricing scheme was derived which induces a potential function for the atomic game on parallel links with linear link costs and symmetric players. Our results were illustrated by an application: multichannel hierarchical rate allocation under unslotted ALOHA for which we explicitly computed the Nash equilibrium for the atomic case.

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