Square Region-Based Coverage and Connectivity Probability Model in Wireless Sensor Networks

Xiaofei Xing[†], Guojun Wang[†‡*], Jie Wu[‡], and Jie Li[§]

[†]Schl. of Info. Sci. & Eng.
Central South University
Changsha, 410083, P. R. China
[*]Corresponding author: csgjwang@mail.csu.edu.cn

[‡]Dept. of Comp. & Info. Sci.
Temple University
Philadelphia, PA 19122, USA

[§]Dept. of Comp. Sci.
Gra. Schl. of Sys. & Info. Eng.
University of Tsukuba
Tsukuba 305-8573, Japan

Abstract—Sensing coverage and network connectivity are two fundamental issues in wireless sensor networks (WSNs). Due to resource constraints of sensor nodes, it may not be possible, or necessary, to provide full coverage and/or connectivity in WSNs. Under a certain coverage and connectivity requirement, the node deployment strategy becomes a challenging issue in randomly deployed networks. In this paper, we propose a *square region-based coverage and connectivity probability model* (SCCP), which reflects the relations among the coverage and connectivity rates, the number of sensor nodes, the sensing and communication ranges of sensor nodes, and the network size. This model can calculate the number of sensor nodes that need to be deployed for maintaining a certain coverage and/or connectivity rate. The simulation results have shown that the error-rate of node deployment is less than 5%, which is defined as the absolute difference between the number of sensor nodes obtained from the theoretical analysis and the number obtained from the simulation, divided by the number of sensor nodes obtained from the theoretical analysis. The proposed model is very useful in estimating the monitoring coverage and connectivity capacity when sensor nodes are distributed randomly and uniformly.

Index Terms—wireless sensor networks (WSNs), coverage, connectivity, random deployment, largest connected component.

I. INTRODUCTION

With the rapid development of wireless communication and micro-electro mechanical systems (MEMS) technologies, wireless sensor networks (WSNs) have attracted a lot of attention due to their wide range of potential applications. A WSN consists of a set of independent sensor nodes, which are self-contained units, made up of cheap, small devices with radio, multiple onboard sensors, and limited computing power. A WSN is often deployed with a large number of sensor nodes to detect, monitor, collect, and report relevant data from surrounding environments. A WSN can be used for various applications, such as battlefield detection, infrastructure security, industrial environment surveillance, habitat monitoring, undiscovered areas exploring, and target tracking [1], [2], [3], [4].

There are some fundamental issues to be solved in WSN applications. Coverage is one of such issues since after deploying a sensor network, we would like to know how well the network can observe a given region of interest. In general,

the goal of coverage is to make sure that each point, or each target of interest in a given physical space, should be within the sensing range of at least one sensor node. In a friendly or accessible environment, sensor nodes can be placed exactly in predefined locations in order to achieve the goal of coverage. This kind of deployment is called *predefined deployment* or *deterministic deployment* [5], [6], [7]. However, in an adverse or inaccessible environment, sensor nodes need to be deployed in an arbitrary manner. The corresponding deployment is called *random deployment* or *stochastic deployment* [8], [9], [10], [11], [12], [13], [19], [20]. This method deploys sensors by means of a vehicle or air-craft.

Besides coverage, connectivity is another fundamental issue in a WSN [14], [15], [22], as it greatly affects whether sensor nodes can communicate with each other, and send data to a sink node. A satisfactory connectivity is highly required in order to achieve robust and smooth communication in a WSN.

Generally, to perform a given task, more sensor nodes are deployed than that required for optimal coverage. On the one hand, this improves the fault tolerance, and is often applied in adverse or inaccessible terrain in that the positions of sensor nodes need not to be predetermined. On the other hand, this also means that distributed algorithms should be designed for such self-organizing sensor networks.

In this paper, we focus on both the coverage and connectivity issues in randomly deployed WSNs. We propose a *square region-based coverage and connectivity probability model* (SCCP), which reflects the relations among the size of the network, the sensing or communication radius of sensors, the number of sensor nodes, and the coverage or connectivity rate of the network. If we know the side length of a monitoring region, the sensing radius, the communication radius, and the required coverage or connectivity rate, we can calculate the numbers of required nodes to satisfy the required coverage and connectivity rates, respectively. Compared with the coverage model proposed in [8], the border effects are considered in our model, so obtaining data by using our model is more accurate. A coverage model in [9] is proposed using circle-shaped network regions. But, its calculation is more complicated than the model proposed in this paper. Furthermore, the number of sensor nodes needed to be deployed in order to satisfy the

different network coverage rates and connectivity rates, is also studied in this paper.

The main results and contributions are summarized as follows:

1) To our best knowledge, this work is the first to tackle the problem of both coverage and connectivity in randomly deployed WSNs, based on square regions considering the border effects.
2) We obtain analytical expressions of the expected coverage and connectivity probability model for randomly deployed WSNs, which can be used to calculate the coverage and connectivity rates more easily.
3) Unlike existing works, the proposed model does not depend on the ratio of the communication radius (r_c) to the sensing radius (r_s).

The rest of this paper is organized as follows: Related work is presented in Section II. In Section III, we discuss the basic assumptions and models used throughout the paper. In Section IV, we describe and analyze the proposed SCCP model in randomly deployed WSNs. The performance of the proposed model is evaluated and compared in Section V. We conclude this paper in Section VI.

II. Related Work

The coverage and connectivity problems have been studied extensively in the literature. Xing et al [10] proposed a coverage configuration protocol (CCP), which gives an important result in regards to the relations between coverage and connectivity. They stated that, if the communication radius of nodes is at least twice the sensing radius, a complete coverage of a convex area implies the connectivity in an arbitrary network. The result can be applied to both 2D planes and 3D space. But, the connectivity of this algorithm depends on the fact that $\frac{r_c}{r_s} \geq 2$, where r_c and r_s denote the communication and sensing radius of the sensor, respectively.

Zhang and Hou [6] presented a decentralized density control algorithm called optimal geographical density control (OGDC), which is based on an important result from [23]. The result is that all crossing points in a given region A, which are intersection points between sensing disks of sensors, and those between a sensing disk of any sensor and the boundary of the region, are covered by at least one other sensor in the region A, then A is completely covered by the sensors. However, the time complexity of this algorithm is $O(n^3)$, where n is the total number of sensor nodes in the network.

Based on the application of large-scale WSN region monitoring in random deployment, Slijepcevic and Potkonjak [21] proposed a most-constrained least-constraining heuristic algorithm. They divided the coverage region into blocks, and each block is covered by nodes in a node set. The goal of this algorithm is to find the maximum amount of different node sets which satisfy certain conditions. By making each block become active node set in turn, it can effectively decrease the nodes' energy consumption, and also prolong the lifetime of the whole network.

Jin et al [9] analyzed the coverage, connectivity, and routing problems under the border effects in circle-shaped regions. They provided a formula to calculate the coverage and connectivity rate of a network. But, the calculations of this model are rather complicated. Based on this work, we propose a more practical model, SCCP, which simplifies the computational complexity of coverage and connectivity rates, and improves the accuracy in terms of the error-rate of node deployment. Furthermore, we quantify the border effects to coverage and connectivity in WSNs.

Gupta et al [16] developed the notion of a connected sensor cover, which is defined as the sensor set that can fully cover the queried region, and constitute a connected communication graph at the same time. They also demonstrated that the calculation of the small connected sensor cover is *NP*-hard, and they proposed both centralized and distributed approximate algorithms. However, the method in [16] requires each individual sensor node to be aware of its precise location in order to check its local coverage redundancy.

Bredin et al [17] studied the problem of placing nodes to achieve k-connectivity at the network setup phase, or to repair a disconnected network. They formulated the problem as an optimization model that strives to minimize the number of additional nodes required to maintain k-connectivity. They showed that the problem is *NP*-hard [18], and proposed two approaches with varying degrees of complexity and closeness to optimality. Both approaches are based on the graph theory. The idea is to model the network as a graph whose vertices are the current or initial set of sensors, and the edges are the existing links among these sensor nodes. Moreover, it is not necessary to achieve k-connectivity among sensors, unless the sink node frequently changes its location in most WSNs.

In summary, due to their energy efficiency, cost effectiveness, easy implementation, and applicability, the different deployment strategies of sensor nodes are an appealing approach to the different surveillance methods in practice. The design of the different random deployment density strategy is a challenging problem, and is affected by a few factors including, the application-specific factors, the sensing capacity, and the number of sensor nodes.

III. Assumptions, Definitions, Problem Formulation, and Notations

A. Assumptions

Assuming the network has the following properties:

1) The sensor network is homogeneous, that is, the sensing range and the communication range of all the sensor nodes are the same, respectively.
2) The communication model and the sensing model of all the sensor nodes are circle-based models, i.e., the sensing region and the communication region are circles.
3) Considering the border effects, all the sensor nodes are deployed uniformly and randomly in a square region Ω with side length l.

B. Definitions

Definition 3.1: A WSN can be modeled by an undirected communication graph $G = (V, E)$, where V is the set of sensor nodes with $|V| = n$ (n is the number of sensor nodes), and E is the edge set of graph G. There is an undirected edge (p, q) in E if and only if node p and node q are within the communication ranges of each other. The degree of node p, denoted as $d(p)$, is the number of neighbors of node p. That is, there are $d(p)$ other nodes whose distance from node p is no greater than the communication radius of the node. The minimum node degree of a communication graph G is denoted as $d_{\min}(G) = \min_{\forall p \in V} \{d(p)\}$.

Definition 3.2: **Region Coverage**. A region is covered if every point within the region is covered by at least one sensor node.

Definition 3.3: **Network Connectivity**. A network is said to be connected, if for every pair of nodes in the communication graph G, there exists a path between them, otherwise it is disconnected. In terms of communication networks, all nodes of a connected network can communicate with each other over one or multi-hops.

Definition 3.4: **Largest Connected Component (LCC)**. Given an undirected graph $G = (V, E)$, we find a minimum size of a subset ψ of vertices, such that the subgraph induced by ψ is connected, and ψ forms the largest connected component in G. In a wireless sensor network, we name this component the largest connected component of the network, LCC for short.

Definition 3.5: **Connectivity Rate of the Network**. Given the area of the largest connected component (A_ψ), and the area of the network region Ω, the connectivity rate of the network (C_ψ) is defined as:

$$C_\psi = \frac{A_\psi}{\Omega}$$

C. Problem Formulation

Assuming sensor nodes are randomly and uniformly deployed in a region of interest Ω, the problem that we expect to solve is to calculate the number of required sensors needed to satsify different coverage rates or connectivity rates of the network.

D. Notations

The notations used in this paper are listed in Table I.

TABLE I
NOTATIONS

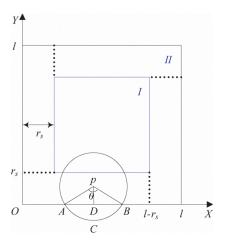| Parameter	Description
l	Side length of the square region
Ω	Area of the network region, i.e., $\Omega = l^2$
n	Number of deployed sensor nodes
r_s	Sensing radius of sensors
r_c	Communication radius of sensors
$E(S)$	Expected coverage area of a sensor
P_{cov}	Network coverage probability provided by n sensors
P_{con}	Network connectivity probability provided by n sensors

Fig. 1. Illustration of a sensor node p that is located in Region *II*. *O* is the origin of a coordinate system. *A* and *B* are the intersection points between the sensing disk of node p and the border of the network region.

IV. SCCP MODEL

In this section, we present our SCCP model in detail. As there is not any necessary connection between the sensing radius and communication radius of a sensor node, we discuss the coverage and connectivity probability models respectively.

A. Network coverage

First, we consider the case that some sensor nodes are deployed randomly in a region of interest. The sensor nodes then form a node set after they are deployed randomly. We denote the coverage area of each node as $E(S)$, so the coverage probability of each node is $E(S)/\Omega$. In the extreme case, if all the sensor nodes are located outside of the network region after deploying n sensor nodes, we can get the coverage probability of the network: $(1 - \frac{E(S)}{\Omega})^n$. Then, the coverage probability of a network (P_{cov}) is demonstrated below when the node set is not empty:

$$P_{cov} = 1 - (1 - \frac{E(S)}{\Omega})^n \qquad (1)$$

Eq. (1) is consistent with the intuition that $\lim_{n \to \infty} E(P_{cov}) = 1$, which indicates that the area is covered fully by the nodes when the number of deployed nodes is large enough.

Below, we calculate the coverage area of a node and its mathematical expectation value, considering the border effects.

As a square region model is widely used in the practical network scenarios, we use a square region as the network monitoring region in this paper. In fact, any irregular region can be seen as one that is composed of many different sizes of squares. Considering the border effects, the network region is divided into two sub-regions, as shown in Fig. 1. The location of any sensor node has two probabilities in this region, with either Sub-Region *I* (simply called Region *I* later) or Sub-Region *II* (simply called Region *II* later). Thus, the coverage expectation value of the node is:

$$E(S) = P(\Omega_I)E[C_{\Omega_I}] + P(\Omega_{II})E[C_{\Omega_{II}}] \qquad (2)$$

where $P(\Omega_I)$ and $P(\Omega_{II})$ denote the probabilities when the sensor is located in Region I and Region II, respectively; $E\left[C_{\Omega_I}\right]$ and $E\left[C_{\Omega_{II}}\right]$ denote the corresponding expected coverage of the node. As the nodes are deployed randomly, we get:

$$P(\Omega_I) = \frac{(l-2r_s)^2}{l^2}, P(\Omega_{II}) = \frac{4r_s(l-r_s)}{l^2} \qquad (3)$$

When the sensor is located in Region I, the covered area of the node is fully included in the region, so we get:

$$E\left[C_{\Omega_I}\right] = \pi r_s^2 \qquad (4)$$

As shown in Fig. 1, when the sensor is located in Region II, the value of $C_{\Omega_{II}}$ is equal to the value that is the sensing area of node p minus the arch area $ACBD$. A and B are the intersection points between the sensing circle and the border of the network. We use variable θ to denote the centre angle formed by A, B, and p, that is $\angle ApB = \theta$. Thus, we get the value of $C_{\Omega_{II}}$ as following:

$$C_{\Omega_{II}} = \pi r_s^2 - S_{ACBD} = \pi r_s^2 - (S_{pACB} - S_{\triangle ApB})$$

$$= \frac{1}{2}r_s^2(2\pi - \theta + \sin\theta) \qquad (5)$$

The expected value of $C_{\Omega_{II}}$ can be calculated as:

$$E\left[C_{\Omega_{II}}\right] = 4 \times \frac{1}{4(l-r_s)r_s} \int_0^{l-r_s} \frac{1}{2}r_s^2(2\pi - \theta + \sin\theta)dx \int_0^{r_s} dy$$

$$= \frac{r_s}{2(l-r_s)} \int_0^{l-r_s} (2\pi - \theta + \sin\theta)dx \int_0^{r_s} dy \qquad (6)$$

where $\theta = 2\arccos\frac{y}{r_s}$.

We summarize the above results by the following two theorems:

Theorem 4.1: Assuming the nodes with the sensing radius r_s are deployed in a square region with side length l, the expected coverage area of the node is shown as below considering the border effects:

$$E(S) = P(\Omega_I)E\left[C_{\Omega_I}\right] + P(\Omega_{II})E\left[C_{\Omega_{II}}\right]$$

$$= \frac{1}{l^2}\left[\pi r_s^2(l-2r_s)^2\right] + \frac{1}{l^2}\left[2r_s^2 \int_0^{l-r_s} (2\pi - \theta + \sin\theta)dx \int_0^{r_s} dy\right] \qquad (7)$$

Proof: Theorem 4.1 can be derived from Eqs. (2)-(6). ∎

Theorem 4.2: Given the side length of a square region l, the sensing radius of the node r_s, and a parameter ε, in order to ensure that the expected network coverage rate is not less than ε, the number of deployed nodes is at least:

$$\frac{\ln(1-\varepsilon)}{\ln(1 - \frac{E(S)}{\Omega})} \qquad (8)$$

where $E(S)$ is shown in Eq. (7).

Proof: According to Eq. (1), the expected network coverage rate must satisfy $P_{cov} = 1 - (1 - \frac{E(S)}{\Omega})^n \geq \varepsilon$. So, we get $n\ln(1 - \frac{E(S)}{\Omega}) \leq \ln(1-\varepsilon)$. As $\ln(1 - \frac{E(S)}{\Omega}) < 0$, we get $n \geq \ln(1-\varepsilon)/\ln(1 - \frac{E(S)}{\Omega})$. Thus, Theorem 4.2 is proved. ∎

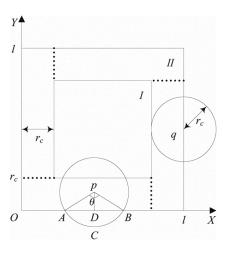Fig. 2. Illustration of a sensor node q that is located on the outer border of Region II. O is the origin of a coordinate system. A and B are the intersection points between the sensing disk of node p and the border of the network region. Node q is located in the border of the network region.

B. Network connectivity

In WSNs, we say that a sensor node completes to do its task, only if it can report data to the sink node by one or multi-hop communication. So, as one significant property of WSNs, we further study the connectivity property in this sub-section, considering the border effects.

According to Definition 3.1, the event $d_{min}(G) > 0$ is a necessary, but not a sufficient condition, such that the communication graph G is a connected graph. Thus, we have $P_{con} \leq P(d_{min}(G) > 0)$. In practice, the upper bound for P_{con} does not have much help if we do not know the density distribution of sensor nodes. In the following, we show that this upper bound is a very high bound, particularly for a high probability, which is certainly most interesting. Therefore, we can draw a conclusion that the value of $d_{min}(G) > 0$ equals to that of P_{con}, approximately.

To prove this conclusion, we apply a result on the property of random geometric graphs by Penrose in [22]. He proved that if n is large enough, then with a high probability, if one starts with an empty graph, and adds the corresponding links as the communication radius of sensor nodes increases, the resulting graph becomes k-connected at the moment it achieves a minimum node degree k. So, for $k=1$, the network becomes connected in the case that r_c is large enough to achieve $d_{min}(G) > 0$ with a high probability. That is:

$$P_{con} = P(d_{min}(G) > 0) \qquad (9)$$

for $P(d_{min}(G) > 0)$, it is almost one. As the distribution of sensor nodes is independent, according to Eq. (1), for any node $p(p \in G)$, we get the limiting value of the connectivity probability of a node: $\lim_{n\to\infty}\left[1 - (1 - \frac{A_p}{\Omega})^n\right] = 1 - e^{-nA_p/\Omega}$, where A_p denotes the effective communication area of node p. The minimum node degree is as follows by [22]:

$$P(d_{min}(G) > 0) = \prod_{\forall p \in G} (1 - e^{-nA_p/\Omega}) \qquad (10)$$

We have $P_{con} = P(d_{min}(G) > 0) = (1 - e^{-n\pi r_c^2/l^2})^n$ without considering the border effects. In practice, as shown in Fig. 2, the value of A_p equals to the effective communication area of node p when it is located in Region I. When node p is located in Region II, the value of A_p equals to its circle area (πr_c^2) minus the arch area $ACBD$. So, Eq. (10) can be calculated as follows:

$$P(d_{min}(G) > 0)$$

$$= (1 - e^{-n\pi r_c^2/l^2})^{4nr_c(l-r_c)/l^2} \times \prod_{\forall p \in \Omega_{II}} (1 - e^{-nA_p/\Omega}) \qquad (11)$$

When node p is located in Region II, we observed the fact that $A_p > A_q$, where node q is located on the outer border of the region $(A_q = \frac{1}{2}\pi r_c^2)$. Thus, we get Eq. (12):

$$\prod_{\forall p \in \Omega_{II}} (1 - e^{-nA_p/\Omega}) > \prod_{\forall p \in \Omega_{II}} (1 - e^{-nA_q/\Omega})$$

$$= (1 - e^{-\frac{1}{2}n\pi r_c^2/l^2})^{4nr_c(l-r_c)/l^2} \qquad (12)$$

Theorem 4.3: Given the side length of a square network region l, the number of nodes n, and the communication radius of node r_c, the lower bound of the network connectivity probability P_{con} is as follows if the border effects are considered:

$$[(1 - e^{-n\pi r_c^2/l^2})(1 - e^{-\frac{1}{2}n\pi r_c^2/l^2})]^{4nr_c(l-r_c)/l^2} \qquad (13)$$

Theorem 4.3 provides a formula for calculating the lower bound of the network connectivity probability, which can be applied when the sensor nodes are deployed randomly. However, in some special cases, such as when the sensor nodes are not deployed uniformly, the above formula can not be applied. For this reason, we propose another method to calculate the connectivity rate of the network.

We all know that the largest connected component can be used to solve the connected set problem in a random graph. Therefore, we provide Definition 3.5 in Section III to calculate the connectivity rate. By extensive simulation, we found that the connectivity rate of the network (C_ψ) equals to the lower bound of the network connectivity probability, approximately.

Finally, we provide a table (as shown in Table II) that reflects the relations between the number of deployed sensor nodes, the coverage rates, and connectivity rates of a network under different network sizes.

V. PERFORMANCE EVALUATION

In this section, we evaluate the performance of the proposed SCCP model using simulation. We first describe our performance metrics and simulation settings. Then, we evaluate the system performance by comparing the proposed model under different network settings.

TABLE II
THE RELATIONSHIP BETWEEN THE NUMBER OF DEPLOYED NODES, COVERAGE RATES, AND CONNECTIVITY RATES UNDER DIFFERENT NETWORK SIZES (r_s=20, r_c=40)

Rate	l=100 cov.[0]	l=100 con.	l=200 cov.	l=200 con.	l=300 cov.	l=300 con.
10%	1	5	4	43	8	124
20%	2	6	7	50	16	131
30%	3	8	11	55	25	142
40%	4	10	15	60	35	152
50%	5	13	21	66	48	165
60%	7	17	27	73	63	183
70%	9	20	36	80	82	205
80%	11	25	48	91	110	242
90%	16	30	68	105	157	292
99%	32	43	136	150	314	350

A. Simulation settings and performance metrics

We evaluate the performance of the proposed SCCP model on a simulator that we implemented in C++. The sensing and communication models of sensor nodes are described in Section III. The parameter settings in our simulation are shown in Table III. In this paper, simulation is conducted under different network sizes to verify whether the number of required sensor nodes is consistent with the number of nodes obtained from theoretical analysis under different coverage and connectivity rates. Under each simulation scenario, 50 runs with different random seeds are executed.

TABLE III
SIMULATION PARAMETERS

Parameter	Description	Value
l	Field size (m)	100, 200, 300
r_s	Sensing radius (m)	20
r_c	Communication radius (m)	40
M	Size of message (bit)	1,000
E_i	Initial energy of each node (J)	2

We used the following metrics to evaluate the proposed SCCP model:

1) Network coverage rate: It is a measure of the quality of coverage.
2) Error-rate of node deployment: It is defined as:

$$r_e = \frac{\sum |N_s - N_t|}{\sum N_t}$$

where N_t and N_s denote the number of deployed sensor nodes calculated by theoretical analysis, and the number of deployed sensor nodes in practice. This metric is a measure of evaluating the absolute difference between the theoretical analysis and simulation results.

B. Comparison under different network sizes

In order to verify whether the number of deployed sensor nodes is consistent with the number of theoretical results under the same coverage and connectivity rates, we provide two simulation figures, which are coverage (as shown in Fig. 3)
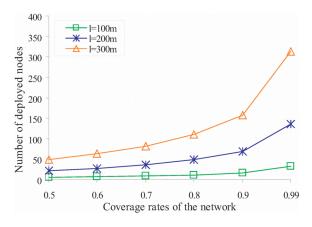
[0]Here cov. =coverage, con. =connectivity

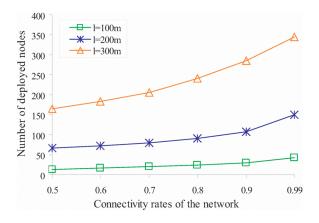Fig. 3. Comparison of coverage under different network sizes.

Fig. 4. Comparison of connectivity under different network sizes.

and connectivity (as shown in Fig. 4), under different network sizes.

Fig. 3 demonstrates the number of deployed sensor nodes under different network sizes to satisfy different network coverage rates. It shows that the larger the network size, the larger the number of required deployed nodes in order to satisfy different network coverage rates. Also, we can see that the higher the network coverage rate, the faster the growth of the number of required deployed nodes. Moreover, the number of deployed sensors obtained from the simulation is almost the same as the theoretical analysis results by comparison.

Fig. 4 plots the number of deployed sensor nodes under different network sizes to satisfy different network connectivity rates. From the figure, we concluded that the larger the network size, the larger the number of required deployed nodes to satisfy a certain network connectivity rate. Similarly to the coverage, the growth of the number of deployed nodes increases more rapidly when the network connectivity rate grows. Moreover, we can observe that when satisfying the same coverage rate and connectivity rate, more nodes need to be deployed to achieve the network connectivity rate than to achieve the network coverage rate for the isolated nodes, or
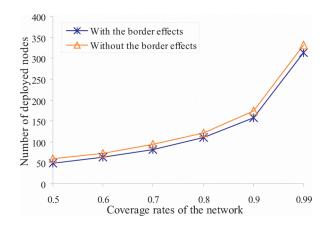

Fig. 5. Comparison of coverage with/without the border effects (l=300).

the sub-network cannot send or forward the data to the sink node in the network.

In order to evaluate the node deployment correctness of the proposed SCCP model, we calculate the value of r_e by using the data obtained from the simulation and Table II. The value of r_e is less than 5% after calculation. We can see that the error-rate of the node deployment between the theoretical analysis and the simulation results is very small. It shows that our proposed SCCP is usable in practice, and bridges the gap between the theory and practice.

C. Comparison with/without considering the border effects

In order to evaluate the border effects, we provide two simulation figures, coverage (as shown in Fig. 5) and connectivity (as shown in Fig. 6), with/without considering the border effects.

Fig. 5 compares the number of deployed sensor nodes to satisfy different coverage rates with/without considering the border effects. From this figure, we can see that with the growth of the network coverage rate, the number of required sensor nodes increases significantly, and the border effects also become greater when the network coverage rate increases.

Fig. 6 presents the number of deployed sensor nodes to satisfy different connectivity rates with/without considering the border effects. Compared with the situation considering the border effects, the number of deployed nodes increases slightly under the situation without considering the border effects. When the network connectivity rate becomes higher, the border effects become greater. By comparing Fig. 5 and Fig. 6, we can also see that the number of deployed sensor nodes needed to satisfy certain coverage or connectivity rates increases slightly when the border effects are not considered.

D. LCC-based network connectivity

In order to validate that our notion of the LCC-based network connectivity rate, as defined in Definition 3.5 using the notion of largest connected component (LCC), is reasonable, we first construct a LCC of the network. By further calculating the area of the LCC of the network, we can get the network
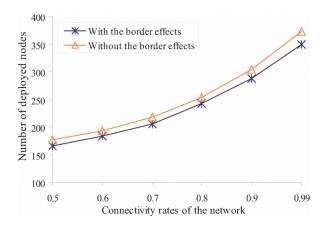
Fig. 6. Comparison of connectivity under different network sizes (l=300).

Fig. 7. Comparison of connectivity rates under different stagies.

connectivity rates as shown in Fig. 7. We compare it with that when the sensor nodes are deployed randomly (simply called the traditional network connectivity). When the same number of sensor nodes are deployed in the network region, we can see that the two network connectivity rates are close to this figure. The reason is that the area of the LCC of the network is slightly less than the total area of the sensor nodes that can connect to the sink node.

Then, we compare the network connectivity rates when the size of the LCC is the same with the number of deployed sensor nodes. As shown in Fig. 8, it reflects that the connectivity rates increase rapidly when the size of the LCC increases. Compared with the LCC-based calculation method on the network connectivity, the number of required sensor nodes by using the traditional deployment method is rather larger when the network connectivity is the same. That is because there are many redundant sensor nodes in the network region using the traditional node deployment method. In order to satisfy the requirement of a certain network connectivity, this figure also shows that the redundant sensor nodes are greatly reduced by constructing the LCC of the network, which can be used to improve the energy efficiency of the network.

VI. Conclusions and Future Work

In this paper, we investigated the problems of coverage and connectivity in randomly deployed WSNs. The SCCP model was proposed considering the border effects. This model simplified the computational complexity of coverage and connectivity properties, and improved the accuracy of deployed sensor nodes under requirements on different coverage and/or connectivity rates. Finally, the effectiveness of our proposal was validated in the performance study via simulation. The proposed probability model is very useful in evaluating the monitoring and connectivity capacity of sensor nodes that are distributed randomly and uniformly.

Future work can be done through a variety of research directions as follows: Our current research assumes that the network region is a square. Real world scenarios require a methodology
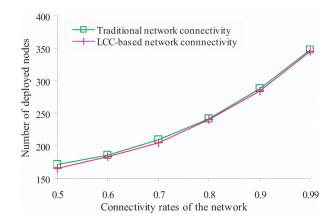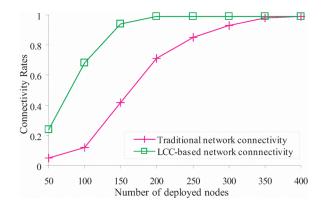

Fig. 8. Comparison of connectivity rates when the number of sensor nodes are the same.

for a network region of a greater complexity. Rectangular-based sensor deployment methods will be investigated in the future. Furthermore, we also plan to study the pairwise-based network connectivity rate, defined as the total number of pairs of sensor nodes in the network that are reachable to each other through direct or multi-hop communication links, over the total number of pairs of sensor nodes in the network. We will also investigate the relations between the LCC-based and pairwise-based network connectivity rates and their impacts on the network performance.

REFERENCES

[1] I. F. Akyildiz, W. Su, Y. Sankarasubramaniam, and E. Cayirci, "Wireless sensor networks: A survey," *Computer Networks*, 38(4): 393-422, March 2002.

[2] A. Ghosha and S. K. Das, "Coverage and connectivity issues in wireless sensor networks: A survey," *Pervasive and Mobile Computing*, 4(3): 303-334, 2008.

[3] G. Wang, M. Z. A. Bhuiyan, and L. Zhang, "Two-Level Cooperative and Energy-Efficient Tracking Algorithm in Wireless Sensor Networks," *Wiley's Concurrency and Computation: Practice and Experience*, DOI: 10.1002/cpe.1503, September 2009.

[4] X. Xing, G. Wang, and J. Wu, "Herd-Based Target Tracking Protocol in Wireless Sensor Networks," *Proc. of WASA-09*, Springer-Verlag Berlin Heidelberg, LNCS 5682, pp. 135-148, August 2009.

[5] X. Bai, D. Xuan, Z. Yun, T. H. Lai, and W. Jia, "Complete optimal deployment patterns for full-coverage and k-connectivity ($k \leq 6$) wireless sensor networks," *Proc. of ACM MobiHoc-08*, pp. 401-410, May 2008.

[6] H. Zhang and J. C. Hou, "Maintaining sensing coverage and connectivity in large sensor networks," *Ad Hoc and Sensor Wireless Networks*, Vol.1, pp. 89-124, March 2005.

[7] R. Iyengar, K. Kar, and S. Banerjee, "Low-coordination topologies for redundancy in sensor networks," *Proc. of ACM MobiHoc-05*, pp. 332-342, March 2005.

[8] B. Liu and D. Towsley, "A study on the coverage of large-scale sensor networks," *Proc. of IEEE MASS-04*, pp. 475-483, October 2004.

[9] Y. Jin, J. Y. Jo, L. Wang, Y. Kim, and X. Yang, "ECCRA: An energy-efficient coverage and connectivity preserving routing algorithm under border effects in wireless sensor networks," *Computer Communications*, 31(10): 2398-2407, June 2008.

[10] G. Xing, X. Wang, Y. Zhang, C. Lu, R. Pless, and C. Gill, "Integrated coverage and connectivity configuration for energy conservation in sensor networks," *ACM Transactions on Sensor Networks (TOSN)*, 1(1): 36-72, August 2005.

[11] G. Xing, R. Tan, B. Liu, J. Wang, X. Jia, and C. Yi, "Data fusion improves the coverage of wireless sensor networks," *Proc. of ACM MobiCom-09*, September 2009.

[12] W. Wang, V. Srinivasan, and K. Chua, "Trade-offs between mobility and density for coverage in wireless sensor networks," *Proc. of ACM MobiCom-07*, pp. 39-50, September 2007.

[13] P. Balister, B. Bollobas, A. Sarkar, and S. Kumar, "Reliable density estimates for coverage and connectivity in thin strips of finite length," *Proc. of ACM MobiCom-07*, pp. 75-86, September 2007.

[14] C. Bettstetter, "On the minimum node degree and connectivity of a wireless multihop network," *Proc. of ACM MobiHoc-02*, pp. 80-91, June 2002.

[15] R. Pietro, L. Mancini, A. Mei, A. Panconesi, and J. Radhakrishnan, "Connectivity properties of secure wireless sensor networks," *Proc. of the 2nd ACM workshop on Security of ad hoc and sensor networks*, pp. 53-58, October 2004.

[16] H. Gupta, Z. Zhou, S. Das, and Q. Gu, "Connected sensor cover: Self-organization of sensor networks for efficient query execution," *IEEE/ACM Transactions on Network (TON)*, 14(1): 55-67, February 2006.

[17] J. Bredin, E. Demaine, M. Taghi Hajiaghayi, and D. Rus, "Deploying sensor networks with guaranteed capacity and fault tolerance," *Proc. of ACM MobiHoc-05*, pp. 309-319, May 2005.

[18] M. R. Garey and D. S. Johnson, "Computers and intractability: A guide to the theory of NP-completeness," San Francisco: *Freeman*, 1978.

[19] M. Cardei and J. Wu, "Energy-efficient coverage problems in wireless ad-hoc sensor networks," *Computer Communications*, 29(4): 413-420, Feburary 2006.

[20] C. Liu, K. Wu, Y. Xiao, and B. Sun, "Random coverage with guaranteed connectivity: Joint scheduling for wireless sensor networks," *IEEE Transactions on Parallel and Distributed Systems*, 17(6): 562-574, May 2006.

[21] S. Slijepcevic and M. Potkonjak, "Power efficient organization of wireless sensor networks," *Proc. of IEEE ICC-01*, Vol. 2, pp. 472-476, May 2001.

[22] M. D. Penrose, "On k-connectivity for a geometric random graph," Wiley: *Random Structure and Algorithms*, 15(2): 145-164, 1999.

[23] P. Hall, "Introduction to the theory of coverage processes," New York: *Wiley Series in Probability and Mathematical Statistics*, pp. 369-394, 1998.