Cooperative Relaying in Sensor Networks

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***Abstract*—Reliable transmission of a discrete memoryless source over a multiple-relay network is considered. Motivated by sensor network applications, it is assumed that the relays and the destination all have access to side information correlated with the underlying source signal. Joint source-channel cooperative transmission is studied in which the relays help the transmission of the source signal to the destination by using both their overheard signals, as in the classical channel cooperation scenario, and the available correlated side information. Decode-and-forward (DF) based cooperative transmission is considered in a network of multiple relay terminals and two different achievability schemes are proposed: i) a regular encoding and sliding-window decoding scheme with no binning at the encoder, and ii) a semi-regular encoding and backward decoding scheme with explicit binning based on the side information statistics. It is shown that both of these schemes lead to the same sufficiency conditions, which are shown to be also necessary in the case of a physically degraded relay network in which the side information signals are also degraded in the same order.**

I. Introduction

A relay network consists of a source-destination pair and dedicated relay terminals that help the transmission of messages from the source to the destination. The classical relay channel model [1] focuses on the maximum channel coding rate that can be achieved with arbitrarily small probability of error. The relay channel has been the subject of study from many different perspectives; however, most papers focus solely on the channel coding aspects of relaying motivated by the improvement in the capacity, reliability or coverage extension provided by the relay terminal. However, in some applications such as sensor networks, the relays might have partial information about the source message by using their own sensing capabilities. This additional side information can be used to improve the end-to-end system performance.

We consider a network with multiple relays, in which all the terminals in the network have access to their own correlated side information. The goal is the reliable (lossless) transmission of the underlying discrete memoryless source signal to the destination, and the problem is to characterize the minimum number of channel uses per source symbol, called the *source-channel rate*, that is required for reliable transmission. This is a joint source-channel coding generalization of the classical relay network problem.

In this model the transmission protocols need to exploit the availability of the side information at the network terminals as well as the overheard channel transmissions. Note that the classical channel cooperation ignores the side information at the terminals. However, this can lead to a significant performance loss depending on the scenario. Consider, for example, a single relay channel in which the side information at the relay is identical to the source output. In this case the relay can cooperate with the source terminal to form a multiple antenna array even if there exists no channel from the source terminal to the relay.

Several channel coding techniques have been proposed for the relay channel [1]. Here, we focus on the decode-and-forward (DF) protocol and propose multiple-relay extensions that exploit the side information at the relays and the destination. In the DF protocol, the relays decode the underlying message, and cooperate with the source terminal to forward it to the destination. While not optimal in general, DF achieves capacity in a physically degraded relay channel [1].

The protocols in the literature for DF relaying are categorized based on the codebook sizes and the decoding strategy. In *irregular encoding and successive decoding* [1], the relay and the source codebooks have different sizes and the destination applies successive decoding. In *regular encoding and sliding-window decoding*, introduced in [2], the source and the relay codebooks have the same size and the destination decodes each source message by using two consecutive channel blocks. Finally, in *regular encoding and backward decoding*, introduced in [3], the destination waits until all channel blocks are received, and decodes the messages starting from the last block and going backwards. DF channel coding is extended to multiple-relay networks in [4], [5], [6], [7] and [8]. While [4] and [8] consider irregular encoding, [5] and [6] study an extension of the regular encoding and sliding window decoding scheme, and finally [7] extends the backward decoding strategy to multiple relays.

We propose two different *joint source-channel cooperation* protocols based on DF relaying. In the first protocol, we consider regular encoding and sliding window decoding for multiple relays [5], [6] without applying explicit binning. The second transmission protocol is based on the nested

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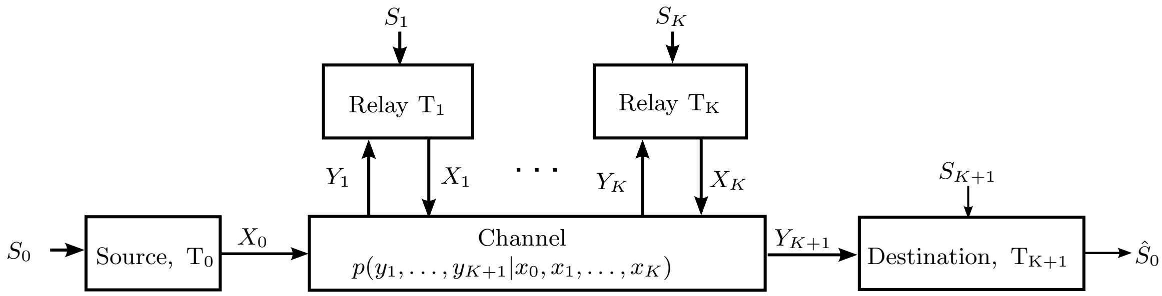

Fig. 1. Illustration of a relay network with correlated side information.

backward decoding scheme of [7] and applies explicit source binning. These two protocols achieve the same source-channel performance while backward decoding introduces a much higher end-to-end delay which increases with the number of relay terminals. We should also remark that the proposed protocols are not expected to achieve the optimal performance in the general setting since our problem is a generalization of the classical relay network problem which remains open. However, we prove that the proposed DF-based protocols achieve the optimal source-channel rate in a physically degraded setting in which both the channel outputs and the side information sequences are degraded in the same order.

The problem of joint source-channel cooperation has been previously studied for a single relay channel in [9], [10] and [11], and for a multiple relay network in [12]. The techniques proposed in all these works are based on DF relaying with different transmission techniques. While semi-regular encoding and backward decoding with explicit binning at the source encoder is proposed in [9], irregular encoding/ successive decoding with and without explicit binning is considered in [11] and [10], respectively. In [12] a regular encoding/ sliding window decoding scheme with explicit binning is considered in the multiple-relay setting.

The rest of the paper is organized as follows: In Section II, we introduce the system model and the problem. The main results of the paper are stated in Section III. In Sections IV and V we provide the sketches for the proofs. The paper is concluded in Section VI.

In this paper we denote random variables by capital letters, sample values by the respective lower case letters, and alphabets by the respective calligraphic letters. For $k \leq n$, the sequence $(X_k, \ldots, X_n)$ will be denoted by X_k^n, while X^n will be used for X_1^n. The complement of a certain element X_i in a vector X^n will be denoted by $X_i^c \triangleq (X_1, \ldots, X_{i-1}, X_{i+1}, \ldots, X_n)$.

II. Problem Setup

We have a relay network composed of $K+2$ terminals (see Fig. 1): terminal T_0 is the source terminal observing the source signal S_0, terminals T_i for $i = 1, \ldots, K$ are the K relay terminals each observing a different correlated side information signal S_i, and terminal T_{K+1} is the destination terminal with its own correlated side information signal S_{K+1}. The underlying discrete memoryless (DM) relay channel is characterized by the conditional distribution

$$p(y_1^n, \ldots, y_{K+1}^n | x_0^n, x_1^n, \ldots, x_K^n) = \prod_{t=1}^{n} p_{Y_1,\ldots,Y_{K+1}|X_0,\ldots,X_K}(y_{1,t}, \ldots, y_{K+1,t}|x_{0,t}, \ldots, x_{K,t}),$$

where $x_{i,t} \in \mathcal{X}_i$ and $y_{i,t} \in \mathcal{Y}_i$, respectively, are the channel input and output of terminal i at time t; and the finite sets $\mathcal{X}_i$ and $\mathcal{Y}_i$ are the corresponding input and output alphabets.

We consider DM independent and identically distributed (i.i.d.) signals $(S_0, \ldots, S_{K+1})$ which are arbitrarily correlated according to a joint distribution $p(s_0, \ldots, s_{K+1})$ over a finite alphabet $\mathcal{S}_0 \times \cdots \times \mathcal{S}_{K+1}$. The sequence $\{S_{0,j}\}_{j=1}^{\infty}$ is denoted as the source sequence while $\{S_{i,j}\}_{j=1}^{\infty}$, $i = 1, \ldots, K+1$ is the side information sequence available at terminal T_i.

Terminal T_0 maps its observation S_0^m to a channel codeword of length-n by the encoding function $f_0^{(m,n)} : \mathcal{S}^m \to \mathcal{X}_0^n$, i.e., $X_0^n = f_0^{(m,n)}(S_0^m)$. The channel input of the terminal T_i at each time instant j, $X_{i,j}$, depends on the previous channel outputs Y_i^{j-1} and its side information S_i^m. Hence, the terminal i has encoding functions $f_i^{(m,n)} = \{f_{i,1}^{(m,n)}, \ldots, f_{i,n}^{(m,n)}\}$ such that

$$X_{i,t} = f_{i,t}^{(m,n)}(Y_{i,1}, \ldots, Y_{i,t-1}, S_i^m),$$

for $1 \leq i \leq K$ and $1 \leq t \leq n$.

The decoder at the destination terminal T_{K+1} maps the channel output Y_{K+1}^n and its side information S_{K+1}^m to the estimate $\hat{S}_0^m$ by the decoding function

$$g^{(m,n)} : \mathcal{Y}_{K+1}^n \times \mathcal{S}_{K+1}^m \to \mathcal{S}_0^m, \tag{1}$$

i.e., $\hat{S}_0^m = g^{(m,n)}(Y_{K+1}^n, S_{K+1}^m)$.

The goal of the network is to transmit the source message S^m to the destination terminal. The probability of error is defined as $P_e^{(m,n)} = Pr\left\{\hat{S}_{K+1}^m \neq S_0^m\right\}$.

Definition 2.1: We say that the source-channel rate r is *achievable* if, there exist a sequence of encoders $f_i^{(m,n)}$, $i = 0, \ldots, K$, and decoders $g^{(m,n)}$ with $r = \frac{n}{m}$, such that the probability of error vanishes, i.e., $P_e^{(m,n)} \to 0$, as $m, n \to \infty$.

Definition 2.2: A discrete memoryless relay network is said to be *physically degraded* if

$$p(y_{i+1}, \ldots, y_{K+1}|y_i, x_0, \ldots, x_K) = p(y_{i+1}, \ldots, y_{K+1}|y_i, x_i, \ldots, x_K) \tag{2}$$

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for all $i = 1, \ldots, K$, or equivalently if

$$(X_0, \ldots, X_{i-1}) \to (Y_i, X_i, \ldots, X_K) \to (Y_{i+1}, \ldots, Y_{K+1})$$

forms a Markov chain for all $i = 1, \ldots, K$.

III. Main Results

We state our main results in this section while the sketches of the proofs are included in the following sections. The first theorem provides sufficient conditions for achieving a source-channel rate of r in a relay network. Let $\pi(\cdot)$ be a permutation on $\{0, \ldots, K+1\}$ and define $\pi(i:j) \triangleq \{\pi(i), \pi(i+1), \ldots, \pi(j)\}$. We also define, for a set $C = \{c_1, \ldots, c_n\}$, $n \in \mathbb{Z}^+$ and $c_i \in \mathbb{Z}^+$, $X_C \triangleq (X_{c_1}, \ldots, X_{c_n})$.

Theorem 3.1: For the DM relay network with correlated relay and destination side information, the source-channel rate r is achievable if, for all $i = 1, \ldots, K+1$,

$$H(S_0|S_{\pi(i)}) < rI(X_{\pi(0:i-1)}; Y_{\pi(i)}|X_{\pi(i)}, \ldots, X_{\pi(K)}), \quad (3)$$

for some permutation $\pi(\cdot)$ with $\pi(0) = 0$ and $\pi(K+1) = K+1$, and some input distribution $p(x_0, \ldots, x_K)$.

We provide two different proofs for the achievability of Theorem 3.1. Both proofs are based on DF relaying in the joint source-channel setting, that is, the source vector S^m is decoded in a lossless fashion by all the terminals in the network. The first proof is based on regular encoding and sliding window decoding without explicit binning at the source terminal. In this scheme, the typical source outcomes are mapped directly to different channel codewords rather than binning the source outputs prior to channel coding. Each relay finds the unique source index for which the corresponding source codeword is jointly typical with its side information and the corresponding channel codewords are jointly typical with the received channel vectors in the preceding blocks. This is a regular coding scheme since all the terminals in the network use a channel codebook of the same size, i.e., the number of typical source outputs.

The second coding scheme, which was studied in [9] for a single relay channel, uses explicit binning at the source encoder and channel codes of different sizes for each terminal in the network. We call it a semi-regular encoding and backward decoding scheme. The source is compressed (by binning) for each separate side information signal in the network, and hence a different rate of information is transmitted to each user; however, the rate of the channel codes for the terminals that have already decoded the message and are cooperating to forward it to the next terminal are the same. This is why we call this coding scheme a semi-regular encoding scheme. For decoding we use nested backward decoding [7].

In the following theorem it is shown that the conditions in Theorem 3.1 are also necessary for a physically degraded relay network with degraded side information sequences.

Theorem 3.2: For a physically degraded relay network in which the side information sequences also form a Markov chain given by

$$S_0 \to S_1 \to \cdots \to S_{K+1},$$

the source-channel rate r is achievable if

$$H(S_0|S_i) < rI(X_0^{i-1}; Y_i|X_i^{K+1}), \quad (4)$$

for all $i = 1, \ldots, K+1$. Conversely, if source-channel rate r is achievable, then there exists an input distribution $p(x_0, x_1, \ldots, x_K)$ such that (4) is satisfied with $<$ replaced by $\leq$.

Proof: See Section V. ■

IV. Proof of Theorem 3.1

A. Regular encoding and sliding window decoding

Fix m and n such that $\frac{n}{m} = r$. Consider source blocks $S_0^m(b)$ for $b = 1, \ldots, B-K$. These will be sent over B channel blocks of the channel each consisting of n channel uses. This corresponds to a source-channel rate of $Bn/(B-K)m$ which gets arbitrarily closely to r as $B \to \infty$.

For the sake of brevity, here we present the transmission scheme for a network with two relays, i.e., $K = 2$. The extension to $K > 2$ follows similarly. We assume that the message is decoded first by T_1 and then by T_2 before finally being decoded by the destination T_3. The rate can be maximized over different decoding orders, hence the consideration of the permutation in Theorem 3.1.

Fix $p(x_0, x_1, x_2)$ such that (3) holds. We use superposition block Markov encoding, sequential decoding at the relay and sliding window decoding at the destination.

Source code generation: Enumerate all typical source outcomes with $s_0^m(w_0)$, $w_0 \in [1, M]$, where $M = 2^{n(H(S_0)+\epsilon)}$. This constitutes the source codebook.

Channel code generation: Generate at random M i.i.d. channel codewords $x_2^n(w_2)$, $w_2 \in [1, M]$, each drawn according to the distribution $\prod_{t=1}^n p(x_{2,t})$.

For each $x_2^n(w_2)$, generate at random M conditionally i.i.d. channel codewords $x_1^n(w_1|w_2)$ each drawn according to the distribution $\prod_{t=1}^n p(x_{1,t}|x_{2,t}(w_2))$ for $w_1 \in [1, M]$.

Finally, generate M conditionally i.i.d. codewords $x_0^n(w_0|w_1, w_2)$ for each pair of

$$(x_1^n(w_1|w_2), x_2^n(w_2))$$

with distribution $\prod_{t=1}^n p(x_{0,t}|x_{1,t}(w_1|w_2), x_{2,t}(w_2))$.

We repeat this code generation process one more time to obtain another independent codebook. These codebooks are used sequentially for transmission over each channel block to create independence.

Encoding: At channel block b, for $b = 1, \ldots, B$, T_0 searches for the source block $s_0^m(b)$ within the source codebook. The corresponding index is denoted by $w_0(b)$, which is set to 1 if the source output does not appear in the codebook. Then T_0 transmits $x_0^n(w_0(b)|w_0(b-1), w_0(b-2))$, where $w_0(b') = 1$ for $b' < 1$ or $b > B-2$.

From the decoding procedure, which will be presented next, at the beginning of block b for $b = 1, \ldots, B$, terminal T_i, $i = 1, 2$, has estimates $\hat{w}_0^i(1), \ldots, \hat{w}_0^i(b-i)$ of the source indices $w(1), \ldots, w(b-i)$, respectively. We set $\hat{w}_0^i(b-i) = 1$ if $b-i < 1$ or $b-i > B-2$. Then in block b, T_1 and T_2 transmit

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$x_1^n(\hat{w}_0^1(b-1)|\hat{w}_0^1(b-2))$ and $x_2^n(\hat{w}_0^2(b-2))$, respectively, where we set $\hat{w}_0^i(b') = 1$ for $b' < 1$.

Decoding: At the end of block b, $b = 1, \ldots, B-2$, relay T_1 declares $\hat{w}_0^1(b) = w$ if there exists a unique index w for which

$$(s_0^m(w), S_1^m(b)) \in A_{S_0,S_1}^{m,\epsilon}, \text{ and}$$
$$(x_0^n(w|\hat{w}_0^1(b-1), \hat{w}_0^1(b-2)), (x_1^n(\hat{w}_0^1(b-1)|\hat{w}_0^1(b-2)),$$
$$x_2^n(\hat{w}_0^1(b-2)), Y_1^n(b)) \in A_{X_0,\ldots,X_2,Y_1}^{n,\epsilon},$$

where $A_X^{m,\epsilon}$ is the set of typical codewords of length m i.i.d. with marginal p_X [13]. Otherwise an error is declared.

Similarly, relay T_i at the end of block b, declares $\hat{w}_0^i(b-i+1) = w$ if there exists a unique index w for which

$$(s_0^m(w), S_i^m(b-i+1)) \in A_{S_0,S_i}^{m,\epsilon} \tag{5}$$

and (6) on the next page is satisfied for all $j = 0, 1, \ldots, i-1$. An error is declared if no or more than one such index is found.

Analysis of probability of error: We find the probability of error at T_k in block b, denoted by P_k^b, assuming that there is no error in the previous blocks, i.e., $\hat{w}_0^k(b'-k-1) = w(b'-k+1)$ for all $k = 1, 2, 3$ and $b' \leq b-1$.

Each terminal T_i, $i = 1, 2, 3$, for $b \geq i$ declares $\hat{w}_i(b-i+1) = w$ if there exists a unique index w that jointly satisfies the joint typicality conditions in (5) and (6) for all the blocks $j = 0, 1, \ldots, i-1$ assuming that the estimates of the previous blocks are correct, i.e., $\hat{w}_0^{i-1-j}(b-i) = w_0(b-i)$ for all $j = 0, 1, \ldots, i-1$.

The probability that the correct index $w_i(b-i+1)$ does not satisfy (5) or (6) for $j = 0, 1, \ldots, i-1$ can be made arbitrarily small by increasing m and n. On the other hand, the probability of an incorrect index satisfying (6) is approximately $2^{-nI(X_{i-j-1};Y_i|X_{i-j},\ldots,X_2)}$. Using the union bound, the probability of an incorrect index satisfying the typicality conditions (5) and (6) for all $j = 0, 1, \ldots, i-1$, can be bounded above by

$$(2^{mM} - 1)2^{-mI(S;S_i)}2^{-nI(X_0^{i-1};Y_i|X_i^2)},$$

which can be made arbitrarily small by fixing $n = rm$ and letting $m \to \infty$ since (3) holds.

B. Semi-regular encoding and backward decoding

In backward decoding for the relay channel [3], while the relay decodes each message block right after it is transmitted, the destination waits until all message blocks are transmitted and decodes them in the reverse order by removing the interference from the decoded messages. In the case of side information [9], the source samples are grouped into blocks and for each source block a bin index is generated for each terminal in the network. The number of bins for each terminal depends on the quality of its side information. We use the multiple relay backward decoding scheme proposed in [7] for the channel coding part. Again we present the transmission scheme for $K = 2$ with a decoding order of T_1, T_2, T_3.

Fix $p(x_0, x_1, x_2)$ such that (3) holds.

Source code generation: Corresponding to each terminal T_i, for $i = 1, 2, 3$, we consider $M_i = 2^{mH(S|S_i)}$ bins, called the T_i bins. All typical source outcomes $s_0^m \in T_{S_0}^n$ are partitioned randomly and uniformly into these bins, independently for each side information sequence, i.e., the distribution into M_i bins for S_i is independent of the distribution into M_j bins for S_j when $i \neq j$. These bin indices, made available to all terminals, form the source codebook.

Channel code generation: For the channel codebook, generate M_3 codewords $x_2^n(j_3)$ for $j_3 \in [1, M_3]$ i.i.d. with $p(x_2^n(j_3)) = \Pi_{t=1}^n p(x_{2,t})$ and index them as $x_2^n(j_3)$. For each $x_2^n(j_3)$, generate M_2 conditionally independent codewords $x_1^n(j_2|j_3)$, $j_2 \in [1, M_2]$, with probability $p(x_1^n|x_2^n(j_3)) = \Pi_{t=1}^n p(x_{1,t}|x_{2,t}(j_3))$ and index them as $x_1^n(j_2|j_3)$. Similarly generate the codebook of size M_1 for each possible combination of $(x_1^n(j_2|j_3), x_2^n(j_3))$, and index them as $x_1^n(j_1|j_2, j_3)$ with $j_1 \in [1, M_1]$.

Encoding: Consider a source sequence $S_0^{B^2m}$ of length B^2m. Partition this sequence into B^2 portions, $s_{0,b}^m$, $b = 1, \ldots, B^2$. Similarly, partition the side information sequences into B^2 length-m blocks $s_i^{B^2m} = [s_{i,1}^m, \ldots, s_{i,B^2}^m]$ for $i = 1, 2, 3$. We will transmit a total of B^2m source samples over a total of $(B+1)^2n$ channel uses. For any fixed (m, n) with $n = rm$, we can achieve a rate arbitrarily close to r by increasing B, i.e., $\frac{(B+1)^2n}{B^2m} \approx \frac{n}{m} = r$.

In block 1, T_0 observes $s_{0,1}^m$. An error is declared if $s_{0,1}^m$ is not typical. For a typical source outcome, it finds the corresponding T_1 bin index $w_{1,1} \in [1, M_1]$. It transmits the channel codeword $x_0^n(w_{1,1}|1, 1)$. The relays T_1 and T_2 simply transmit $x_1^n(1|1)$ and $x_2^n(1)$, respectively. The bin index of the jth block of the typical source output sequence with respect to T_i bins is denoted by $w_{j,i}$. In block 2, T_0 transmits the channel codeword $x_0^n(w_{2,1}|w_{1,2}, 1)$. The relays T_1 and T_2 transmit $x_1^n(\hat{w}_{1,2}^1|1)$ and $x_2^n(1)$, respectively, where $\hat{w}_{1,2}^1$ is the T_2 bin index estimate of $s_0^m(1)$ at the relay T_1.

In the following blocks $b = 2, \ldots, B$, the source terminal transmits the channel codeword $x_0^n(w_{b,1}|w_{b-1,2}, 1)$ where $w_{b,i} \in [1, M_i]$. In block $B+1$, T_0 transmits $x_0^n(1|w_{2,B}, 1)$.

The first relay T_1 estimates the source block $s_{0,b-1}^m$ at the end of block $b-1$ and finds the corresponding T_2 bin index $\hat{w}'_{b-1,2} \in [1, M_2]$. At block b, for $b = 2, \ldots, B+1$, T_1 transmits the channel codeword $x_1^n(\hat{w}'_{2,b-1}|1)$.

In the following $B+1$ blocks, the source terminal transmits $x_0^n(w_{B+1,1}|1, w_{1,3}), \ldots, x_0^n(w_{2B,1}|w_{2B-1,2}, w_{B-1,3})$, $x_0^n(1|w_{2B,2}, w_{B,3})$. The relay T_1 transmits $x_1^n(1|\hat{w}_{1,3}^1)$, $x_1^n(\hat{w}_{B+1,2}^1|\hat{w}_{2,3}^1), \ldots, x_1^n(\hat{w}_{2B,2}^1|\hat{w}_{B,3}^1)$. Having estimated the source blocks $s_{0,1}^m, \ldots, s_{0,B}^m$ at the end of channel block $B+1$ by backward decoding, the second relay T_2 transmits $x_2^n(\hat{w}_{1,3}^2), \ldots, x_2^n(\hat{w}_{B,3}^2)$.

They continue similarly for a total of $B+1$ channel blocks of $(B+1)n$ channel uses each such that no new source block is encoded in the last channel block of $(B+1)n$ channel uses.

Decoding and error probability analysis: The relay T_1 decodes the source signal by sequentially reconstructing source block $s_{0,b}^m$ at the end of channel block b. Assume that T_1 knows $s_{0,b-1}^m$ at the end of block $b-1$ with high probability. Hence,

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$$(x_{i-1-j}^n(w|\hat{w}_0^i(b-i),\ldots,\hat{w}_0^i(b-j-2)),\ldots,x_2^n(\hat{w}_0^i(b-j-2)),Y_j^n(b-j))\in A^{n,\epsilon}_{X_{i-1-j},\ldots,X_2,Y_i} \tag{6}$$

it can find the T_2 bin index $w_{b-1,2}$. Using this information and its received signal y_1^n, the T_1 channel decoder will attempt to decode $w_{b,1}$, i.e., the T_1 bin index corresponding to $s_{0,b}^m$. This is then given to the T_1 source decoder. With the T_1 bin index and the side information $s_{1,b}^m$, the T_1 source decoder estimates $s_{0,b}^m$. The estimation error can be made arbitrarily small for large enough m and n satisfying $n = rm$ since

$$H(S_0|S_1) < rI(X_0;Y_1|X_1,X_2).$$

The relay T_2 starts backward decoding after channel block $B+1$ and reconstructs source blocks $s_{0,1}^m,\ldots,s_{0,B}^m$ at the end of channel block $B+1$. This is same as the backward decoding used by the destination terminal in the single relay setup [9], and is successful with high probability if

$$H(S_0|S_2) < rI(X_0,X_1;Y_2|X_2).$$

Decoding at the destination is also done using backward decoding, but the destination waits till the end of channel block $(B+1)^2$. It first tries to decode s_{0,B^2}^m using the received signal at channel block $(B+1)^2$ and its side information s_{2,B^2}^m. Going backwards from the last channel block, the destination follows the usual backward decoding technique and decodes the source blocks backwards with high probability since

$$H(S_0|S_3) < rI(X_0,X_1,X_2;Y_2).$$

V. Proof of Theorem 3.2

The converse for degraded relay networks follow from the cut-set bound. Consider the set $\mathcal{S} = \{T_0,\ldots,T_{i-1}\}$ and assume that the terminals in $\mathcal{S}$ all have access to the source vector S_0^m, hence they can cooperate for transmitting S_0^m. On the other hand, we assume the remaining terminals can cooperate perfectly by using all the available side information vectors $S_i^m,\ldots,S_{K+1}^m$ as well as their received channel outputs $Y_i^m,\ldots,Y_{K+1}^m$. This reduces to a point-to-point scenario for which the following is a necessary condition for reliable transmission:

$$H(S_0|S_i,\ldots,S_{K+1}) \le rI(X_0^{i-1};Y_i,\ldots,Y_{K+1}|X_i^{K+1}).$$

From the degradedness assumption of the side information vectors we have

$$H(S_0|S_i,\ldots,S_{K+1}) = H(S_0|S_i),$$

and from the physically degraded channel assumption we have

$$I(X_0^{i-1};Y_i,\ldots,Y_{K+1}|X_i^{K+1}) = I(X_0^{i-1};Y_i|X_i^{K+1}).$$

We complete the proof of the theorem by considering the sets corresponding to $i = 1,\ldots,K+1$.

VI. Conclusion

We have considered the reliable transmission of a discrete memoryless source signal over a cooperative multiple-relay network in which the relays and the destination all have access to a different side information signal correlated with the source signal. We have developed a multiple-relay extension of the decode-and-forward source-channel relaying scheme. To be able to use the side information at each terminal for decoding we have developed two separate block Markov encoding schemes; the first scheme does not use source binning and is based on sliding window joint source-channel decoding, while the second scheme applies explicit source binning and uses separate source and channel decoders based on backward decoding.

Moreover, we have proven the optimality of DF relaying in the joint source-channel setting for a physically degraded relay channel with degraded side information. This can be used to model, for example, a scenario in which the relays and the destination are located with increasing distances from the source terminal, and both the received signal quality and the quality of the sensed source signal at the terminals degrade with distance.

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