A Coupled Multichannel Filter Bank and Sniffer Spectrum Analyzer

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ABSTRACT:

The FFT, the efficient algorithm for implementing the DFT, enjoys great acceptance as the signal processing tool for spectrum analysis, for channelized receivers, and for fast convolution. In the first applications, spectrum analysis, the FFT is supported by a set of weights, the window, applied to data multiplicatively. In the second application the FFT is supported by a set of weights, the filter, applied to data convolutionally. Both operations accomplish the same task; that of spectral decomposition with controlled spectral response. In reality, the two operations are identically the same since the sliding windowed FFT is in fact a particular implementation of a resampling filter bank. Since the two processes are the same, when a system includes both a channelizer and a spectrum analyzer that steers the channelizer to spectral areas of interest the two can be combined or coupled to share their computational burden. In this paper we illustrate the benefit of this merged option.

I. INTRODUCTION:

When we do spectrum analysis we gather successive N point sequences of a time series and have each sequenced processed by an N-point DFT. The raw power spectral estimates from each DFT are ensemble averaged to reduce the variance of the estimates. The collection and processing of successive intervals is known as a sliding windowed DFT. Figure 1 shows the successive windowed intervals presented to the DFT.

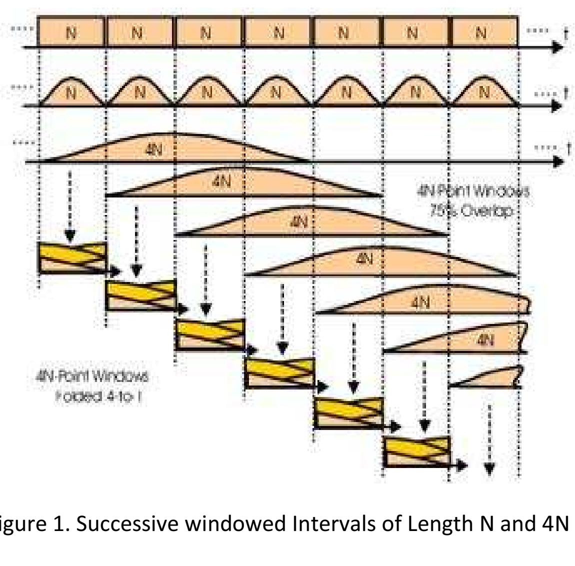

Figure 1. Successive windowed Intervals of Length N and 4N

Turning on and off of a signal sequence by the default rectangle window place boundaries on the signal not related to the signal itself but rather to the observation process. The spectral side-lobes of the window's Fourier transform permit signal energy at one frequency to influence and bias the observed energy at another. This effect, known as spectral leakage, is related to the discontinuity caused by the gating process. We note that the DFT describes the input sequence as a weighted sum of N sinusoids harmonically related to the signal collection interval of length N. The IDFT describes the periodic extension of the input sequence. If the sequence is not periodic, the periodic extension exhibits discontinuities in many order derivatives at the wrap-around boundary. To minimize spectral artifacts these discontinuities must be suppressed.

We suppress the discontinuities at the boundary by applying a multiplicative weighting function called a window [1] that gently and smoothly brings the weighted sample values and all order derivatives at the boundary to near zero values. The amplitude of the spectral leakage terms are best visualized through the spectral side lobes of the Fourier series of the window. Figure 2 shows the spectra of an N-point Rectangle window with its equally spaced zero crossings that alias to DC when down sampled N-to-1 in a non-overlapped sequence of windowed DFTs. When we apply a window to each interval, as shown in figure 1, to reduce spectral side lobe levels to near -50, -70, or -90 dB, we increase the main lobe width by a factor of 2, 3, or 4 respectively.

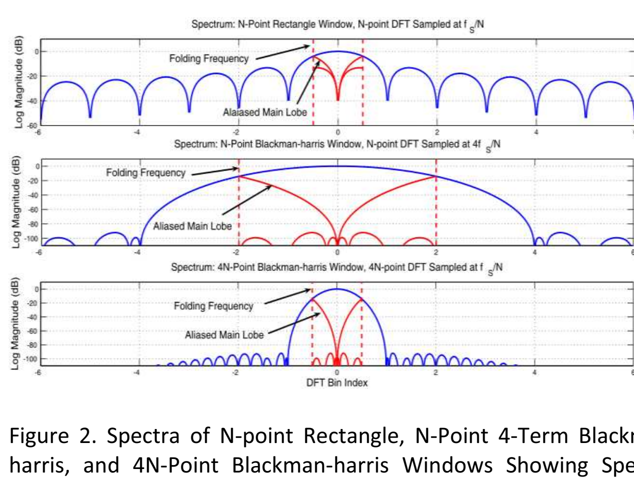

Figure 2. Spectra of N-point Rectangle, N-Point 4-Term Blackman-harris, and 4N-Point Blackman-harris Windows Showing Spectral folding of Main Lobe Down-Sampled N-to-1, N/4-to-1, and N-to-1

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To satisfy the Nyquist criteria, the 4-times wider main lobe width requires us to sample the output of the transform 4-times as often, hence 4-to-1 overlap. Desiring to obtain the side lobe suppression of the good window without increasing the main lobe width, we can increase the window length from N to 4N and still maintain the same 4-to-1 overlap of the intervals required by the Nyquist criterion. The 4N point windowed sequence could be offered to a 4N point DFT but that would increase the number of output frequency bins as well as the processing workload. In our mind's eye we can present the 4N points to the 4N point DFT and then take every 4-th spectral output which would match the spectral sample spacing of the N-point DFT. The 4-to-1 down-sampling of the DFT spectra causes 4-fold aliasing of its windowed time series. We recognize the 4-fold aliasing as a polyphase partition of the window sequence [2, 3]. This polyphase interpretation of the time domain aliasing is shown in equation (1). Here the inner summation is the polyphase partition of the window.

$$
\begin{aligned}
H_{4N}(k) &= \sum_{n=0}^{4N-1} d(n) w_{4N}(n) e^{-j\frac{2\pi}{4N}nk} \\
H_{4N}(4k) &= \sum_{n=0}^{4N-1} d(n) w_{4N}(n) e^{-j\frac{2\pi}{4N}n4k} \\
&= \sum_{n=0}^{4N-1} d(n) w_{4N}(n) e^{-j\frac{2\pi}{N}nk} \\
&= \sum_{n=0}^{N-1} \left[\sum_{s=0}^{3} d(n+4s) w_{4N}(n+4s) \right] e^{-j\frac{2\pi}{N}nk}
\end{aligned} \tag{1}
$$

We know now that the preferred windowing procedure for an N-point DFT uses a window of at least length 4N which is folded or partitioned into a 4-path polyphase filter. We showed in figure 2 the folding of the spectral main lobe when the DFT is down-sampled N-to-1, i.e. 1-ouput sample for every N-input samples. This folding corrupts the spectral content of the pass band frequency span. We can avoid this corruption by increasing the sample rate from N-to-1 to N/2-to-1. This means that the N-point polyphase window and DFT should operate in the same manner as a polyphase channelizer. In a companion paper [4] we described the minor change in the commutator control required to operate the channelizer in the N/2-to-1 down-sample mode. The change in spectral folding due to the increased sample rate is illustrated in figure 3. The alias free spectra available from the N/2-to-1 down-sampled output of a channelizer and of a folded sliding windowed spectrum analyzer DFT allows us to cascade their functions to obtain successively narrower channel bandwidths and higher resolution spectral analysis. We address this option in the next section.

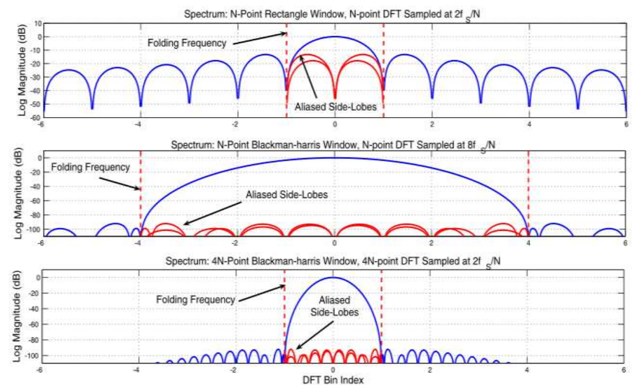

Figure 3. Spectra of N-point Rectangle, N-Point 4-Term Blackman-harris, and 4N-Point Blackman-harris Windows Showing Spectral folding of Main Lobe Down-Sampled N/2-to-1, N/8-to-1, and N/2-to-1

II. STEERED DIGITAL DROP RECEIVERS:

We recently participated in a design and implementation of a system that coupled a high resolution spectrum analyzer to a set of digital drop receivers for a signal collection and monitoring activity. A simplified block diagram of the system is shown in figure 4. The input sample rate to the system is 90 MHz. The 720 path polyphase filter and phase rotators offer spectral center spacing of 125 kHz. The prototype filter in the polyphase partition has a bandwidth of 250 kHz and the output sample rate of the channelizer is 500 KHz. The available adjacent channels overlap by 50%. This overlap assures us that signal spectra of interest, with bandwidths less than 125 kHz and with arbitrary center frequency will be contained in at least one channel. The channelizer does 180-to-1 down sampling to obtain the 500 kHz output rate. All 720 center frequencies are aliased to DC in the polyphase partition. The channel selector can direct the channelizer to un-alias up to 16 channels by appropriate phase aligned summations that match the unique phase profiles of the selected aliased signal set.

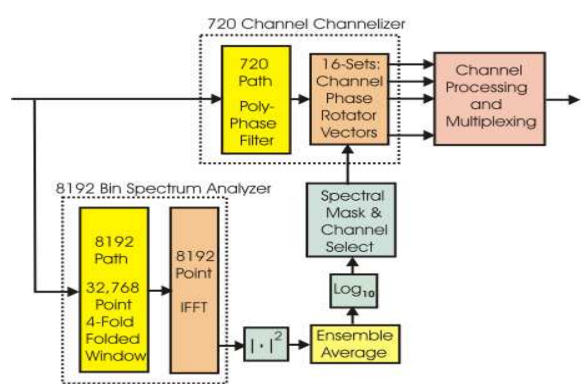

Figure 4. Block Diagram Spectral Sniffer Steered Digital Drop Receiver

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The channel selector in figure 4 monitors the smoothed spectral estimates obtained from the 8192 point spectrum analyzer running in parallel with the channelizer. The spectrum analyzer has a polyphase partitioned set of 32,768 weights that define its spectral characteristics. The equivalent channel filters of the spectrum analyzer are very nearly the spectrum shown in the bottom plot of figure 3. The resolution bandwidth of the spectral sniffer is 90 MHz/8192 which equals 10.986 kHz, a width we will approximate by 11 kHz.

It is an interesting exercise to estimate the workload of the two spectral engines. We do so by first determining the usage rates for the two processing engines and then estimating the workload per cycle. Let's start with the large 8192 point windowed transform. The large transform is performed every time 8192 points are delivered to it. This happens 90,000,000 samples/sec divided by 8192 samples/transform or approximately 10,986 8k point transforms per second. The number of real multiplications required for the 8192 point IFFT is approximately 160,000 per IFFT. The real multiply workload to apply the 32,768 point window is approximately 131,000, a burden almost as large as the IFFT. Thus the workload per spectrum analyzer cycle is about 292,000 multiplies and the workload per second is an astounding 3.2*10^9 multiplies per second.

We now estimate the workload for the 720 path channelizer. The channelizer operates every time 180 new data samples are presented to it which of course matches the 500 kHz rate of each output channel. In its present form the polyphase filter has two real taps per path for the filter workload of 2880 real multiplies per cycle. The channel select phase rotators require 720 complex multiplies per channel which for the set of 16 channels require some 46,080 real multiplies, a much larger number than that required for the polyphase filter segment. The total workload per channel bank cycle is approximately 48,960. When this workload is folded into the cycle time of the channelizer, the workload rises to another astounding number, 24.48*10^9 multiplies per second. This workload is about 8 times the workload of the companion spectrum analyzer.

II. STEERED CHANNELIZER DROP RECEIVER:

In the previous section we determined that the digital drop receiver had the higher cost of the two processes. We start this section by trying to reduce its cost. Our first approach is to replace the complex inner products performing the phase coherent sums for each of the 16 selected channels with an efficient 720 point IFFT. In general, the FFT becomes more efficient than the set of inner products when the number of inner products exceeds $\log_2(N)$. When N is 720, this cross over point occurs between 9 and 10 channels. We have need for an efficient 720 point FFT. The number 720 is highly composite with factors 5, 9, and 16. These factors are primes or powers of primes which means they lend themselves well to the prime-factor or Good-Thomas partition of the DFT. The Prime factor algorithm maps the one dimensional DFT to uncoupled multi-dimension, in this case a 3-dimension DFT. The uncoupling means there are no twiddle factors or phase spinners to be applied to the intermediate arrays between directions, or corner turns as described in the literature. All we need to implement the 720 point FFT is a set of efficient 5-point, 9-point, and 16 point FFTs. The most efficient version of these is the Winograd Fourier transform algorithm. The literature [5] has addressed the 720 point Good-Thomas, Winograd algorithm and describes the workload for complex input data as less than 2,400 multiplies. We fold in the 4-path polyphase filter of 2889 real multiplies to obtain a workload for the full 720 channel channelizer of 5,300 real multiples which is less than 11 percent of the workload to form the 16 digital drop receivers. The IFFT that builds all the channels reduces the work load by nearly an order of magnitude. The whole channelizer workload is 5,300 multiplies per channelizer cycle which is the 500 kHz output sample rate. The total workload is then 5,300*500*10^3 or 2.65 *10^9 multiplies per second which is about 11% of the digital drop receivers'. The block diagram of the alternate channelizer structure with its alternate spectrum analyzer sniffer is shown in figure 5.

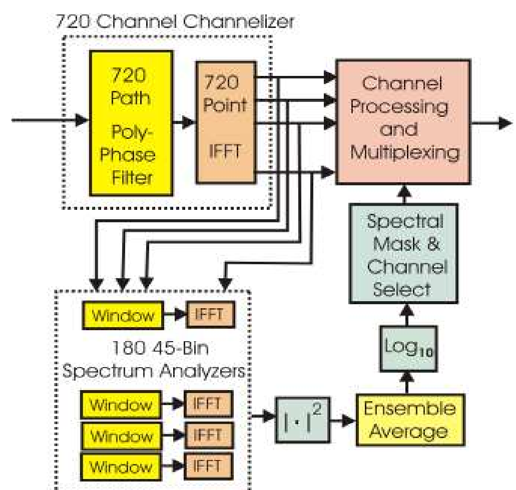

Figure 5. Block Diagram: Spectral Sniffer Selected Channelized Receiver

Now that we have access to the output of the full filter bank we can consider applying a set of short transforms to each filter bank time series to obtain the high resolution spectral decomposition. In reality we don't need the decomposition of every filter output. Remember

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that when we designed the channel filters they were 50% overlapped so we can likely obtain the spectral record from alternate channels of the channelizer. In addition, we probably only have interest in less than half of the 90 MHz bandwidth. Thus rather than process the time series from each of the 720 channels we likely can obtain valid spectral estimates from 180 of them.

We want the frequency resolution from the short channel based transforms to be comparable to that of the 8192 point transform. This resolution was about 11 kHz. The data rate out of each channel is 500 kHz so to obtain the same 11 kHz resolution the transform length needs to be 500/11 or near length 45. Interestingly, we have a 45 point transform embedded in the 720 point transform so this is a good match. The 45 point Good-Thomas Winograd transform requires 190 real multiples to process complex input signals. If we include a 4-fold polyphase window to shape the frequency response of the 45-point transform we have another 180 points of multiplication for both I and Q input samples for an additional 380 multiplies per transform cycle. Here again the shaping filter requires more work than does the transform and the total work per windowed 45-point transform is 570 multiplies per spectral estimation cycle. We perform the estimation cycle every time 45 new samples are presented to us and at the 500 kHz rate; this occurs 500 kHz/45 or 11,111 times per second. The workload for 180 channels at 570 multiplies per channel at 11,111 cycles per second is 1.14*10^9 multiplies per second. This work rate is less than half the rate of the 8192 point spectrum analyzer's rate.

III. SIMULATION DEMONSTRATION:

To demonstrate the validity and performance of the cascade coupled channelizer and spectrum analyzer we wrote a MATLAB simulation of a channelizer and coupled spectrum analyzer. The simulation contained a 180 channel channelizer with 100 dB dynamic range polyphase filter operating in the 90-to-1 down-sample mode. The cascade spectral analyzer that formed the spectral decomposition of each channel time series was a windowed 128 point FFT with -100 dB side-lobe levels. This corresponds to a 32 point FFT in the 4-fold polyphase version of the window. The input test signal consisted of a sum of complex sinusoids offset from each channel center frequency with two located at the boundaries of the channel bandwidths.

Figure 6 shows the spectrum of the input test signal composed of 8 equal amplitude complex sinusoids. Figure 7 shows an estimate of the signal power spectrum obtained as the short term average power output from each of the 180 filters in the channelizer.

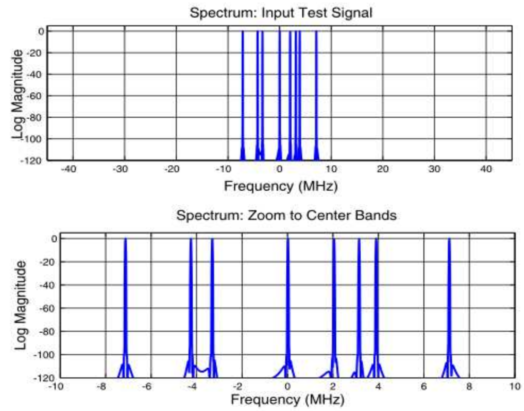

Figure 6. Spectrum and Zoom Detail: Input Test Signal

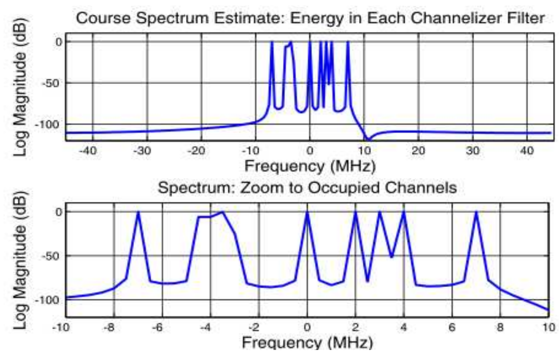

Figure 7. Course Spectrum and Zoom Detail: Power Output from 180 Channelizer Filters

Figure 8, a very busy and detailed figure, shows the power spectra formed by the spectrum analysis of 60 of the 180 separate channel output series. The integers in the upper right hand corner of each subplot are the frequency index of the channelizers' 180 Point IFFT. Figures 9 and 10 are expanded segments of this figure to permit a more detailed examination of the channel spectra and the design considerations.

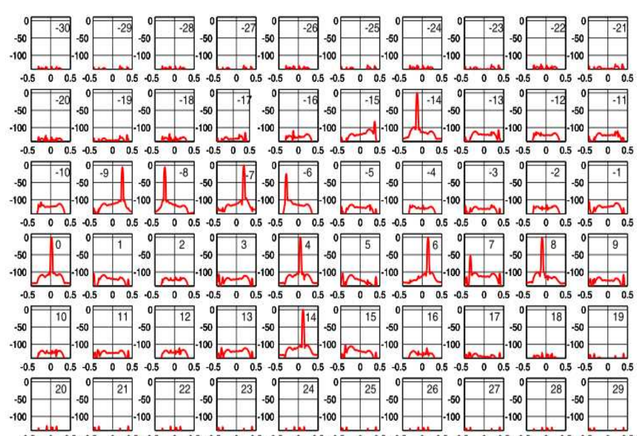

Figure 8. Power Spectra from Channels -30 to +29 of the 180 Channel Channelizer

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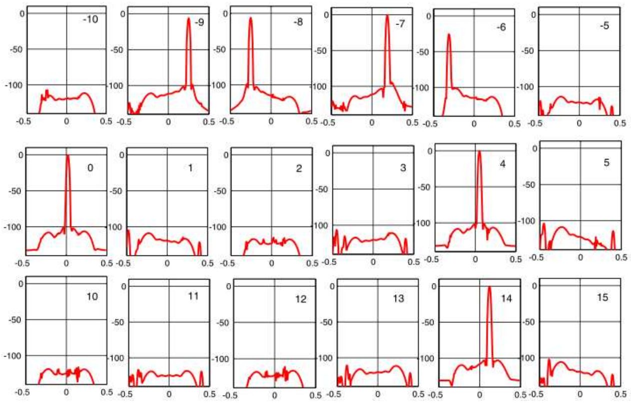

Figure 9. Power Spectra from Selected Channels in Channelizer: Subplots Extracted from Left Center of Figure 8

The first detail we note in the subplots of figure 9 is the all the spectra are band-limited with bandwidths confined to the interval -0.4 to +0.4 in the normalized interval -0.5 to +0.5. What we see here is the channelizer shaping filter response to the out of band spectral components suppressed down to the stop band levels of the channelizers' prototype low pass filter. We also see that the output sample rate of each channelizer output is twice the channel bandwidth. This was the intent and consequence of running the 180 path channelizer in the 90-to-1 down-sampling mode.

The next item we note from the subplots is the effect of the bandwidth selected for the channelizer. Since this simulation was not for the 720 channel system described earlier in this paper, the channels widths were not designed for 50% overlap but rather designed to cross at their 6-dB points. This choice permits perfect reconstruction of spectra from adjacent channels to extract signals whose spectra brackets two adjacent channels. We note that the spectral lines in the subplots with IFFT indices -9 and -8 are the same spectral line residing at the channel band-edge. The spectral lines in the subplots with IFFT indices -7 and -6 are also seen to be the same line.

The subplots of figure 10 present details of a different segment of the spectral plots of figure 8. Here too we see the band-width of the channel filter and the deep level of channel attenuation, greater than 100 dB. Of interest in this set of spectra are the subplots with indices 6, 7, and 8. The spectral lines seen at the edges of the index 7 subplot are the line located near the edge of index 6 subplot and the line located in the pass-band of index 8 subplot. Similar residual coupling between adjacent bands can be seen in other adjacent sub plots.

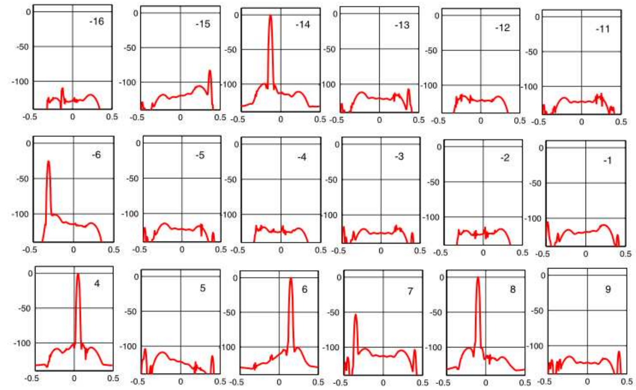

Figure 10. Power Spectra from Selected Channels in Channelizer: Subplots Extracted from Right Center of Figure 8

IV. CLOSING COMMENTS:

We have described some important considerations in the design of polyphase channelizers and polyphase-folded window spectrum analyzers. These include the need for oversampling their respective channels [6] to avoid band edge folding and options to permit extraction of signals whose spectrum brackets channelizer band edges. We have also described and compared two techniques to implement a channelizer and a companion spectral sniffer. We have concluded that there are implementation advantages to couple the spectral sniffer to the channelizer outputs rather than to the channelizer input.

REFERENCES:

1. fred harris, "On the Use of Windows for Harmonic Analysis with the Discrete Fourier Transform", Proceedings of the IEEE, Vol. 66, No. 1, January 1978.
2. fred harris, "On Reconstructing Filtered Time Series from a Sequence of Overlapped, Windowed, and Folded Fast Fourier Transforms", 19-th Annual Asilomar Conference on Signals, Systems, and Computers, Pacific Grove, CA, November 1985.
3. fred harris, "On the Relationship between Multirate Polyphase FIR Filters and Windowed, Overlapped FFT Processing", 23-rd Annual Asilomar Conference on Signals, Systems, and Computers, Pacific Grove, CA, 30-October to 1-November 1989.
4. fred harris, Bob McGwier, and Benjamin Egg, "A Versatile Multichannel Filter Bank with Multiple Channel bandwidths", CrownCom 2010,9-11 June 2010, Cannes, France.
5. C.S. Burrus and T.W. Parks, "DFT/FFT and Convolution Algorithms", John Wiley & Sons, 1985.
6. fred harris, "Multirate Signal Processing for Communication Systems", Prentice-Hall, 2004.

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http://dx.doi.org/10.4108/ICST.CROWNCOM2010.9237