Non-cooperative Optimal Game-Theoretic Opportunistic Dynamic Spectrum Access

Oliver Yu, Emir Saric
Department of Electrical and Computer Engineering
University of Illinois at Chicago, Chicago, IL 60607

Abstract— **This paper considers the problem of competitive sharing of open spectrum resources between collocated spread spectrum based secondary systems. The problem is formulated as a strategic form game where the objective of each player (secondary system) is to maximize its own payoff defined in terms of resource utilizations. The necessary and sufficient conditions for the existence of the optimal Nash equilibrium solution are derived for the specified payoff functions. Using tools of the non-cooperative game theory, the Payoff-Enriched Adaptive Learning (PEAL) methodology is proposed to enable each secondary system to iteratively adapt spectrum access strategy in response to the observed interference from other secondary systems. The self-learning adaptations of PEAL require neither signaling nor time synchronization between autonomous secondary systems. It is shown through extensive numerical evaluations that the PEAL adaptations converge to the theoretical Nash equilibrium in a finite numbers of trials.**

I. Introduction

Growing demand for wireless services and scarce radio spectrum has led to the emergence of new frequency agile radio technologies that enable flexible utilization of spectrum resources under the framework of software defined radio (SDR) [1]. One such technology is the multi-carrier spread spectrum (MC-SS) modulation that distributes data over a large number of carriers [2]. Ability to spread data over carriers spaced apart at different frequencies makes it a very agile technology that could utilize arbitrary open spectrum holes. However, despite recent advancements, full potential of SDRs cannot be exploited given the current policies set by the national regulatory bodies. For instance, in the United States, Federal Communications Commission (FCC) divides the spectrum into number of fixed non-overlapping bands and limits their usages to licensed primary systems with regulated levels of radio emissions.

Past measurements conducted by the FCC had shown that at any given time, on average 70% of the licensed spectrum was not used by the primary systems [3]. This creates large pool of dynamic "open spectrum" holes that could be utilized if regulations were relaxed. Recognition of this fact by the FCC has driven evolution of SDR towards cognitive radio (CR) [4] approach of enabling opportunistic utilization of open spectrum by un-licensed users without infringing the rights of licensed spectrum holders. CR is a paradigm in which wireless network domain or user node adapts transmission parameters in response to dynamically changing system parameters (e.g. spectrum availability) without interfering with licensed users.

Many current open spectrum sharing protocols are developed under the assumption that the distributed unlicensed users are cooperative and seek to obtain optimal spectrum resource allocation. For examples, schemes in [5]–[8] consider cooperative distributed approach to obtain globally optimal spectrum resource allocation. The scheme in [5] proposes a distributed random access protocol designed to maximize the utilization of the open spectrum by requiring each secondary system to cooperate. Similarly, schemes in [6] and [7] present the benefits of the distributed collaborative approach in accessing the open spectrum. The scheme in [8] proposes a methodology for enabling two distinct wireless systems to share spectrum resources via reactive coordination, so as to improve the overall throughput.

In reality, the unlicensed users are likely to be independent competing entities with neither interaction nor cooperation between them. As competing "players", their primary interest is in optimizing their own payoffs (objectives) rather than those of the system. This paper is concerned with the problem of competitive sharing of open spectrum resources between collocated unlicensed entities of secondary systems and users. We extend the concept of open spectrum sharing to unlicensed secondary systems, which are autonomous wireless network domains with administrative authorities. The secondary systems can act as wireless service providers to subscribed users. For the rest of the paper, secondary systems can refer to either unlicensed network domains or unlicensed users in the limiting cases.

To enable distributed utilization of the open spectrum resources under the autonomous open spectrum sharing (AOSS) paradigm, this paper proposes Payoff-Enriched Adaptive Learning (PEAL) methodology. Under the AOSS paradigm, secondary systems are treated as competing entities that are opportunistically seeking to access open spectrum resources. Using tools of the non-cooperative game theory, the proposed PEAL methodology enables a (spread spectrum based) secondary system to adaptively make open spectrum access decisions independently of those of the others, so as to maximize its own payoff (i.e., objective). The payoff—defined in terms of resource utilization—is allowed to be adaptively weighted to adjust resource valuation due to the varying demand within underlying secondary system. Optimal spectrum access strategies for the specified payoff functions are obtained in accordance with the Nash equilibrium solution. To iteratively reach the theoretical equilibrium solution in an environment where the "common knowledge" of opponents' payoffs is not assumed, the proposed PEAL methodology enables each player to iteratively adapt their spectrum access strategies in response to the observed history of opponents' decisions. The PEAL methodology is based on spread

Digital Object Identifier: 10.4108/ICST.BROADNETS2009.7301
http://dx.doi.org/10.4108/ICST.BROADNETS2009.7301

spectrum or code division multiple access (CDMA) of open spectrum. Consequently, the open spectrum resource can be considered as "elastic" sharing resource, which can be shared by multiple entities subjected to potential performance degradation perceived by the entities.

Related works of non-cooperative spectrum access for intra-domain users have been considered in [9]–[12] under the framework of distributed power control in a spread spectrum based system. These schemes adapt a game-theoretic approach under which users choose their transmit powers so as to maximize their payoffs. Even though these schemes are developed under the non-cooperative paradigm, they are not easily extendable to the "opportunistic" open spectrum sharing environment where the payoff of a secondary system domain significantly differs from that of a user in a single system domain. Specifically, the schemes in [9]–[12] do not allow for adaptive weightings of payoffs needed to appropriately scale resource valuations in response to dynamically changing state of a secondary system (e.g. changing resource demand characterized by a varying number of users within its domain).

The rest of this paper is organized as follows. System model and the formal statement of the problem are presented in Section II. Proposed scheme is described in details in Section III. In Section IV, numerical results obtained under various scenarios are presented. Finally, conclusions are drawn in Section V.

II. System Model and Problem Statement

A. System Model

This paper considers the problem of non-cooperative resource-sharing between collocated spread-spectrum based secondary wireless systems seeking to opportunistically utilize open spectrum that is left unused by the licensed entity. By the term *collocated* is meant both geographical and spectrum coexistence (i.e. they occupy the same geographical region or cell, and seek to share the same frequency spectrum). The *secondary* refers to the fact that these systems do not own licensing rights to the spectrum, but rather it belongs to some other entity referred hereto as the *primary* system. The shared open spectrum is then the unused portion of this licensed band so that it is time-variable as it depends on the usage by the primary system.

At time-instant *n*, the open spectrum consists of $W(n)$ Hz of frequency bandwidth. It is assumed that the amount of open spectrum $W(n)$ is known to the secondary systems through some cognitive sensing process.

Secondary systems are assumed to employ a spread spectrum modulation and are able to spread data over the whole $W(n)$ Hz—even if this band is not contiguous (i.e. it consists of disjointed subsets) in which case employment of a multi-carrier based SDR technology, such as proposed in [2] is required. The topology of each secondary system is infrastructure-based (e.g. cellular), that is, each one has a single entity designated as an agent (controller) responsible for resource allocation within its domain as shown in the high-level system diagram of

Fig. 1. In addition, secondary systems are identified via a unique pseudo-random code.

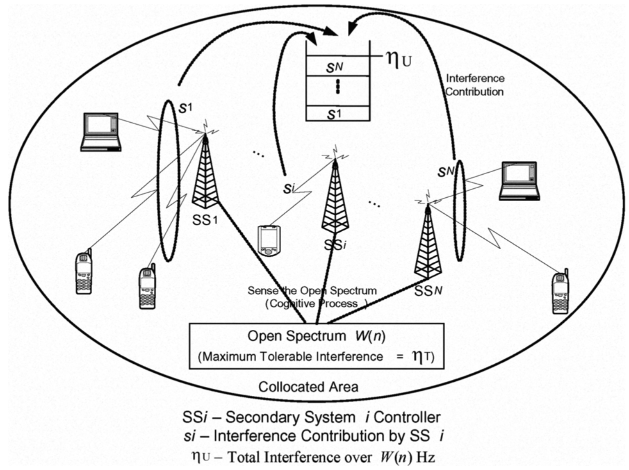

Fig. 1 AOSS System Model

The *horizontal sharing*, or (open spectrum) sharing between secondary systems is accomplished via distributed AOSS paradigm. Consequently, at each time *n*, (nodes of) secondary systems spread their information payload over the whole $W(n)$ Hz of open spectrum. By doing this, each secondary system contributes some amount of interference level over the open spectrum—the exact amount of which is determined by the actual power level and transmission rate chosen by the corresponding controller. For the given amount of maximum tolerable interference over the open spectrum, the objective of each secondary system is to choose the optimal level of interference contribution so as to maximize its own payoff. Thus, the optimization, as elaborated later, is in a *non-cooperative* way without regard for the system as a whole. No signaling or any binding agreements exist between secondary systems when making interference contribution decisions.

B. Problem Statement

For the system model defined in the last subsection (consisting of *N* spread spectrum based secondary systems, refer back to

Fig. 1), the mathematical formulation of the problem (in an arbitrary time-slot *n*) is stated next. For the presentational clarity, since arbitrary time is considered, the time-slot index *n* is dropped from all relevant quantities in the rest of the paper. Let the maximum tolerable interference over *W* Hz of frequency spectrum be denoted by η_T. Note that in an interference-limited spread spectrum system η_T denotes the interference ceiling—any additional interference above it leads to outage that disables the usage of wireless links. Outage is the condition arising when the signal to interference ratio of a communication link is less than the minimum needed to keep the bit-error-rate (BER) below some desired target. For the given BER targets, the utmost interference level η_T depends on the available spectrum *W* and maximum transmit powers of the secondary systems. It could be derived by following standard analysis procedure for the spread spectrum system as in [13].

The problem facing each secondary system (controller) is to determine the optimal amount of interference contribution over *W* Hz of frequency spectrum (i.e. amount of resource usage) so as to maximize its own payoff. Let the ongoing

Digital Object Identifier: 10.4108/ICST.BROADNETS2009.7301
http://dx.doi.org/10.4108/ICST.BROADNETS2009.7301

aggregated interference by N secondary systems (in time-slot n) measured over W Hz of open frequency spectrum be denoted as η_U. Notice that for a viable communication the following resource constraint needs to be satisfied $\eta_U \leq \eta_T$. Thus, when making its interference contribution decision, a secondary system needs to account for possible decisions of other secondary systems (its opponents) contending for the common resource. Each secondary system is assumed to be rational—making the best decision for itself. Notice that it is not in its best interest to be greedy and choose unreasonably large interference contribution, as even slight additional contribution by other players could violate the resource constraint and disable the communication for all. Rationality will naturally lead to some equilibrium decisions that satisfies the resource constraint and from which no secondary system would have an incentive to deviate as it could not further increase its payoff.

III. Proposed PEAL Methodology

The proposed Payoff-Enriched Adaptive learning (PEAL) methodology enables a (spread spectrum based) secondary system to iteratively choose the best strategy, so as to maximize its own payoff when accessing the open spectrum. The optimal strategy for particular payoff functions is determined via the game theoretic approach. PEAL defines a distributive learning procedure to achieve this optimal strategy when the opponents' payoff functions are not known—as is anticipated under a realistic open spectrum access scenario.

A. *Game Theoretic Formulation*

An N-player strategic form game G with player (decision maker) set $P = \{1, 2, \ldots, N\}$ can be described with an ordered $2N$-tuple ([14]),

$$G = \{S_1, \ldots, S_N, \pi_1, \ldots, \pi_N\} \quad (1)$$

where S_i is the non-empty strategy set of player i, and $\pi_i : S_1 \times S_2 \times \ldots \times S_N \rightarrow \Re$ is the payoff function of player i which assigns to each N-tuple $(s_1, \ldots, s_N)$ of strategies a real number $\pi_i(s_1, \ldots, s_N)$. Elements s_i of strategy set S_i are referred to as strategies of player i.

The non-cooperative strategic form game G is played as follows. After pre-play communications where no binding agreements can be made (i.e. no cooperation), each player i chooses simultaneously and independently of other players, a strategy $s_i \in S_i$. Once this is done, for the given outcome of the game denoted by N-tuple $(s_1, \ldots, s_N)$ of strategies, each player i receives a payoff $\pi_i(s_1, \ldots, s_N)$. The Cartesian product $S_1 \times S_2 \times \ldots \times S_N$ of the strategy sets is known as the set of possible outcomes of the game. Note that the payoff $\pi_i(s_1, \ldots, s_N)$ for a player i could represent some monetary gain/loss or some other objective/cost function that is of significance to the player.

Many models of conflict can be dealt with the tools of the game theory. The horizontal spectrum sharing problem is formulated as an N-player strategic form (non-cooperative) game G. The game is played in every time-slot n (n = 1, 2, …). The open spectrum available at a time-slot can be viewed as *common* property resource that is to be utilized by N spread spectrum based secondary systems under the framework of the AOSS paradigm (see Fig. 2).

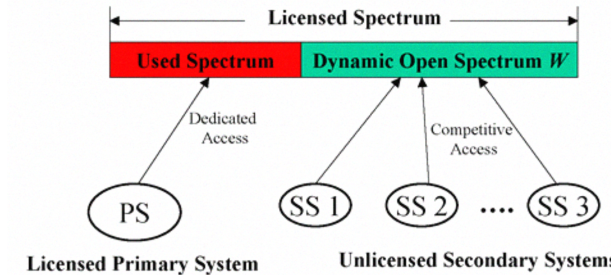

Fig. 2 Common Property Resource: Game Theoretic Formulation

Consequently, the players of the game G correspond to the (controllers of) N spread spectrum based secondary systems seeking resources (i.e. access to the open spectrum) in time-slot n. Secondary systems—labeled as 1, 2, …, N—are clustered in the player set $P = \{1, 2, \ldots, N\}$. As mentioned earlier, the decision facing each player (i.e. secondary system controller) is to select what amount of common resource to utilize in the time-slot. The amount of resources utilized by player i is specified by the level of interference contribution over W Hz of frequency bandwidth. Consequently, let the interference contribution by player i be its selected strategy s_i. Thus, the strategy set of a player i (in time-slot n) is denoted by the open interval $S_i = [0, \eta_T]$, where η_T denotes the maximum tolerable interference (in the time-slot n) over W Hz of frequency bandwidth.

To complete the formulation of the strategic form game G, it is required to define the payoff functions $\pi_i(s_1, \ldots, s_N)$ for each player i. The cost to player i contributing s_i units of interference depends not only on the amount s_i contributed by the player but also on the amount $(\eta_U - s_i)$ contributed by the other players (the ongoing aggregated interference η_U was defined earlier and for the game G given as $\eta_U = \sum_{i=1}^{N} s_i$). This is reasonable as the system becomes more loaded, transmission performance over wireless link would decrease (e.g. increased BER). Let the cost be denoted by $C_i(s_i, \eta_U - s_i)$. For a given η_U, let the utility that a player i receives from selecting s_i units of interference be denoted by $u_i(s_i, \eta_U)$. Thus, given the outcome of the game $(s_1, \ldots, s_N)$, the payoff of player i is in general terms expressed as,

$$\pi_i(s_1, \ldots, s_N) = u_i(s_i, \eta_U) - C_i(s_i, \eta_U - s_i) \quad (2)$$

To analyze the game G and obtain the optimal strategies under the AOSS methodology, particular payoff functions are needed and are defined in the next subsection.

B. *Game Analysis*

For the problem at hand, defining particular payoff functions requires some economical considerations as elaborated next. It is reasonable to assume that the marginal cost of using a resource increases with the total amount of resource used. Thus, as the secondary systems contribute more interference (i.e. use more resource) the marginal cost of contributing additional interference goes up. Stated

Digital Object Identifier: 10.4108/ICST.BROADNETS2009.7301
http://dx.doi.org/10.4108/ICST.BROADNETS2009.7301

mathematically, the cost $C_i(s_i, \eta_U - s_i)$ in (2) is a convex function that satisfies the following properties, for all i, $s_i > 0$, and $\eta_U > 0$,

$$\frac{\partial C_i(s_i,\eta_U - s_i)}{\partial s_i} > 0, \quad \frac{\partial C_i(s_i,\eta_U - s_i)}{\partial \eta_U} > 0,$$
$$\frac{\partial^2 C_i(s_i,\eta_U - s_i)}{\partial s_i^2} > 0, \quad \frac{\partial^2 C_i(s_i,\eta_U - s_i)}{\partial \eta_U^2} > 0 \qquad (3)$$

Moreover, without loss of generality, the cost $C_i(s_i, \eta_U - s_i)$ is assumed to be a separable quadratic function of its arguments, and hence let it be defined as,

$$C_i(s_i,\eta_U - s_i) = \left[s_i^{\,2} + (\eta_U - s_i)^2 \right] \qquad (4)$$

On the other hand, it is economically sound to assume that the utility function $u_i(s_i, \eta_U)$ is increasing in s_i and decreasing in η_U (for all i). This means that as the value of interference contribution s_i increases, the utility to player i increases (e.g. higher throughput is achieved for fixed η_U). But, as the value of ongoing aggregated interference in the system η_U is increased, the utility drops (e.g. throughput is reduced for a fixed interference contribution s_i). Hence, for the problem at hand, let the utility function $u_i(s_i, \eta_U)$ be defined as,

$$u_i(s_i,\eta_U) = (\eta_T - \eta_U)\cdot s_i \qquad (5)$$

Moreover, in an opportunistic open spectrum environment, it is reasonable to assume that the payoff of player i, or SS_i, depends on the number of users n_i seeking resources within its domain. Increasing number of users implies higher resource demand and hence potential for a higher payoff. Consequently, as the number of users n_i changes, so does the game G, as reflected by the corresponding scaling on the payoffs. Thus, for a given number of users n_i, the particular payoff $\pi_{i|n_i}(\cdot)$ for SS_i ($i = 1, 2, \ldots, N$) is expressed as,

$$\begin{aligned}\pi_{i|n_i}(s_1,s_2\ldots,s_N) &= a(n_i)\cdot u_i(s_i,\eta_U) - b(n_i)\cdot C_i(s_i,\eta_U - s_i) \\ &= a_i(n_i)\cdot(\eta_T - \eta_U)\cdot s_i - b_i(n_i)[s_i^2 + (\eta_U - s_i)^2]\end{aligned} \qquad (6)$$

where $a_i(n_i)$ and $b_i(n_i)$ denote the utility and cost (weighting) factor of secondary system (i.e., player) i when the number of users within its domain is n_i.

Let the utility to cost factor ratio of player i (SS_i) with n_i users within its domain be defined as $UCR_i(n_i) = a_i(n_i)/\, b_i(n_i)$. As the number of users n_i in SSi is dynamically changing, the particular $UCR_i(n_i)$ and the corresponding payoff function $\pi_{i|n_i}(\cdot)$ in accordance with (6) would also change dynamically . The exact functional relation between the number of users n_i and the corresponding $UCR_i(\cdot)$ is player-subjective depending on many economical factors. It is reasonable to assume that $UCR_i(n_i)$ is increasing in n_i. As the resource demand (i.e., n_i) increases, the utility would be given higher weighting [i.e., higher $UCR_i(\cdot)$], but nevertheless, the utility should level off for very large n_i due to the increased interference.

1) Nash Equilibrium Solution:

Given the game G with the particular payoffs determined by the current number of users n_i, the Nash equilibrium is an outcome $(s_1^*, s_2^*, \ldots, s_n^*)$ such that it satisfies,

$$\begin{aligned}&\pi_{i|n_i}(s_1^*, s_2^*, \ldots, s_{i-1}^*, s_i^*, \ldots, s_N^*) \geq \\ &\qquad \pi_{i|n_i}(s_1^*, s_2^*, \ldots, s_{i-1}^*, s_i, \ldots, s_N^*), \;\; \forall s_i \in S_i\end{aligned} \qquad (7)$$

Remark: Note that the Nash equilibrium is the most likely outcome of the game. In general it is not the point of global optimum, but nevertheless it is an outcome from which no player has an incentive to deviate. Hence in a competitive environment like the open spectrum sharing system defined earlier, given the common knowledge about the game, it is optimal for each player to play it.

When a unique outcome satisfies (7), there is one particular NE of the game referred to as the *pure strategy* NE. However, it has been shown that the pure strategy NE does not always exist. Under that situation, randomization of strategy selections via so-called *mixed strategies* approach is required. This approach assigns probabilities $p_i(\cdot)$ (i.e. probability densities) over (pure) strategies $s_i \in S_i$. The structure of the payoff functions $\pi_{i|n_i}(\cdot)$ determines the existence of pure strategy NE. More on this is elaborated next.

If each strategy set S_i is an open interval of real numbers [as is the case in the AOSS game formulation of the last subsection], the (pure strategy) Nash equilibrium of the game G with payoff functions $\pi_{i|n_i}(\cdot)$ as in (6) satisfies the following system of N equations. The outcome $(s_1^*, s_2^*, \ldots, s_n^*)$ is the NE of the game G if it is the solution of the following system of N equations,

$$\begin{aligned}&\frac{\partial \left[\pi_{i|n_i}(s_1^*, s_2^*, \ldots, s_N^*) \right]}{\partial s_i} = 0, \;\; i = 1,2,\ldots,N \\ &\frac{\partial^2 \pi_{i|n_i}(s_1^*, s_2^*, \ldots, s_N^*)}{\partial^2 s_i} < 0 \\ &\textit{subject to} \sum_{j=1}^{N} s_i^* = \eta_U \leq \eta_T\end{aligned} \qquad (8)$$

The NE test of (8) follows from the definition in (7) and in accordance with the optimization theory. Note from (6) that the selection of factors $a_i(n_i)$and $b_i(n_i)$ determines the particular payoffs and hence the pure strategy NE. The following theorem states the conditions for the existence of the NE and characterizes it in terms of $a_i(n_i)$and $b_i(n_i)$. The NE outcome is the strategy for player i adapted by the proposed PEAL methodology.

Theorem 1: For the game G with payoff functions $\pi_{i|n_i}(\cdot)$ as in (6), in accordance with the test in (8), the pure strategy NE $(s_1^*, s_2^*, \ldots, s_n^*)$ always exists if $a_i(n_i)$and $b_i(n_i)$ for all i. It is given by the simultaneous solution of the following N equations,

$$s_i^* = \frac{\eta_T - \sum_{j=1, j\neq i}^{N} s_j^*}{(2 + 2\cdot b_i(n_i)/a_i(n_i))}, \;\; i = 1,2,\ldots,N \qquad (9)$$

Digital Object Identifier: 10.4108/ICST.BROADNETS2009.7301
http://dx.doi.org/10.4108/ICST.BROADNETS2009.7301

Proof: Given in the Appendix.

2) *PEAL Evolution to Nash Equilibrium*

To achieve the equilibrium requires the assumption that each secondary system (i.e., player) has a complete knowledge of the opponents' payoffs. In a cognitive, "ad-hoc" like environment, a secondary system may only know its own payoff and it is unreasonable to assume any more knowledge than that. Thus, it might be impossible for each secondary system to obtain the NE by solving (8) directly.

To mitigate for this problem, PEAL procedure enables a secondary system to adapt its strategy in response to the observed strategies of its opponents so as to iteratively approach the NE. Each player thus monitors the strategy actions of the other players and makes strategy adaptation decisions sequentially. At each iteration (trial) step, each player selects the strategy that maximizes its (weighted) payoff function given the "current" strategy actions observed from other players. It is assumed that the algorithm converges before the number of users n_i is changed. PEAL procedure is outlined as follows (iteration or trial step is denoted as k):

PEAL procedure for SS i

1. Initialization;
   $k=1$(*k is the iteration index*);
2. Observe the total interference of all other secondary systems, $\sum_{j=1, j\neq i}^{N} s_j^{'}(k)$
3. Based on the monitored interference, calculate the payoff,
   $\pi_{i|n_i}(s_1(k), s_1(k), ..., s_{i-1}(k), s_i(k), s_{i+1}(k), ..., s_N(k)) = \pi_{i|n_i}(s_i(k), \eta_T - s_i(k))$;
4. ***if*** the payoff could be increased then selects the new strategy in accordance with (9);
   $$s_i^*(k) = \frac{\eta_T - \sum_{j=1, j\neq i}^{N} s_j^{'}(k)}{(2 + 2 \cdot b_i(n_i)/a_i(n_i))}$$
   else, keep current in-use strategy;
   $$s_i^*(k) = s_i^*(k-1);$$
5. $k = k+1$;
6. Go to 2.

IV. Numerical Results

In this section, performance of the proposed PEAL methodology for horizontal open spectrum sharing is evaluated. First, the convergence property is studied under the scenario whereby the "common knowledge" of payoff functions $\pi_{i|n_i}(\cdot)$ is not assumed a priori. Then, the effect of changing individual *UCR* on the payoffs of all secondary systems is investigated. Also, the end-result of changing the number of players N (i.e., secondary systems) on the performance of the proposed PEAL methodology is examined. Finally, the PEAL methodology is compared to the overall (globally) optimal cooperative approach to spectrum sharing.

A. PEAL Convergence

The analysis is performed under a scenario whereby the "common knowledge" of payoff functions $\pi_{i|n_i}(\cdot)$ is not assumed a priori. Each player knows only its own payoff $\pi_{i|n_i}(\cdot)$ that is given in accordance to (6). Convergence property of the PEAL methodology is evaluated in an environment consisting of N = 5 players (i.e., secondary systems). Two experiments with uniform UCR_i = 0.1 and UCR_i = 10 respectively are conducted for any player i.

In accordance with the PEAL methodology, the player adapts the strategy (i.e., interference contribution) in response to the observed strategies of its opponents. Fig. 3 and Fig. 4 respectively, show the PEAL induced strategy evolution $s_i(k)$ for $N = 5$ secondary systems in the aforementioned experiments. The index k denotes the trial (iteration) number.

The convergence points or the pure strategy Nash equilibriums (unknown to the players) for the low and high *UCR* experiments, obtained from (9), are $(s_1^*, s_2^*, ..., s_n^*)$ = (0.1, 0.1, 0.1, 0.1, 0.1) and $(s_1^*, s_2^*, ..., s_n^*)$ = (0.16, 0.16, 0.16, 0.16, 0.16) respectively. Observe from both Fig. 3 and Fig. 4 that in both experiments, the strategies of all secondary systems converge to the respective Nash equilibrium points.

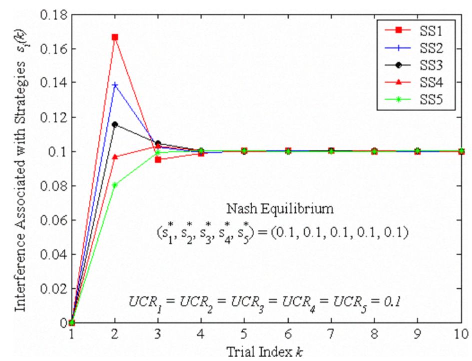

Fig. 3 PEAL Methodology Convergence to Nash Equilibrium with Low Uniform UCR_i (Less Weight to Utility)

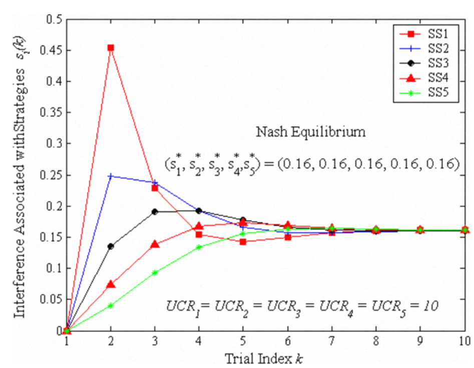

Fig. 4 PEAL Methodology Convergence to Nash Equilibrium with High Uniform UCR_i (More Weight to Utility)

Comparing the two experiments, it is observed that the higher *UCR* implies larger interference contribution at Nash equilibrium. The reason is that higher *UCR* imposes higher weighting on the utility due to the increased internal resource demand (players can economically sustain higher interference). This, in turn, leads to more aggressive spectrum access strategies, as the potential of higher payoffs for the given interference contribution is increased. Fig. 3 shows that it takes about 4 trials to obtain the Nash equilibrium in the first experiment (UCR_i = 0.1). On the other hand, as shown in Fig. 4, it takes more than 8 trials to reach Nash equilibrium point in the second experiment (UCR_i = 10). The reason for the increase in the number of trials needed for the convergence is

Digital Object Identifier: 10.4108/ICST.BROADNETS2009.7301
http://dx.doi.org/10.4108/ICST.BROADNETS2009.7301

again, the more aggressive interference contribution under higher *UCR* that increases the search region and leads to higher fluctuations and more trials to reach the stable Nash equilibrium point.

Experiment with non-uniform UCR_i values is also conducted and the corresponding strategy evolution $s_i(k)$ of each secondary system i is illustrated in Fig. 5. In this experiment, each secondary system employs different *UCR* value (actual *UCR* values are shown in Fig. 5). The pure strategy Nash equilibrium point for this game (unknown to the players) is $(s_1^*, s_2^*, ..., s_n^*)$ = (0.22, 0.19, 0.14, 0.10, 0.09). Observe from Fig. 5 that players' strategies converge to the pure strategy Nash equilibrium in about 6 trials (iterations).

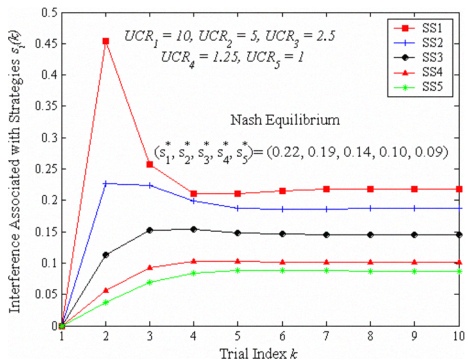

Fig. 5 PEAL Methodology Convergence to Nash Equilibrium with Non-uniform UCR_i

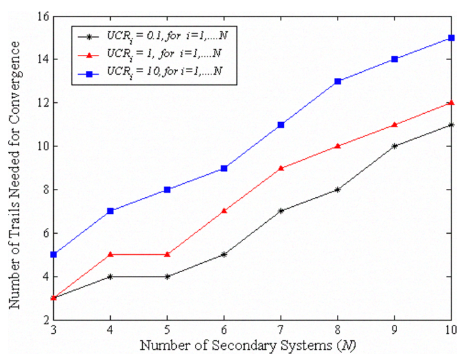

Fig. 6 Effect of Varying Number of Secondary Systems on PEAL Convergence to Nash Equilibrium

Convergence of the PEAL methodology is mainly affected by the number of secondary systems N contending for the resources. To investigate the effect of varying number of secondary systems, the experiment with uniform *UCRs* is repeated for different N. Fig. 6 shows the number of trials needed for the convergence vs. number of secondary systems N. Each curve in Fig. 6 corresponds to the particular uniform UCR_i value (namely, 0.1, 1 and 10 respectively). It can be observed from Fig. 6, as expected, that as the number of secondary systems N increases, the number of iteration steps (trials) required for convergence to the NE point increases almost linearly. Linear increase implies that the PEAL is a scalable methodology. Moreover, for the economical reasons stated earlier, as shown in Fig. 6, for a fixed N, the number of trials required for the strategies to converge increases as the UCR_i is increased.

B. *Effect of Changing Individual UCR on Payoffs*

The effect of changing *UCR* of one secondary system on payoffs of all secondary systems is studied. Consequently, experiment is conducted for different UCR_1 values of secondary system 1 in a game G consisting of $N = 5$ secondary systems. The UCR_i of other secondary systems is kept fixed (the actual values used are shown in the Fig. 7). As illustrated in Fig. 7, the payoff of secondary system 1 increases for higher UCR_1, while the payoff of others decreases. This is because a higher UCR_1 implies that the utility of employing some strategy s_1 (i.e. interference contribution) is weighted more than the corresponding cost incurred by this strategy. Therefore, to maximize its own payoff for higher *UCR*, the secondary system would end up contributing more interference so as to achieve a higher payoff. However, due to fixed available resource, other secondary systems need to decrease their strategies, in terms of interference contribution, accordingly to keep the system stable. Consequently, the payoff of those secondary systems falls as their interference contributions decreases. Recall that in the actual system the determination of UCR_i for a player (i.e., secondary system) i depends on the actual demand within its domain (e.g. number of users) and requires an economical cost/benefit analysis. More on this is elaborated in the following subsection.

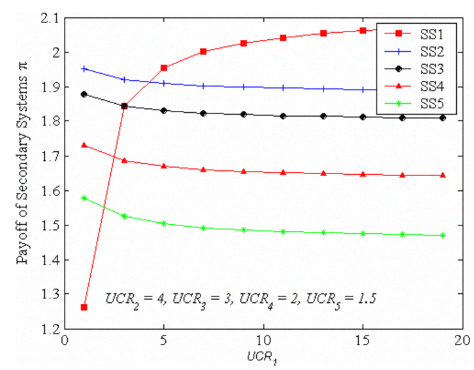

Fig. 7 Effect of Changing Individual UCR_i on Payoffs of all Secondary Systems

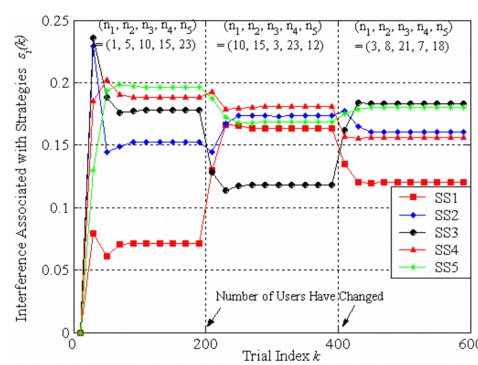

Fig. 8 PEAL Methodology Convergence to Nash Equilibrium with Time-varying Number of Users

Digital Object Identifier: 10.4108/ICST.BROADNETS2009.7301
http://dx.doi.org/10.4108/ICST.BROADNETS2009.7301

C. *Effect of Random Resource Demand (Time-Varying Number of Users)*

Fig. 8 shows the PEAL strategy adaptations (converging to Nash Equilibriums) in a dynamic environment with time-varying number of users (n_i). Concave $UCR_i(n_i)$ mapping is assumed for all secondary systems. As shown in Fig. 8, PEAL adaptations still converge to Nash Equilibrium. Observe from Fig. 8 that strategies temporarily diverge from the Nash equilibrium point when the number of users n_i changes (at k = 200 and k = 400 respectively). However, as evident from Fig. 10, the PEAL strategies are re-adapted and converge to the new Nash Equilibrium quickly.

D. *Comparison to Global Optimum*

For the evaluation purposes, the proposed PEAL methodology is compared to the (unrealistic) cooperative approach whose objective is to maximize the *total payoff* of all secondary systems (i.e., all players are in a coalition so as to benefit the overall system). The cooperative optimization problem is formulated as follows,

$$\max_{s_1, s_2, \ldots, s_N} \sum_{i=1}^{N} \pi_{i|n_i}(s_1, s_2, \ldots, s_N), \text{subject to} \sum_{j=1}^{N} s_i = \eta_U \le \eta_T \quad (13)$$

The solution of the global optimization problem $(s_1^{opt}, s_2^{opt}, \ldots, s_n^{opt})$, while unrealistic, is the best collective resource allocation possible. As such it achieves the best resource utilization. Fig. 9 compares the total payoff (i.e., summation of all individual payoffs) of this cooperative approach to that of the proposed PEAL approach that complies within the framework of the realistic AOSS paradigm. Observe from Fig. 9 that the cooperative approach merely outperforms the proposed PEAL methodology by achieving only an increase of 0.2 units of total payoff. Hence, the proposed non-cooperative PEAL scheme is able to provide comparable performance to the cooperative approach but under the AOSS paradigm.

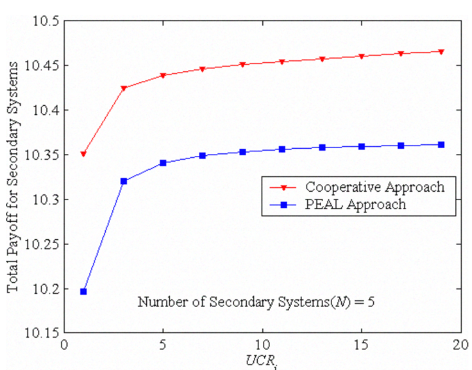

Fig. 9 Total Payoff: Cooperative Approach versus PEAL Approach

V. Conclusion

In this work, the PEAL methodology was proposed for competitive horizontal sharing of open spectrum resources between spread spectrum based secondary systems. Using the game theoretic approach, the proposed PEAL methodology prescribes rules for a distributed spectrum access strategy selection under the AOSS paradigm. Secondary system adaptively determines the optimal access strategy in terms of the prescribed amount of interference contribution over the open spectrum. The strategies are obtained for the pre-specified payoff functions. When selecting the optimal strategy, the objective of a secondary system is to maximize its own payoff. To reach the equilibrium in an environment where the "common knowledge" of opponents' payoffs is not assumed, the proposed PEAL methodology enables each player to adapt their spectrum access strategy in response to the observed history of opponents' decisions. It is shown in the paper that the PEAL prescribed strategy converges to the optimal equilibrium solution.

Appendix (Proof of the Therom)

According to the NE test in (8), the sufficient and necessary condition for the existence of pure strategy NE given payoffs in (6) is,

Constraint 1:

$$\frac{\partial \pi_{i|n_i}(s_1^*, s_2^*, \ldots, s_N^*)}{\partial s_i} = a_i(n_i)\cdot(\eta_T - \sum_{j=1, j\ne i}^{N} s_j - 2\cdot s_i^*) - 2b_i(n_i)\cdot s_i^* = 0 \quad (14)$$

Solving the above equation for s_i^*,

$$s_i^* = \frac{\eta_T - \sum_{j=1, j\ne i}^{N} s_j^*}{(2 + 2\cdot b_i(n_i)/a_i(n_i))}, \quad i = 1, 2, \ldots, N \quad (15)$$

Moreover, from (8) it also follows,

Constraint 2:

$$\frac{\partial^2 \pi_{i|n_i}(s_1^*, s_2^*, \ldots, s_N^*)}{\partial^2 s_i} = -2a_i(n_i) - 2b_i(n_i) \quad (16)$$

Since $a_i(n_i) > 0, b_i(n_i) > 0$, $\frac{\partial^2 \pi_{i|n_i}(s_1^*, s_2^*, \ldots, s_N^*)}{\partial^2 s_i} < 0$ always satisfies.

The strategy set of player *i* is denoted by the open interval $S_i = [0, \eta_T]$ and subject to following (resource) constraint,

$$\text{Constraint 3: } \sum_{j=1}^{N} s_i^* = \eta_U \le \eta_T \quad (17)$$

From (15) it follows,

$$(2 + 2\frac{b_i(n_i)}{a_i(n_i)}\cdot s_i^*) + \sum_{j=1, j\ne i}^{N} s_j^* = \eta_T \quad (18)$$

Simplifying,

$$\sum_{j=1}^{N} s_j^* + (1 + 2\frac{b_i(n_i)}{a_i(n_i)})\cdot s_i = \eta_U + (1 + 2\frac{b_i(n_i)}{a_i(n_i)})\cdot s_i^* = \eta_T \quad (19)$$

But if $a_i(n_i) > 0, b_i(n_i) > 0$ and $s_i \ge 0$, it follows that resource constraint in (17) is satisfied,

$$\eta_U = \sum_{j=1}^{N} s_j^* = \eta_T - (1 + 2\frac{b_i}{a_i})s_i^* \le \eta_T \quad (20)$$

Digital Object Identifier: 10.4108/ICST.BROADNETS2009.7301
http://dx.doi.org/10.4108/ICST.BROADNETS2009.7301

Thus, given that $a_i(n_i) > 0, b_i(n_i) > 0$ (sufficient condition), the Nash Equilibrium exists and can be obtained by solving N linear equations in (15). This concludes the proof.

REFERENCES

[1] J. Mitola, III. "Software Radio Architecture: A Mathematical Perspective," *IEEE J. Select. Areas Commun.,* vol. 17, no. 4, pp. 514-538, April 1999.

[2] N. Yee, J-P Linnartz, and G. Fettweis, "Multi-Carrier CDMA in Indoor Wireless Radio Networks," *IEICE Transactions on Communications*, vol. 77, no. 7, pp. 900-904, 1994.

[3] *Notice for Proposed Rulemaking (NPRM 03 322), Facilitating Opportunities for Flexible, Efficient, and Reliable Spectrum Use Employing Cognitive Radio Technologies,* ET Docket No. 03 108, Federal Communications Commission, Dec. 2003.

[4] S. Haykin, "Cognitive Radio: Brain-Empowered Wireless Communications," *IEEE J. Select. Areas Commun.,* vol. 23, no. 2, pp. 201-220, Feb. 2005.

[5] Y. Xing, R. Chandramouli, S. Mangold, and S. Shankar N., "Dynamic Spectrum Access in Open Spectrum Wireless Networks," *IEEE J. Select. Areas Commun.,* vol. 24, no. 3, pp. 626-637, March 2006.

[6] G. Ganesan, and Y. Li, "Agility Improvement through Cooperative Diversity in Cognitive Radio," *Proc. IEEE GLOBECOM* 2005.

[7] H. Zheng, and C. Peng. "Collaboration and Fairness in Opportunistic Spectrum Access," *Proc. IEEE ICC 2005.*

[8] X. Jing, S. C. Mau, and R. Matyas. "Reactive Cognitive Radio Algorithms for Co-Existence between IEEE 802.11b and 802.16a Networks," *Proc. IEEE GLOBECOM* 2005.

[9] C. W. Sung and W. S. Wong, "A Non-cooperative Power Control Game For Multirate CDMA Data Networks," *IEEE Trans. Wireless Commun.*, vol. 2, no. 1, pp. 186–194, Jan. 2003.

[10] Z. Han and K. J. Liu, "Non-cooperative Power-Control Game And Throughput Game Over Wireless Networks," *IEEE Trans. Commun.*, vol. 53, no. 10, pp. 1625–1629, Oct. 2005.

[11] H. V. P. F. Meshkati, M. Chiang and S. C. Schwartz, "A Game-Theoretic Approach To Energy-Efficient Power Control In Multicarrier CDMA Systems," *IEEE J. Select. Areas Commun.*, vol. 24, no. 6, pp. 1115–1129, June 2006.

[12] J. O. Neel, J. H. Reed, R. P. Giles, "Convergence of Cognitive Radio Networks," *Proc. IEEE WCNC* 2004.

[13] O. Yu, E. Saric, and A. Li, "Fairly-Adjusted Multimode Dynamic Guard Bandwidth Admission Control over CDMA Systems," *IEEE J. Select. Areas Commun.*, vol. 24, no. 3, pp. 579-592, March 2006.

[14] D. Fudenberg, and J. Tirole, Game Theory. Cambrige, Massachusetts: The MIT Press, 1991.

Digital Object Identifier: 10.4108/ICST.BROADNETS2009.7301
http://dx.doi.org/10.4108/ICST.BROADNETS2009.7301