

# Multiuser Diversity Analysis in Spectrum Sharing Cognitive Radio Networks

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**Abstract**—Cognitive radio has recently been proposed as an efficient paradigm to solve the spectrum scarcity in modern communication systems. In this paper, we consider spectrum sharing cognitive radio networks that utilize spectrum bands licensed to primary network. Spectrum sharing cognitive radio networks operate by ensuring that interference power at primary receiver remains below a certain threshold. We analyze the performance of the spectrum sharing cognitive radio networks with multiuser selection. Analytical and simulation results show that multiuser diversity gain in each network is substantially larger than that in the conventional wireless networks.

**Index Terms**—Cognitive radio, Spectrum sharing, Multiuser diversity

## I. INTRODUCTION

With the rapid growth of wireless applications and systems, the lack of available spectrum has hindered the deployment of novel communication technologies. Cognitive radio has been proposed as a promising technology to improve spectrum utilization and to provide highly reliable communications [1]. Various cognitive radio models have been proposed in cognitive radio research literature [2]. Interweave cognitive radio paradigm makes use of efficient spectrum sensing algorithms to detect the unused spectrum, licensed to primary network, and use it for own transmission when primary is not transmitting. On the other hand, spectrum sharing paradigm of cognitive radio allows cognitive users to transmit concurrently with primary users, provided interference power at primary receiver is kept below a certain threshold, known as interference temperature. Such spectrum sharing results in more efficient spectrum utilization compared to conventional interweave approach where cognitive users are allowed to use spectrum only when primary users are absent.

There have been several studies on spectrum sharing cognitive radio model where capacity of cognitive radio network has been analyzed for Gaussian and fading channels [3]-[4]. Most of these studies dealt with single primary and single cognitive user. On the other hand, presence of multiple primary and cognitive users brings new issues related to user scheduling and medium access control. Multiuser diversity has been utilized in conventional wireless networks due to fluctuations of fading channels of different users [5]. The user with best channel gain is selected to transmit at any given time, resulting in improved long term system throughput.

There have been very few works on multiuser diversity in cognitive radio networks where interference power threshold at primary receiver adds a new constraint to selection of cognitive user for transmission. In [7], a new form of multiuser diversity, named multiuser interference diversity is investigated for various types of spectrum sharing based cognitive radio networks. The variation of multiuser diversity gain in spectrum sharing systems in different scenarios was studied in [8]. In this paper, we quantify the multiuser diversity gain in the multiple access (MAC) and broadcast (BC) spectrum sharing cognitive radio networks. We derive analytical expressions for the probability density distribution of receiver SNR as well as multiuser diversity gain for each configuration.

The rest of the paper is organized as follows. Section II provides the system model for the fading MAC and BC cognitive radio networks. Sections III and IV then present the results on the statistics of the received signal-to-interference-plus-noise ratio (SINR) at cognitive receiver for each scenario, and determine its probability density function (PDF). Multiuser diversity gain expressions are derived in each case. In Section V, Monte Carlo simulations of multiuser diversity gains for MAC and BC cases are performed and simulation results are provided. Finally, Section VI concludes the paper.

## II. SYSTEM MODEL

We first consider a fading multiple access channel cognitive radio network, shown in Fig.1 where  $K$  cognitive users (CRs) transmit to the single cognitive receiver by sharing the spectrum, licensed to primary user. We assume single primary user in this case. Primary and cognitive users are equipped with single antenna each. Independent and identically distributed (i. i. d.) Rayleigh flat fading is assumed in each wireless link. The channel coefficients are all zero-mean circularly symmetric complex Gaussian (CSCG) random variables with unit variance  $\sim \mathcal{CN}(0, 1)$ . Therefore, the channel gains are all i.i.d. exponential random variables.  $h_{pp}$ ,  $h_{pc}$ ,  $h_{cpk}$  and  $h_{cck}$  are denoted as channel power gains between primary transmitter and primary receiver, primary transmitter and cognitive receiver,  $k^{th}$  cognitive transmitter and primary receiver,  $k^{th}$  cognitive transmitter and cognitive receiver respectively. We assume that transmit powers of all cognitive transmitters are equal, denoted by  $P$ . The transmit power in primary link is

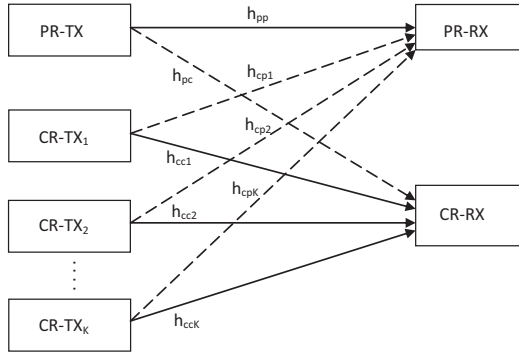


Fig. 1. Multiple access (MAC) spectrum sharing cognitive radio network

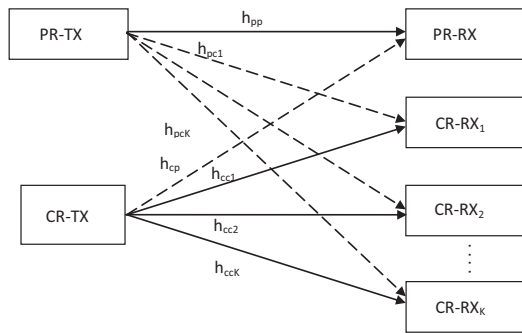


Fig. 2. Broadcast channel (BC) spectrum sharing cognitive radio network

assumed to be  $Q$ . We assume block fading channel model for all channels involved. It is assumed that additive noises at all cognitive receivers are independent CSCG random variables each distributed as  $\sim \mathcal{CN}(0, 1)$ . It is also assumed that the CR receiver is aware of the realizations of  $h_{pc}$ ,  $h_{cck}$ ,  $h_{cpk}$ ,  $k = 1, \dots, K$ . The PR link works with an interference power guaranteed to be below  $\Gamma$ . In each block transmission, the CR receiver chooses one of the CR transmitters which has the largest received SINR. The selected user is informed by base station to transmit throughout this fading block. This scheduling is fair for all CR transmitters in a long term, since the channel gains are assumed to be i.i.d..

Next, we consider broadcast channel spectrum sharing cognitive radio network in Fig. 2 where secondary transmitter transmits to  $K$  cognitive users. We again assume a single primary user in this case. We use the same notation,  $h_{cck}$ , to denote the channel power gain from the cognitive base station to  $k^{th}$  cognitive user. It is assumed that additive noises at all cognitive receivers are independent CSCG random variables each distributed as  $\sim \mathcal{CN}(0, 1)$ . The transmit power at primary and cognitive transmitter are assumed to be  $Q$  and  $J$  respectively.

### III. MULTIUSER DIVERSITY IN THE CR MAC

In the CR MAC scenario, we first analyze the statistics of the received SINR,  $\gamma^{MAC}$ , at the CR base station, and quantify the performance gains available in the above multiuser scheduling system. We determine the PDF of  $\gamma^{MAC} = \max_k \frac{[h_{cck} \min(P, \Gamma/h_{cpk})]}{1 + Qh_{pc}}$  and then find the multiuser diversity gain.

#### A. Determining the PDF

In this case,

$$\begin{aligned} \gamma^{MAC} &= \max_k \frac{[h_{cck} \min(P, \Gamma/h_{cpk})]}{1 + Qh_{pc}} \\ &= \frac{\max_k [h_{cck} \min(P, \Gamma/h_{cpk})]}{1 + Qh_{pc}}. \end{aligned} \quad (1)$$

We deliberately separate the random variable  $\gamma^{MAC}$  as follows

$$\gamma^{MAC} \triangleq \beta^{MAC} \cdot \bar{C}_{MAC}, \quad (2)$$

where  $\bar{C}_{MAC}$  is a constant and  $\beta^{MAC}$  is a random variable. We then determine the PDF of  $\beta^{MAC}$ , where

$$\beta^{MAC} = \frac{\max_k [h_{cck} \min(P, \Gamma/h_{cpk})]}{[1 + Qh_{pc}] \bar{C}_{MAC}}.$$

If we set  $\bar{C}_{MAC} = 1$ , then our derivations apply to  $\gamma^{MAC}$ . For simplicity, define  $\theta_k^{MAC} = h_{cck} \cdot \min(P, \Gamma/h_{cpk})$ . Then,

$$\begin{aligned} \Pr\{\theta_k^{MAC} \leq x\} &= \Pr\{h_{cck}P \leq x, P \leq \Gamma/h_{cpk}\} + \\ &\quad \Pr\{h_{cck}\Gamma \leq h_{cpk}x, P > \Gamma/h_{cpk}\} \\ &= 1 - (1 - e^{-\Gamma/P})e^{-x/P} - \frac{\Gamma}{\Gamma + x}e^{-\frac{\Gamma+x}{P}} \end{aligned} \quad (3)$$

Now we calculate the cumulative distribution function (CDF) of  $\beta^{MAC}$ .

$$\begin{aligned} F_{\beta^{MAC}}(x) &= \Pr\{\beta^{MAC} \leq x\} \\ &= \Pr\left\{\max_k \theta_k^{MAC} \leq x(1 + Qh_{pc})\bar{C}_{MAC}\right\} \\ &= \sum_{k_1=0}^K \sum_{k_2=0}^{K-k_1} \frac{K!}{k_1!k_2!(K-k_1-k_2)!} (1 - e^{-\Gamma/P})^{k_1} \\ &\quad (e^{-\Gamma/P})^{k_2} (-1)^{k_1+k_2} I_{MAC,k_1,k_2}(x), \end{aligned} \quad (4)$$

where

$$\begin{aligned} I_{MAC,k_1,k_2}(x) &= \frac{\Gamma^{k_2}}{Q\bar{C}_{MAC}x} e^{\frac{(k_1+k_2)\Gamma}{P}} e^{\frac{\Gamma+\bar{C}_{MAC}x}{Q\bar{C}_{MAC}x}} \\ &\quad \cdot \left[ \frac{1}{Q\bar{C}_{MAC}x} + \frac{k_1+k_2}{P} \right]^{k_2-1} \\ &\quad \cdot \Gamma[-(k_2-1), (\Gamma + \bar{C}_{MAC}x)] \\ &\quad \cdot \left( \frac{k_1+k_2}{P} + \frac{1}{Q\bar{C}_{MAC}x} \right), \end{aligned} \quad (5)$$

where in (5), we have used the following [11, (6.5.3)],

$$\Gamma(a, z) \triangleq \int_z^\infty e^{-t} t^{a-1} dt.$$

$$\begin{aligned}
f_{\beta^{MAC}}(x) &= \sum_{k_1=0}^K \frac{K!}{k_1!(K-k_1)!} \left[ -\left(1 - e^{-\Gamma/P}\right) \right]^{k_1} f_{1,k_1}(x) \\
&+ \sum_{k_1=0}^{K-1} \frac{K!}{k_1!(K-1-k_1)!} \left[ -\left(1 - e^{-\Gamma/P}\right) \right]^{k_1} \left(-e^{-\Gamma/P}\right) [f_{3,k_1}(x) - f_{2,k_1}(x)] \\
&+ \sum_{k_1=0}^{K-2} \sum_{k_2=2}^{K-k_1} \frac{K!}{k_1!k_2!(K-k_1-k_2)!} \left[ -\left(1 - e^{-\Gamma/P}\right) \right]^{k_1} \left(-e^{-\Gamma/P}\right)^{k_2} [f_{5,k_1,k_2}(x) - f_{4,k_1,k_2}(x)], \tag{6} \\
&(x \geq 0)
\end{aligned}$$

where

$$\begin{aligned}
f_{1,k_1}(x) &= -e^{-\frac{k_1 \bar{C}_{MAC} x}{P}} \cdot \frac{k_1 \bar{C}_{MAC} [P + PQ + k_1 Q \bar{C}_{MAC} x]}{[P + k_1 Q \bar{C}_{MAC} x]^2}, \\
f_{3,k_1}(x) &= \frac{\Gamma P}{\bar{C}_{MAC} Q x^2} \cdot e^{-\frac{(k_1+1) \bar{C}_{MAC} x}{P}} \cdot \frac{\Gamma}{[\Gamma + \bar{C}_{MAC} x][P + (k_1+1)Q \bar{C}_{MAC} x]} \\
&\quad - e^{\frac{(k_1+1)\Gamma}{P}} e^{\frac{\Gamma + \bar{C}_{MAC} x}{Q \bar{C}_{MAC} x}} \left[ \frac{\Gamma}{Q \bar{C}_{MAC} x^2} + \frac{\Gamma^2}{Q^2 \bar{C}_{MAC}^2 x^3} \right] \\
&\quad \cdot \Gamma \left[ 0, (\Gamma + \bar{C}_{MAC} x) \left( \frac{k_1+1}{P} + \frac{1}{Q \bar{C}_{MAC} x} \right) \right], \\
f_{2,k_1}(x) &= (k_1+1) e^{-\frac{(k_1+1) \bar{C}_{MAC} x}{P}} \cdot \frac{\Gamma \bar{C}_{MAC}}{[\Gamma + \bar{C}_{MAC} x][P + (k_1+1)Q \bar{C}_{MAC} x]}, \\
f_{5,k_1,k_2}(x) &= \left( \frac{\Gamma}{\Gamma + \bar{C}_{MAC} x} \right)^{k_2} \cdot e^{-\frac{(k_1+k_2) \bar{C}_{MAC} x}{P}} \cdot \frac{\Gamma P}{Q \bar{C}_{MAC} x^2} \cdot \frac{1}{P + (k_1+k_2)Q \bar{C}_{MAC} x} \\
&\quad - \left( \frac{\Gamma}{Q} \right)^{k_2} \cdot e^{\frac{\Gamma(k_1+k_2)}{P}} \cdot e^{\frac{\Gamma + \bar{C}_{MAC} x}{Q \bar{C}_{MAC} x}} \cdot \left[ \frac{(k_1+k_2)Q}{P} + \frac{1}{\bar{C}_{MAC} x} \right]^{k_2-2} \\
&\quad \cdot \Gamma \left[ -(k_2-1), (\Gamma + \bar{C}_{MAC} x) \left( \frac{k_1+k_2}{P} + \frac{1}{Q \bar{C}_{MAC} x} \right) \right] \cdot f_{T,k_1,k_2}(x), \\
f_{T,k_1,k_2}(x) &= \frac{k_2}{\bar{C}_{MAC}^2 x^3} + \frac{\Gamma(k_1+k_2)}{P \bar{C}_{MAC}^2 x^3} + \frac{\Gamma}{Q \bar{C}_{MAC}^3 x^4} + \frac{(k_1+k_2)Q}{P \bar{C}_{MAC} x^2}, \\
f_{4,k_1,k_2}(x) &= \left( \frac{\Gamma}{\Gamma + \bar{C}_{MAC} x} \right)^{k_2} \cdot e^{-\frac{(k_1+k_2) \bar{C}_{MAC} x}{P}} \cdot \frac{(k_1+k_2) \bar{C}_{MAC}}{P + (k_1+k_2)Q \bar{C}_{MAC} x}.
\end{aligned}$$

$$\Gamma(0, z) \triangleq \int_z^\infty e^{-t} t^{-1} dt \triangleq \mathbf{E}_1(z),$$

By straightforward differentiations, we obtain the PDF of  $\beta^{MAC}$  as given by (6).

### B. Multiuser diversity gain in the CR MAC

In a traditional wireless link with only fading and noise and without interference, the *diversity gain* and *coding gain* characterize the performance of a wireless link at high signal-to-noise ratio (SNR) [9]. Since now we have interference in our system, it is interesting to see how our system behaves asymptotically with different settings of parameters. First of all, in order to apply the results in [9], we define [see (2)]

$$\gamma^{MAC} \triangleq \beta^{MAC} \cdot \bar{C}_{MAC},$$

where  $\bar{C}_{MAC}$  is a constant but can be increased to infinity with a specific system parameter, e.g.,  $P$  here.  $\beta^{MAC}$  is a random variable, which satisfies the conditions required in [9, p. 1390, AS3]). The statistics of  $\beta^{MAC}$  should remain unchanged when  $\bar{C}_{MAC}$  goes to infinity. In addition, given  $\beta^{MAC}$ , the value of  $\gamma^{MAC}$  should increase *linearly* with  $\bar{C}_{MAC}$ , at least when  $\bar{C}_{MAC}$  is large. If (2) holds, then we can possibly determine the *diversity gain* and *coding gain* achieved by a system which produces  $\gamma^{MAC}$ .

Now we have a close look at  $\gamma^{MAC}$ . Consider  $\mathbb{E}[\min(P, \Gamma/h_{cpk})]$ , since it denotes the average transmit power in the scenario under our investigation. we find that with the linear increase of  $P$ ,  $\bar{C}_{MAC}$  increases with  $P$  only in a log scale. Conversely, if  $\mathbb{E}[\min(P, \Gamma/h_{cpk})]$  increases

linearly,  $P$  will increase exponentially (asymptotically). The above simple calculation reveals that the power dynamics in  $\gamma^{MAC}$  is different from that of a traditional system without interference (see, e.g. [9]).

We now define the multiuser diversity gain in the CR MAC as

$$G^{MAC} = \frac{\mathbb{E}(\beta^{MAC})|_{K \text{ users } (K>1)}}{\mathbb{E}(\beta^{MAC})|_{1 \text{ user}}}, \quad (7)$$

which can be shown to be as in (8) on the next page.

Note that we adopt the above definition to quantify the result in [7, Theorem 3.1]. It is interesting to note that this gain is not affected by the transmit power of the primary user,  $Q$ .

#### IV. MULTIUSER DIVERSITY IN THE CR BC

In the CR BC scenario, we again analyze the statistics of the received SINR at the CR receivers, and quantify the performance gains available. We first determine the PDF and then determine the multiuser diversity gain.

##### A. Determining the PDF

$$\gamma^{BC} = \max_k \frac{h_{cck} \min(J, \Gamma/h_{cp})}{1 + Q \cdot h_{pck}}$$

Again, we purposely separate  $\gamma^{BC}$  as

$$\gamma^{BC} = \beta^{BC} \cdot \bar{C}_{BC}, \quad (9)$$

where  $\bar{C}_{BC}$  is a constant, and  $\beta^{BC}$  is a random variable, with

$$\beta^{BC} = \left[ \max_k \frac{h_{cck}}{1 + Q h_{pck}} \right] \frac{\min(J, \Gamma/h_{cp})}{\bar{C}_{BC}}.$$

Define

$$\theta^{BC} = \max_k \frac{h_{cck}}{1 + Q h_{pck}}.$$

The CDF of  $\beta^{BC}$  can be expressed as

$$\begin{aligned} F_{\beta^{BC}}(x) &= \Pr\{\beta^{BC} \leq x\} \\ &= \Pr\{\theta^{BC} \cdot J \leq \bar{C}_{BC} \cdot x, J \leq \Gamma/h_{cp}\} + \\ &\quad \underbrace{\Pr\{\theta^{BC} \cdot \Gamma/h_{cp} \leq \bar{C}_{BC} \cdot x, J > \Gamma/h_{cp}\}}_{I_{BC,2}(x)} \end{aligned} \quad (10)$$

Thus, the CDF of  $\beta^{BC}$  has been obtained as  $F_{\beta^{BC}}(x) = I_{BC,1}(x) + I_{BC,2}(x)$ . After taking the derivative of  $F_{\beta^{BC}}(x)$ , we can obtain the pdf of  $\beta^{BC}$  as given in (11) on the next page.

##### B. Quantifying the gains from multiuser selection in the CR BC

We define the multiuser diversity gain in the CR BC as

$$G^{BC} = \frac{\mathbb{E}(\beta^{BC})|_{K \text{ users } (K>1)}}{\mathbb{E}(\beta^{BC})|_{1 \text{ user}}}, \quad (12)$$

which can be shown to be as in (13). It is somewhat surprising to see that this gain is not affected by the transmit power of the cognitive user,  $J$ , or the interference temperature,  $\Gamma$ .

## V. SIMULATION RESULTS

In this section, we provide numerical results for the MAC and BC spectrum sharing cognitive radio networks. In Fig. 3, we show the variation of multiuser diversity gain with number of users in the CR MAC. Both Monte Carlo simulations and calculations using (8) are employed. Analytic and simulation results match well. It is observed that  $G^{MAC}$  is increasing when  $P$  is increased from 13 dB to 20 dB, while  $\Gamma$  is fixed.

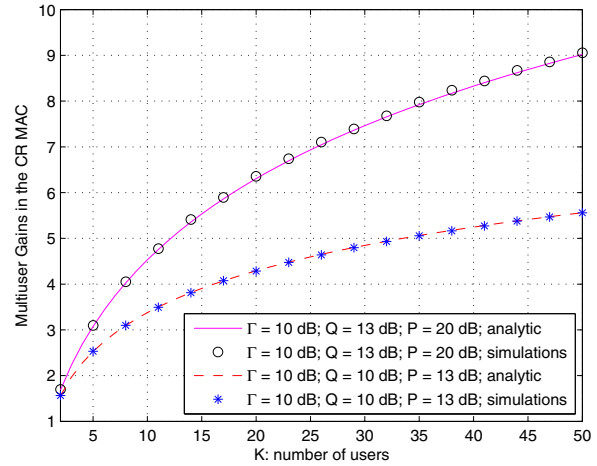


Fig. 3. Multiuser diversity gain in the CR MAC.

Fig. 4 gives simulation results of the multiuser gain in the CR BC. Both Monte Carlo simulations and calculations using (13) are employed. Analytic and simulation results match well. It is observed that  $G^{BC}$  is increasing when  $Q$  is increased from 10 dB to 13 dB.

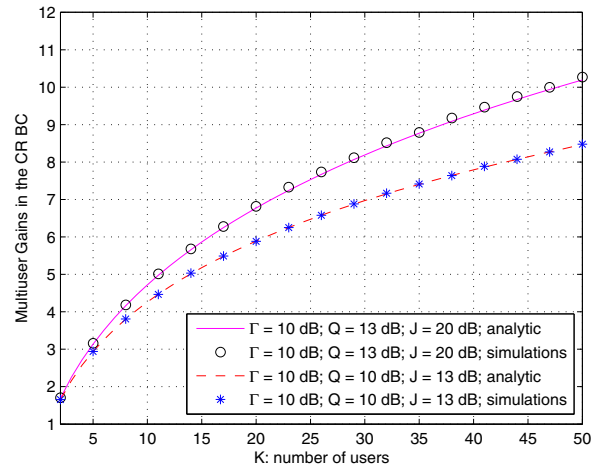


Fig. 4. Multiuser diversity gain in the CR BC.

## VI. CONCLUSION

In this paper, We analyzed the multiuser diversity performance of multiple access and broadcast spectrum sharing

$$G^{MAC} = \frac{\sum_{\substack{k_1, k_2 \\ k_1 + k_2 \neq 0}} \binom{K}{k_1, k_2} \left(e^{\frac{\Gamma}{P}} - 1\right)^{k_1} (-1)^{k_1 + k_2 + 1} \Gamma^{k_2} \left(\frac{k_1 + k_2}{P}\right)^{k_2 - 1} \Gamma \left[-(k_2 - 1), \frac{\Gamma(k_1 + k_2)}{P}\right]}{(1 - e^{-\Gamma/P}) P + \Gamma \cdot \mathbf{E}_1(\Gamma/P)}. \quad (8)$$

$$f_{\beta BC}(x) = (1 - e^{-\frac{\Gamma}{J}}) \cdot K \cdot e^{-\frac{\bar{C}_{BC} x}{J}} \cdot \frac{\bar{C}_{BC} [(Q+1)J + Q\bar{C}_{BC} \cdot x]}{(Q\bar{C}_{BC} \cdot x + J)^2} \left[1 - \frac{J \cdot e^{-\frac{\bar{C}_{BC} x}{J}}}{J + Q\bar{C}_{BC} \cdot x}\right]^{K-1} + \sum_{k=0}^K \binom{K}{k} (-1)^k [g_{1,k}(x) - g_{2,k}(x)], \quad (11)$$

where

$$g_{1,k}(x) = e^{-\frac{k\bar{C}_{BC} \cdot x + \Gamma}{J}} \frac{\Gamma \cdot J^k}{(k\bar{C}_{BC} \cdot x + \Gamma)(Q\bar{C}_{BC} \cdot x + J)^k} \cdot \frac{\Gamma}{Q\bar{C}_{BC} \cdot x^2} - e^{\frac{1}{Q}(k + \frac{\Gamma}{\bar{C}_{BC} \cdot x})} \cdot \frac{\Gamma \cdot (k\bar{C}_{BC} \cdot x + \Gamma)^{k-2}}{(Q\bar{C}_{BC} \cdot x)^k} \left[ k\bar{C}_{BC} + \frac{k\Gamma}{x} + \frac{k\Gamma}{Qx} + \frac{\Gamma^2}{x^2 Q\bar{C}_{BC}} \right] \cdot \Gamma \left[ -(k-1), \left( \frac{k\bar{C}_{BC} \cdot x}{\Gamma} + 1 \right) \left( \frac{\Gamma}{J} + \frac{\Gamma}{Q\bar{C}_{BC} \cdot x} \right) \right],$$

$$g_{2,k}(x) = e^{-\frac{k\bar{C}_{BC} \cdot x + \Gamma}{J}} \frac{\Gamma \bar{C}_{BC} \cdot J^k}{(k\bar{C}_{BC} \cdot x + \Gamma)(Q\bar{C}_{BC} \cdot x + J)^k} \cdot \frac{k}{J}.$$

$$G^{BC} = \sum_{k=1}^K \binom{K}{k} \left( -\frac{k \cdot e^{1/Q}}{Q} \right)^{k-1} \frac{\Gamma [-(k-1), k/Q]}{\mathbf{E}_1(1/Q)}. \quad (13)$$

cognitive radio networks CR. We first studied the statistics of receiver SINR and determined analytical expressions for probability density function in each system. We then found analytical expressions for multiuser diversity gains. It is observed that multiuser diversity gain in both systems is substantially larger than that in conventional wireless networks. It is also observed that multiuser gain in the CR MAC is not affected by the transmit power of the primary user,  $Q$ , whereas in the CR BC, it is not affected by the transmit power of the cognitive user,  $J$ , or the interference temperature,  $\Gamma$ .

#### ACKNOWLEDGEMENT

This work was supported by the Future and Emerging Technologies (FET) Programme within the Seventh Framework Programme for Research of the European Commission under FET-Open grant number CROWN-233843. The authors would also like to thank Dr Minhua Ding for her significant contribution in the mathematical formulations while she was with the CROWN project at ECIT, Queen's University Belfast, UK.

#### REFERENCES

[1] S. Haykin, "Cognitive radio: Brain empowered wireless communications," *IEEE Trans. Sel. Areas Commun.*, vol. 23, no. 2, pp. 201-220, Feb. 2005.

[2] A. Goldsmith, S. A. Jafar, I. Maric and S. Srinivasa, "Breaking spectrum gridlock with cognitive radios: An information theoretic perspective," *Proc. IEEE*, vol. 97, no. 3, pp. 894-914, May 2009.

[3] A. Ghasemi and E. S. Sousa, "Fundamental limits of spectrum-sharing in fading environments," *IEEE Trans. Wireless Commun.*, vol. 6, no. 2, pp. 649-658, Feb. 2007.

[4] R. Zhang, S. Cui and Y. C. Liang, "On ergodic sum capacity of fading cognitive multiple-access and broadcast channels," *IEEE Trans. Inf. Theory*, vol. 55, no. 11, pp 5161-5178, Nov. 2009

[5] P. Viswanath, D. Tse and R. Laroia, "Opportunistic beamforming using dumb antennas," *IEEE Trans. Inf. Theory*, vol. 48, no. 6, pp. 1277-1294, Jun. 2002.

[6] H. Wang, J. Lee, S. Kim and D. Hong, "Capacity of secondary users exploiting multispectrum and multiuser diversity in spectrum-sharing environments," *IEEE Trans. Veh. Tech.*, vol. 59, no. 2, pp. 1030-1036, Feb. 2010.

[7] R. Zhang and Y. C. Liang, "Investigations on multiuser diversity in spectrum sharing based cognitive radio networks," *IEEE Commun. Letters*, vol. 14, no. 2, Feb. 2010

[8] T. W. Ban, W. Choi, B. C. Jung and D. K. Sung, "Multi-user diversity in a spectrum sharing system," *IEEE Trans. Commun.*, vol. 8, no. 1, pp. 102-106, Jan. 2009

[9] Z. Wang and G. B. Giannakis, "A simple and general parameterization quantifying performance in fading channels," *IEEE Trans. Commun.*, vol. 51, no. 8, pp. 1389-1398, Aug. 2003.

[10] A. Ribeiro, X. Cai, and G. B. Giannakis, "Symbol error probabilities for general cooperative links," *IEEE Trans. Wireless Commun.*, vol. 4, no. 3, pp. 1264-1273, May 2005.

[11] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, 1972.