

# Beamforming in Underlay Cognitive Radio: Null-Shaping Constraints and Greedy User Selection

Eduard Jorswieck and Rami Mochaourab  
 Communications Theory, Communications Laboratory  
 Dresden University of Technology, Dresden, Germany  
 e-mail: {Jorswieck, Mochaourab}@ifn.et.tu-dresden.de

**Abstract**—In a multiuser cognitive radio setting, multiple secondary systems coexist with multiple primary systems. We assume the secondary systems consist of transmitter-receiver pairs, and each transmitter is equipped with multiple antennas while all receivers use a single antenna. In this setting, the secondary transmitters are to operate under the constraints of producing no interference at the primary users. Such constraints on the secondary systems are referred to as null-shaping constraints. Three results are derived in this work. First, the Pareto optimal operation points for the secondary systems under null-shaping constraints are characterized by real-valued parametrization of the transmission strategies. Second, we show that all points on the Pareto boundary of the secondary systems achievable rate region without interference constraints can be achieved as the outcome of a noncooperative game by imposing certain virtual null-shaping constraints. Third, motivated by the surprising result that all Pareto efficient points can be achieved as a Nash equilibrium, we consider the problem of selecting a subset of the noncooperative secondary users for operation under the objective of maximizing their achievable sum rate. A low complexity suboptimal greedy secondary user selection algorithm is proposed and its performance is illustrated by simulations.

## I. INTRODUCTION

In a cognitive radio scenario, the systems are capable of detecting their environment and reconfiguring their operation accordingly. These capabilities are feasible due to measuring and feedback mechanisms in the network [1]. Consider a network composed of licensed primary users. The offered radio resources might not be utilized completely by these systems such that more users can be supported in the network. Additional users, having cognitive radio capabilities, can be also supported in the network. These users are called secondary users, and they can use the resources licensed to the primary users under the condition of not imposing quality of service (QoS) degradation to these systems. A limited QoS degradation to the primary users in an underlay cognitive radio setting [2] is described by interference temperature constraints (ITC) [1]. These constraints can be imposed by the primary users on the secondary users. In [3], a single primary user is considered as a leader in a Stackelberg game with the secondary users as followers. The primary user determines the prices for utilizing the frequency tones under which the

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secondary users automatically fulfill the ITCs. Hence, the primary user has the power to determine the constraints which the secondary users abide.

The ITCs are distinguished in soft- and peak-power-shaping constraints [4]. These constraints refer to the maximum average power and average peak power tolerated at the primary receivers, respectively. In our case, the two types of constraints are equivalent since we consider only single stream beamforming which is shown in [5] to be optimal in the multiple-input single-output (MISO) interference channel (IFC). Reference [6] considers the setting of a single secondary transmitter sharing the same spectral band with multiple primary users. The authors provide optimal transmit beamforming strategies under ITCs for the secondary transmitter. Furthermore, reference [7] characterizes the Pareto boundary of the MISO IFC through controlling the ITCs on the receivers. Each Pareto rate tuple is achieved in a decentralized manner, where each transmitter maximizes its rate independently limited to ITCs at not designated receivers. Moreover, convex optimization techniques for solving cognitive radio problems are studied in [8].

In this work, we assume that the primary systems do not tolerate any interference from the secondary systems, i.e., the ITCs are fixed to be zero. In this case, the constraints are called null-shaping constraints [4]. The outline and main results of this paper are as follows:

- 1) (Section III) Given a set of existing primary users, the Pareto boundary of the secondary users' achievable MISO IFC rate region is characterized under the null-shaping constraints. The Pareto efficient strategies can be performed by the secondary users if they cooperate.
- 2) (Section IV) Motivated by distributed (noncooperative) operation of the secondary systems, we turn our interest to the design of null-shaping constraints that improve the efficiency of the noncooperative systems. We characterize the null-shaping constraints, corresponding to virtual primary users, such that all points on the Pareto boundary of the MISO IFC rate region without constraints are achieved by the noncooperative secondary systems. This result shows that imposing null-shaping constraints on the secondary users can be sufficient to improve their noncooperative outcome.
- 3) (Section V) Following the previous result, we investigate the noncooperative secondary user selection problem to increase the achievable sum rate. The selection should

activate only a subset of the existing secondary systems for operation. Assuming that the secondary users are noncooperative and null-shaping constraints corresponding to existing primary users exist, we provide a secondary user selection algorithm that improves the sum performance of the systems. The algorithm is greedy such that in each iteration step, activating a secondary system has to increase the sum rate of the selected secondary systems set.

## II. PRELIMINARIES

### A. Notations

Column vectors and matrices are given in lowercase and uppercase boldface letters, respectively. The notation  $x_{k,\ell}$  describes the  $\ell$ th component of vector  $\mathbf{x}_k$ . The Euclidean norm of  $\mathbf{a}$ ,  $\mathbf{a} \in \mathbb{C}^{N \times 1}$ , is written as  $\|\mathbf{a}\|$ . The absolute value of  $b$ ,  $b \in \mathbb{C}$ , is denoted by  $|b|$ . The eigenvector which corresponds to the  $i$ th eigenvalue of the matrix  $\mathbf{Z}$  is denoted by  $\mathbf{v}_i(\mathbf{Z})$ . The eigenvector corresponding to the largest eigenvalue of a matrix  $\mathbf{Z}$  is specified as  $\mathbf{v}_{\max}(\mathbf{Z})$ . We always assume that the eigenvalues are ordered in nondecreasing order such that  $\mu_i(\mathbf{Z}) \leq \mu_{i+1}(\mathbf{Z})$ . The orthogonal projector onto the orthogonal complement of the column space of  $\mathbf{Z}$  is written as  $\Pi_{\mathbf{Z}}^\perp = \mathbf{I} - \mathbf{Z}(\mathbf{Z}^H \mathbf{Z})^{-1} \mathbf{Z}^H$ .

### B. System Model

We consider a setting in which  $K$  secondary cognitive transmitter-receiver pairs, also referred to as users, coexist with  $L$  active primary systems. Define the set of secondary users as  $\mathcal{S} \triangleq \{1, \dots, K\}$  and that of the primary users as  $\mathcal{P} \triangleq \{1, \dots, L\}$ . Each secondary transmitter is equipped with  $N$  transmit antennas and each receiver with a single antenna. All systems share the same bandwidth such that the setting leads to a MISO IFC [9]. Unlike the primary users, the secondary users are considered to have cognitive capabilities. These systems can adapt their operation according to different situations. Therefore, we are only interested in this paper in the operation of the secondary systems. The primary users can be considered as a set of receivers that have the power to influence the operation of the secondary users.

### C. Channel Model

The quasi-static block flat-fading vector channel from secondary transmitter  $k$  to secondary receiver  $j \in \mathcal{S}$  is denoted by  $\mathbf{h}_{kj} \in \mathbb{C}^{N \times 1}$ . The channel vectors from secondary transmitter  $k$  to primary receiver  $j \in \mathcal{P}$  is denoted by  $\mathbf{z}_{kj} \in \mathbb{C}^{N \times 1}$ . The beamforming vector of secondary transmitter  $k$  is represented with  $\mathbf{w}_k \in \mathbb{C}^{N \times 1}$ . Moreover, each transmitter has a total power constraint of  $P \triangleq 1$ . Generalization to different power constraints at each transmitter can be done without affecting the results in the next sections. This leads to the constraint:  $\|\mathbf{w}_k\|^2 \leq 1, k \in \mathcal{S}$ . The set of feasible transmit beamforming vectors of transmitter  $k, k \in \mathcal{S}$ , is defined as

$$A_k \triangleq \{\mathbf{w} : \|\mathbf{w}\|^2 \leq 1\}. \quad (1)$$

Noise plus interference originating from primary users is assumed independent and identically distributed (i.i.d.) complex

Gaussian with zero mean and variance  $\sigma^2$ . We define the transmit signal-to-noise ratio (SNR) as  $1/\sigma^2$ . Each secondary transmitter is assumed to have perfect local channel state information (CSI), i.e., it has perfect knowledge of the channel vectors only between itself and all secondary and primary receivers. The achievable rate of secondary user  $k$  is

$$R_k(\mathbf{w}_1, \dots, \mathbf{w}_K) = \log_2 \left( 1 + \frac{|\mathbf{h}_{kk}^H \mathbf{w}_k|^2}{\sigma^2 + \sum_{l \in \mathcal{S} \setminus \{k\}} |\mathbf{h}_{lk}^H \mathbf{w}_l|^2} \right), \quad (2)$$

where the receivers are assumed to treat interference from secondary transmitters as additive noise. Thus, each transmitter need not know the channels incident at its receiver in order to allocate the transmission rate in (2). The achievable rate region defined as

$$\mathcal{R} \triangleq \{(R_1, \dots, R_K) : \mathbf{w}_k \in A_k, k \in \mathcal{S}\}, \quad (3)$$

is the set of all rate tuples achieved by feasible beamforming vectors.

*Definition 1:* A rate tuple  $(R_1, \dots, R_K)$  is Pareto optimal if there is no other tuple  $(Q_1, \dots, Q_K)$  with  $(Q_1, \dots, Q_K) \geq (R_1, \dots, R_K)$  and  $(Q_1, \dots, Q_K) \neq (R_1, \dots, R_K)$  (the inequality is componentwise).

The Pareto boundary of the achievable rate region  $\mathcal{R}$  is the set consisting of all Pareto optimal points.

## III. PARETO BOUNDARY FOR GIVEN CONSTRAINTS

In this section, we characterize the beamforming vectors that achieve Pareto optimal points under null-shaping constraints. In other words, given the channels to primary users, the secondary transmitters are to form a null in the direction of these channels, i.e.,

$$|\mathbf{z}_{k\ell}^H \mathbf{w}_k| = 0, \quad \text{for all } k \in \mathcal{S}, \ell \in \mathcal{P}. \quad (4)$$

Hence, the set of feasible transmission strategies for secondary transmitter  $k \in \mathcal{S}$  is

$$\tilde{A}_k \triangleq \{\mathbf{w} : \|\mathbf{w}\|^2 \leq 1, |\mathbf{z}_{k\ell}^H \mathbf{w}| = 0 \text{ for all } \ell \in \mathcal{P}\}. \quad (5)$$

The achievable rate region under null-shaping constraint is defined as

$$\tilde{\mathcal{R}} \triangleq \{(R_1, \dots, R_K) : \mathbf{w}_k \in \tilde{A}_k, k \in \mathcal{S}\}. \quad (6)$$

In [10], null-shaping constraints on secondary users are considered in a noncooperative MIMO cognitive radio game. The orthogonal projector onto the null space of the primary user's channel is used to transform the secondary user's rate maximization problem to one that is convenient to study. Next, we use a similar application of this projection. Define the matrix containing the channels from secondary transmitter  $k$  to all primary users as  $\mathbf{Z}_k \triangleq [\mathbf{z}_{k1}, \dots, \mathbf{z}_{kL}]$ .

*Proposition 1:* ([11, Corollary 1]) Assume  $N \geq K + L$ . All beamforming vectors which fulfill the null-shaping constraints in (4) and achieve the Pareto boundary of  $\tilde{\mathcal{R}}$  in (6) are

$$\mathbf{w}_k(\boldsymbol{\lambda}_k) = \mathbf{v}_{\max} \left( \sum_{\ell=1}^K \lambda_{k,\ell} \mathbf{e}_{k,\ell} \Pi_{\mathbf{Z}_k}^\perp \mathbf{h}_{k\ell} \mathbf{h}_{k\ell}^H \Pi_{\mathbf{Z}_k}^\perp \right), \quad k \in \mathcal{S}, \quad (7)$$

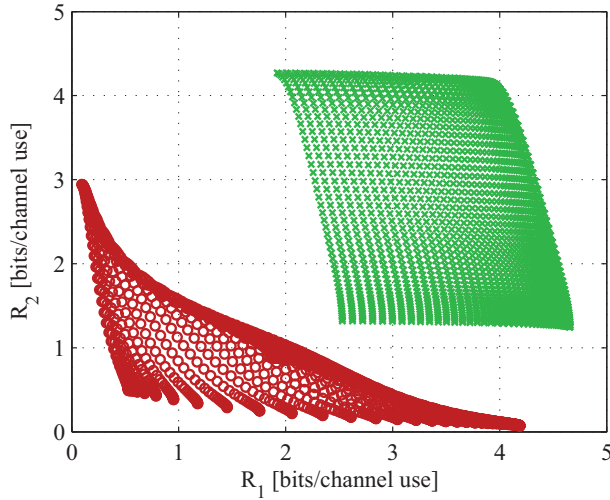


Fig. 1. The Pareto boundary of  $\mathcal{R}$  in (3) is marked with crosses. The Pareto boundary of  $\tilde{\mathcal{R}}$  in (6) is marked with circles. The plots are made for single random channel realization with  $N = 3$  and SNR = 10 dB.

where

$$e_{k,\ell} = \begin{cases} +1 & \ell = k \\ -1 & \text{otherwise} \end{cases}, \quad (8)$$

and  $\lambda_k \in \Lambda_K$ . The set  $\Lambda_K$  is defined as

$$\Lambda_K \triangleq \left\{ \lambda \in [0, 1]^K : \sum_{\ell=1}^K \lambda_\ell = 1 \right\}. \quad (9)$$

We assume  $N \geq K + L$  in Proposition 1 for two reasons: First, the constraints in (4) can be satisfied by an active secondary transmitter when  $N \geq L$ . Second, for  $N \geq K$  full power transmission is optimal to achieve Pareto optimal points [12, Section III.A]. For  $N < K$ , a transmitter has to vary its transmission power for specific beamforming vectors in order to achieve Pareto efficient operating points [12, Section III.B].

In Fig. 1, we show a comparison of the rate regions with and without null-shaping constraints. These regions correspond to  $\mathcal{R}$  and  $\tilde{\mathcal{R}}$  defined in (3) and (6), respectively. The setting has two secondary transmitters equipped with three antennas and a single primary user. The Pareto boundary of  $\mathcal{R}$  is marked with crosses which correspond to beamforming vectors characterized in [13, Theorem 2].

For the case in which null-shaping constraints are imposed on the secondary users, the Pareto boundary of  $\tilde{\mathcal{R}}$  is achieved by beamforming vectors characterized in Proposition 1. By varying the parameters  $\lambda_1$  and  $\lambda_2$  in (7) between zero and one in a 0.02 step-length, the Pareto boundary of  $\tilde{\mathcal{R}}$  is plotted in Fig. 1 with circle markers. The region  $\tilde{\mathcal{R}}$  is certainly always smaller and within the region  $\mathcal{R}_2$  because the feasible transmission strategies set is smaller,  $A_k \subseteq A_k$  for all  $k \in \mathcal{S}$ . Notice that each secondary transmitter uses three antennas. Due to the null-shaping constraint, one spacial transmit dimension is reduced for each secondary transmitter such that two spacial dimensions are left available to operate in.

Given certain null-shaping constraints, beamforming vectors described in Proposition 1 achieve Pareto efficient rate tuples

for the secondary users. In the next section, we study whether through the choice of virtual null-shaping constraints the secondary users could operate on the Pareto boundary of  $\mathcal{R}$ .

#### IV. CONSTRAINTS ACHIEVING PARETO BOUNDARY

In this section, we investigate the design of null-shaping constraints such that the secondary users in a noncooperative game achieve the Pareto boundary of  $\mathcal{R}$  defined in (3). We denote this game in strategic form [14, Part I] by

$$\langle \mathcal{S}, (A_k)_{k \in \mathcal{S}}, (R_k)_{k \in \mathcal{S}} \rangle, \quad (10)$$

where  $A_k$  is defined in (1) and  $R_k$  is defined in (2). The players of this game are assumed to be rational, i.e., seek to maximize their utility. The described game is a game of complete information such that its outcome is a Nash equilibrium (NE). In a setting where null-shaping constraints on the secondary users do not exist, the unique NE strategy of each system is maximum-ratio combining [15]

$$\mathbf{w}_k^{\text{NE}} = \frac{\mathbf{h}_{kk}}{\|\mathbf{h}_{kk}\|}, \quad k \in \mathcal{S}. \quad (11)$$

The rate tuple corresponding to the NE strategy profile is not necessarily Pareto optimal [15]. Of interest is to design constraints on the noncooperative systems such that NE rate tuples lie on the Pareto boundary of the rate region achieved with no constraints.

*Proposition 2:* Assume  $N \geq K$ . Define the matrix

$$\tilde{\mathbf{Z}}_k(\lambda_k) = [\tilde{\mathbf{z}}_1(\lambda_k), \dots, \tilde{\mathbf{z}}_{K-1}(\lambda_k)], \quad (12)$$

where

$$\tilde{\mathbf{z}}_i(\lambda_k) = \mathbf{v}_i \left( \sum_{\ell=1}^K \lambda_{k,\ell} e_{k,\ell} \mathbf{h}_{k\ell} \mathbf{h}_{k\ell}^H \right), \quad (13)$$

with  $\lambda_k \in \Lambda_K$  defined in (9) and  $e_{k,\ell}$  defined in (8). All points on the Pareto boundary of the rate region  $\mathcal{R}$  defined in (3) can be reached by the beamforming vectors

$$\mathbf{w}_k(\lambda) = \frac{\Pi_{\tilde{\mathbf{Z}}_k(\lambda_k)}^\perp \mathbf{h}_{kk}}{\left\| \Pi_{\tilde{\mathbf{Z}}_k(\lambda_k)}^\perp \mathbf{h}_{kk} \right\|}. \quad (14)$$

This result follows directly from [12, Corollary 1] and generalizes the result in [11, Proposition 1] where only the two-user case has been considered. Note that the beamforming vector in (14) is the unique NE for transmitter  $k$  which abides by the null-shaping constraints. The noncooperative game in (10) is only between the secondary users. The null-shaping constraints in (12) are imposed on the secondary users by an authority which is not included as a player in the game. This authority in game theoretic terms is represented by an arbitrator [16].

Interestingly, the null-shaping constraints are sufficient to characterize the Pareto boundary of the rate region  $\mathcal{R}$  without constraints. Thus, Pareto optimal points can be obtained in with noncooperative strategies described in (14). This result is related to the result in [7] in that both methods achieve the same Pareto boundary in NE, and both methods utilize interference constraints on the transmitters. However, ITCs for each receiver are considered in [7], and we consider null-shaping constraints that correspond to  $K - 1$  virtual primary

**Result:** Greedy User Selection  
**Input:** set of Secondary Users,  $\mathcal{K}$   
initialize  $\mathcal{U} = \{\}, \mathcal{T} = \mathcal{K}, n = 0, SR(\{\}) = 0;$   
**while**  $n \leq |\mathcal{K}|$  **do**  
     $n = n + 1;$   
     $k_n = \arg \max_{u \in \mathcal{T}} SR(\{u\} \cup \mathcal{U});$   
    **if**  $SR(\mathcal{U}) > SR(\{k_n\} \cup \mathcal{U})$  **then**  
        **break;**  
    **else**  
         $\mathcal{T} = \mathcal{T} \setminus \{k_n\};$   
         $\mathcal{U} = \mathcal{U} \cup \{k_n\};$   
    **end**  
**end**  
**Output:** Set of selected users,  $\mathcal{U}$

**Algorithm 1:** Greedy User Selection Algorithm.

receivers. In [7], each point on the boundary is determined iteratively while here we provide the constraints and the corresponding strategies in closed form.

Next, we continue to consider the noncooperative operation of the secondary users. However, the null-shaping constraints imposed on these users correspond to the existing primary users as described in (4). Our objective is to increase the sum rate for the noncooperative secondary users by secondary user subset-selection.

## V. GREEDY NONCOOPERATIVE USERS SELECTION

In this section, each transmitter chooses its NE transmission strategy, such that the null-shaping constraints in (4) are satisfied. The noncooperative game is described by

$$\langle \mathcal{S}, (\tilde{A}_k)_{k \in \mathcal{S}}, \tilde{\mathcal{R}} \rangle, \quad (15)$$

where  $\tilde{A}_k$  is defined in (5) and  $\tilde{\mathcal{R}}$  is defined in (6). The unique NE strategy of a secondary transmitter is

$$\mathbf{w}_k^{\text{NE}} = \frac{\prod_{\perp} \mathbf{z}_k \mathbf{h}_{kk}}{\|\prod_{\perp} \mathbf{z}_k \mathbf{h}_{kk}\|}, \quad k \in \mathcal{S}, \quad (16)$$

where  $\mathbf{Z}_k \triangleq [z_{k1}, \dots, z_{kL}]$ . We address the issue of selecting a subset of the users  $\mathcal{U} \subseteq \mathcal{S}$  to operate in a noncooperative game. This scheme is motivated by the fact that when interference dominates in the network, turning some links off could increase the achievable sum rate of the remaining systems through interference reduction. Note that the remaining users  $\mathcal{S} \setminus \mathcal{U}$  are to be inactive when not selected. The selection mechanism could be done by an authority or an arbitrator. For a set of secondary users  $\mathcal{U}$ ,  $\mathcal{U} \subseteq \mathcal{K}$ , the achievable sum rate is written as

$$SR(\mathcal{U}) = \sum_{k \in \mathcal{U}} \log_2 \left( 1 + \frac{|\mathbf{h}_{kk}^H \mathbf{w}_k^{\text{NE}}|^2}{\sigma^2 + \sum_{\ell \in \mathcal{U} \setminus \{k\}} |\mathbf{h}_{\ell k}^H \mathbf{w}_\ell^{\text{NE}}|^2} \right), \quad (17)$$

where  $\mathbf{w}_k^{\text{NE}}, k \in \mathcal{U}$ , is given in (16). Finding the optimal user selection has very high complexity and could be done by an exhaustive search over all possible combinations of the

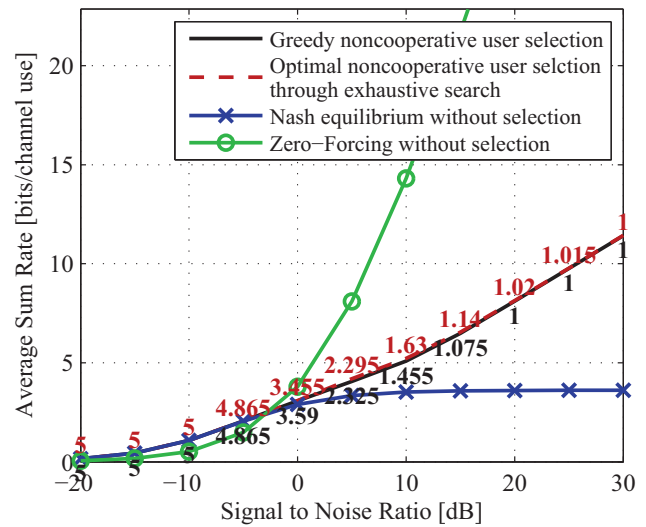


Fig. 2. Achievable average sum rates for 5 secondary users with 6 transmit antennas and a single primary user.

user subsets. We propose a low complexity suboptimal user selection algorithm, described in Algorithm 1. This algorithm is influenced by the algorithm proposed in [17] for the user selection in a downlink scenario with zero-forcing beamforming at the transmitter. See [17] for details on the complexity of the algorithm. Starting with an empty secondary user set, in each iteration step of Algorithm 1, a secondary user that contributes highest sum rates to the set is added. The algorithm terminates when the sum rate decreases with an additional user or when all the users have been selected. The number of selected users and their performance depends on the SNR.

In Fig. 2, the achievable average sum rate of the secondary systems in different operation schemes is compared for increasing SNR. The simulations are performed for a single existing primary user and 5 secondary users with each transmitter using 6 transmit antennas. The average sum rate is taken over 200 random samples of each channel. The performance of Algorithm 1 is given under Greedy noncooperative user selection, and the average number of selected users is written under the curve. The optimal noncooperative user selection is obtained through exhaustive search, and the corresponding performance is plotted with dashed line. The average number of supported users for this scheme is written over the curve. The exhaustive search algorithm searches between  $K$  factorial user combinations. Note that both algorithms select a subset of the secondary users that are noncooperative. The performance of the suboptimal algorithm is very close to the optimal one. The gap between the optimal and suboptimal algorithm curves is largest in the intermediate SNR regime between 0 and 20 dB. Moreover, it is noticed that the gap increases as the number of transmit antennas increases. In the low SNR regime, the number of supported users is highest and decreases for increasing SNR values. In the high SNR regime only a single transmitter is eventually selected. Similar curves are given in [18], where opportunistic beamforming in the MISO broadcast channel is analyzed. There, for small SNR,

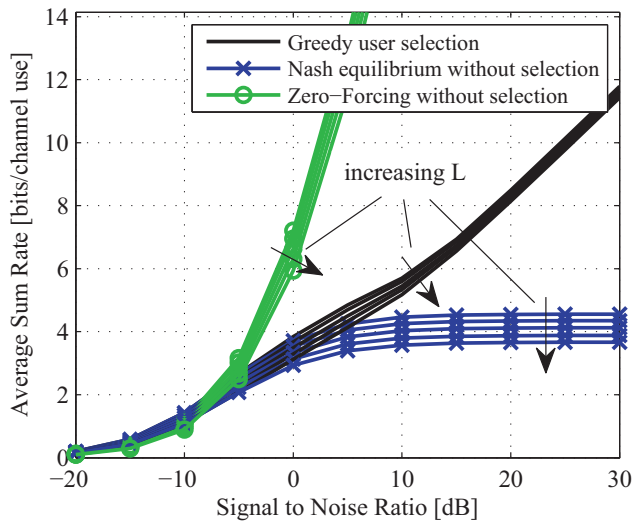


Fig. 3. Achievable average sum rates for 5 secondary users with 10 transmit antennas. Increasing the number of primary users  $L$  reduces the average sum rate of the secondary users.

all spacial dimensions are exploited and users are scheduled, whereas for high SNR, only one user on one beam is scheduled due to interference.

The curve with cross markers in Fig. 2 describes the performance when all noncooperative secondary transmitters are always chosen. This operation scheme has good performance in the low SNR regime. At high SNR however, the average sum rate saturates and approaches a constant value due to multiuser interference. In selecting fewer users to operate as in Algorithm 1, better performance is achieved. The curve with circle markers describes the average sum rate achieved when all secondary users perform zero-forcing transmission strategies. This transmission strategy performs best in the high SNR regime and achieves maximum multiplexing gain. A single transmitter is capable of performing this strategy if its number of transmit antennas is greater or equal to the number of primary receivers plus the number of secondary receivers. This transmission scheme is not denoted as optimal because it is not achieved in the game described for this section in (15).

In Fig. 3, the achievable average sum rate of the secondary systems is compared for increasing SNR and increasing number of existing primary users. Again, the average sum rate is taken over 200 random channel vector samples. The number of primary users is varied between 1 and 5 and the number of transmit antennas is 10. Including additional primary users, the average sum rate of the secondary systems always decreases. It is observed, however, that for a single channel sample the performance of the systems could increase with increasing the number of primary users. This observation supports the result given in Proposition 2 that null shaping constraints could improve the performance of the noncooperative systems.

## VI. CONCLUSIONS

In underlay hierarchical cognitive radio environments, primary systems have higher priority for available resources

than secondary systems. We consider such a setting in which the primary users tolerate no interference from secondary systems, i.e., null-shaping constraints are to be imposed on the secondary systems. First, we characterize the beamforming vectors which satisfy the null-shaping constraints and achieve Pareto optimal points on the achievable rate region. Then, motivated by the noncooperative operation of the secondary systems, we investigate and characterize the design of virtual null-shaping constraints that brings the noncooperative operation point to the Pareto boundary of the rate region without constraints. In the third part, noncooperative secondary user selection is investigated in order to maximize the achievable sum rate. A suboptimal algorithm is provided that selects a subset of the secondary users to operate such that the sum rate of the selected secondary systems is increased. Simulation results illustrate the gain in performance through user selection.

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