

# Association in Multi-agent Simulations of Dynamic Random Social Networks.

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## ABSTRACT

Social groups form where individuals who are attracted to each other - usually by a common interest - interact and form clusters. These groups exist within structural networks that rely on the patterns of links between members through which communication and resource transfer occurs. Individual influence impacts on emergent characteristics of a group, for example, global opinion and collective behaviour. However, individuals join and leave groups, thus changing the system's dynamics. What impact do these structural changes have on the emergence of sub-groups? Here our interest is in the association of members around a particular ideology and real social network systems provide our bio-inspired simulation models. We address the effects of dynamic structural changes to randomly connected networks on global behaviour and the emergence of subgroups that associate with specific states. Results from multi-agent simulations demonstrate that social cohesion and collection of nodes around particular states are dependent on group dynamics and can have an impact on social management that effects social order and stability.

## Keywords

Multi-agent systems, social networks, simulation, complex systems.

## 1. INTRODUCTION

How do society's values, norms, and ideals become embedded in social structure? How do some people network better than others? How do people become associated with ideological groups? Such questions are of interest to social network researchers.

Social structures are typical of complex adaptive systems whose characteristics may be based around common principles. Characterised by large numbers of agents that interact and communicate through patterns of connections called networks<sup>[22]</sup>, complex behaviour is at the root of many natural and artificial

phenomena. However, real-world multi-agent systems are not effectively studied by traditional methods<sup>[15,16]</sup>, as emergent properties make them unstable and unpredictable<sup>[4,14,18]</sup>.

Clustering is evident in many systems as a mechanism for coping with complexity where hierarchies of clusters reduce internal interactions and constrain behaviour<sup>[19]</sup>. Social groups can be described in of terms clusters, alliances and networks of interactions. Common ideals, interests, and the like, link individuals together and the patterns of connectivity and information exchange between members influence the collective opinion of social networks<sup>[15]</sup>.

Opinion is usually diverse and spread across many different ideas, attitudes, and preferences. Often public opinion converges on ideas that resist change<sup>[32]</sup> stabilized by social comparison<sup>[17]</sup>. One research area, Memetics<sup>[3,6,8,27,28]</sup>, suggest that ideas themselves influence selection. Previous work<sup>[34,35,36]</sup> supports that group opinion is influenced by the number of connections between individuals<sup>[20]</sup>, communication between peers<sup>[29]</sup>, individual power<sup>[9]</sup> and susceptibility to influence from other ideas. Diversity also means that disagreement may result in the formation of sub-groups whose members share similar points of view<sup>[11]</sup>. Smaller disenfranchised or "fringe" groups can collect around radical opinions or ideas that are not representative of the majority. People who share common interests do not always remain static. Members leave and join groups; they break and make connections between individuals and other groups with similar philosophies or ideologies; individuals and groups change focus on what their main objectives might be.

Structural analysis<sup>[41]</sup> examines relationships in complex interactions of social members in the context of the social system in which they act<sup>[33]</sup>. Social transitions result from network topology and information exchange between connected network members<sup>[23,26,32]</sup>. A natural and effective means of representing topology and interaction is through network models<sup>[38]</sup>.

Multi-agent network systems, including random graphs, scale-free networks and small-world networks<sup>[1,2,7,11,14,39,40]</sup> are typically modelled using Graph Theory (a mature research discipline) to describe the general rules of agents' behaviour, and topological rules of agents' interconnectivity and communication patterns<sup>[25]</sup>.

Multi-agent simulations allow us to investigate mechanisms and patterns that emerge<sup>[11,15,41]</sup> from the interaction of explicitly defined states of individual agents and the causal processes that change these states over time<sup>[21]</sup>. They provide an opportunity to study complex systems *in silico*<sup>[15]</sup>.

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In previous studies<sup>[34,35,36]</sup> we modeled agents that had a binary choice of state in static network structures (fixed population and links) and determined that network structural characteristics impact on cohesion, communication and global behaviour.

Here we investigate the behaviour of simulated large-scale societies under the influence of exponential growth and decay. In particular, we look for factors that influence the emergence of clusters or subgroups that share common beliefs or values as a result of the interaction between peers within a dynamic social network. In a multi-agent network diffusion simulation<sup>[5,30,31,38]</sup> we construct a dynamic random network structure where agents have multiple instead of binary choices. We address these specific questions. How do individual states and connectivity parameters in dynamic random network structures impact on global behaviour? Do clusters of agents that adopt specific states emerge?

## 2. METHODS

The essential elements of this study are modelled from considering real-world bio-inspired circumstances: former primary school students commencing at a high school; formation of a new political party; establishment of a neighbourhood collective in a new suburb; a new inter-national trading collaborative between nations in a region (e.g. ASEAN); anti-social group formation within traditional societal boundaries; and the like. However, these elements are abstracted as variables in a simulated social network.

The dynamic model used in these multi-agent simulations is an extension of earlier work with static networks<sup>[34,35,36,37]</sup> where the static nature of the networks is due to fixing the population size and the bi-directional dyadic links between nodes. The dynamic model can be likened to the development of a new special-interest network where the fluid membership is establishing its strategy and policy directions based on member ideology. Here, the model incorporates real-world behaviours including: birth and death of agents as exponential growth and decay; fracturing of existing links and reconnections to new or existing agents; individual ideologies or opinions manifested as agent states; power of agents over each other; and susceptibility of agents to change their opinion. These bio-inspired parameters influence the emergence of patterns of connection, the formation of groups with like opinion and the majority opinion of the network members as interaction between them occurs.

The simulated social network is initialised with 50 peer nodes (representing a startup group) that are randomly assigned a preferred state from a range of up to 10 different states, each representing a belief or value. Communication is established by initialising links between pairs of nodes in the connection pattern for a random network with Boolean idealisation<sup>[24]</sup>. However, in contrast, here we also allow randomly selected nodes to enter or leave the network according to exponential growth ( $0.2 < P_g < 0.4$ ) and decay probability ( $P_d = 0.1$ ) factors at each evolution of the network. Initially, the model was tested using a larger range of values for each of  $P_g$  and  $P_d$ . Those at the lower end were selected largely due to the rapid population growth of the model.

As existing nodes are deleted, their edges are deleted. As new nodes are added, new links between these new and existing nodes are established.

Network evolution involves four tasks, executed in the following order:

1. *interaction*: connected pairs of nodes interact with each other, changing their opinions based on susceptibilities and influences,
2. *growth*: a node is added to the network and is connected to other nodes already in the network,
3. *decay*: nodes are disconnected from other nodes and removed from the network, and
4. *reconfiguration*: edges are added and removed between nodes according to the random pattern of connectivity.

The maximum population is limited to 5000 nodes and the model to 500 evolutions, based on observations of the model's behaviour in initial test runs and computational performance constraints. Stable behaviour was observed after around 200 evolutions (see Figure 4). Each node is also randomly initialised with characteristics that mimic the intrinsic sociability factors of an individual in society, for example, levels of power or influence and susceptibility to change opinion (values from 0.0 to 1.0). They express how sociable the person is, that is, how likely they are to participate in a given social group.

The network structure is represented as a graph  $\{V, E\}$  with a population of  $N$  agents with  $L$  connections selected at random from the  $N(N - 1)/2$  possible non-directed connections, with a fixed probability (see<sup>[34]</sup>) of 0.03. It is at this probability that critical behaviour with respect to the formation of global opinion occurs<sup>[34]</sup>. Each agent has a state at time  $t$ :  $\phi_i(t) \in \{0, \dots, (S - 1)\}$ , where  $S$  is the total number of states available. Parameters of the model are the number of nodes  $N(t)$ , and probabilities of connection ( $P_k$ ), growth ( $P_g$ ), decay ( $P_d$ ), susceptibility ( $\sigma_i = \langle 0 \dots I \rangle$ ) and influence ( $\gamma_i = \langle 0 \dots I \rangle$ ).

Asynchronous update of the system is applied to reflect real-world behaviour of social networks<sup>[10,21]</sup>. At time  $t$  (every iteration) an agent is randomly selected from the set of agents and updated according to:

$$\left. \begin{aligned} \phi_i(t+1) &= \phi_j(t) \quad \text{iff} ((\gamma_i < \gamma_j) \wedge (\sigma_i > \sigma_j)) \\ \phi_j(t+1) &= \phi_i(t) \quad \text{iff} ((\gamma_i \geq \gamma_j) \wedge (\sigma_i \leq \sigma_j)) \end{aligned} \right\} i \neq j \quad (1)$$

(adapted from<sup>[10]</sup>)

That is, if the influence of node  $i$  is less than that of node  $j$  and the susceptibility of node  $i$  is greater than that of node  $j$  then the state of node  $i$  at time  $(t+1)$  will change to the state of node  $j$  at time  $t$ . Conversely, if the influence of node  $i$  is greater than or equal to that of node  $j$  and the susceptibility of node  $i$  is less than or equal to that of node  $j$  then the state of node  $j$  at time  $(t+1)$  will change to the state of node  $i$  at time  $t$ . Interaction is limited such that  $i$  is not equal to  $j$ .

After initialisation, the simulation is allowed to evolve over 500 discrete time steps. We measure social cohesion as aggregation around agent states, that is, number of nodes having the same state. This provides an indication of sociability or social

participation. The simulation is run 10 times for each set of parameters and the results averaged.

We determine an Adoption Ratio that provides measure of the relationship between the expected number of nodes adopting an idea for a given network population and the actual value over time as the nodes interact. It is not a measure of clustering in relation to links between nodes (i.e. the number of nodes that are connected to another node). Here the expected value is equivalent to the number of nodes at time  $t$  divided by the number of states  $S$ . For example, if there were 200 nodes and 10 states the expected value would be 20 nodes adopting each state. The Adoption Ratio  $AR$  is calculated according to:

$$AR = \frac{\frac{1}{T} \left[ \sum_{t=1}^T \max(n_1(t), n_2(t), \dots, n_i(t)) \right]}{\frac{N(t)}{S}} \quad (2)^{[37]}$$

Where:

$n_i(t)$  is the number of nodes adopted by state  $i$  at time  $t$ ,

$N(t)$  is the number of nodes in the population at time  $t$ ,

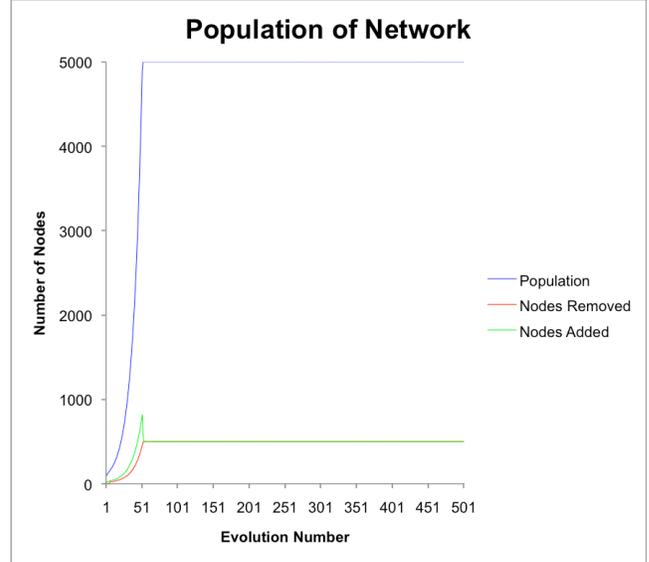
$T$  is the total number of time steps, and

$S$  is the number of states available.

### 3. RESULTS

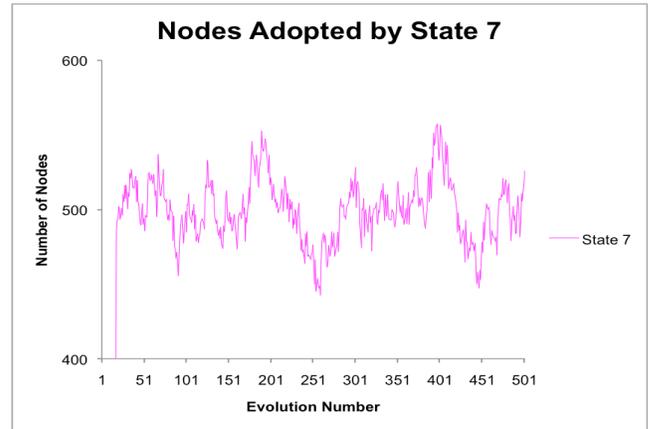
Selected results presented below are characteristic of the behaviour of the social network simulated in these experiments. These results are constrained by the need to construct two-dimensional representations of multi-dimensional complexity.

As the model evolves, the number of agents  $N(t)$  grows exponentially with time and the maximum population of 5000 agents is reached within 25 to 50 evolutions (Figure 1). In figure 1, the maximum number of nodes added (816) occurs at evolution 50 while the maximum number of nodes removed (500) occurs at evolution 52. At this point equal numbers of nodes are added and removed (500) at each time step for the duration of the experimental run. Similar behaviour occurs for all different values of the growth factor  $P_g$ , decay factor  $P_d$ , connectivity  $P_k$ , and number of states  $S$ . As the growth factor  $P_g$  increasingly exceeds the decay factor  $P_d$  (that is the separation between the values), the number of evolutions required to reach the maximum population  $N$  reduces - an intuitive result. After the population stabilises, the interaction between nodes becomes more pronounced. It demonstrates consistent model behaviour with respect to the dynamics of network construction.

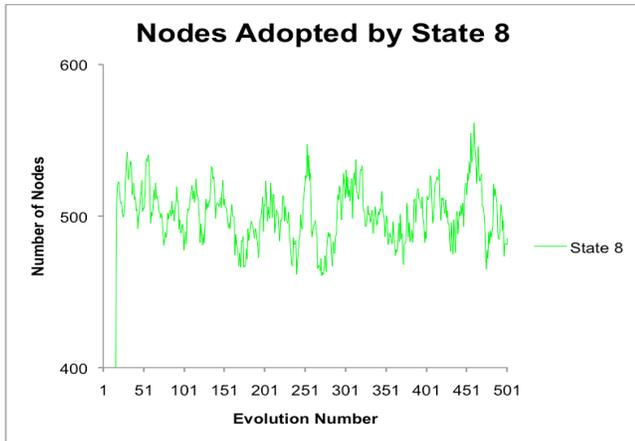


**Figure 1. Population growth and nodes added/removed for  $P_g = 0.2$ ,  $P_d = 0.1$ ,  $P_k = 0.003$  and  $n_s = 10$ , showing the maximum population  $N = 5000$  reached within 50 time steps.**

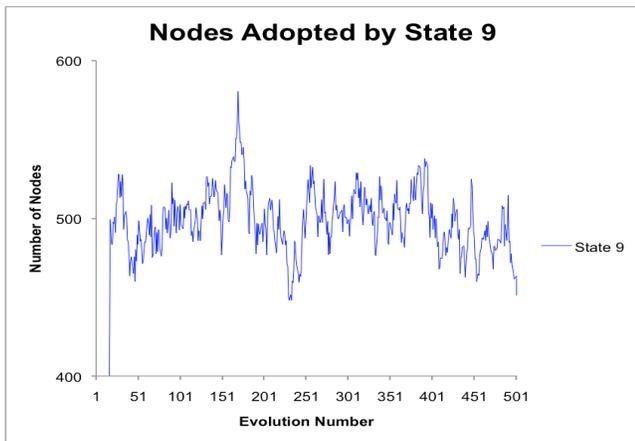
As the network population grows, nodes and links are added and deleted according to growth ( $P_g$ ) and decay ( $P_d$ ) factors. The nodes interact through their links to other nodes (Equation 1) and they change allegiance to the states with which they were initiated. There is a significant change of node "loyalty" to specific states where the maximum number of nodes adopting one or another state is not consistent. That is, aggregation of the nodes around specific states occurs to varying degrees. However, after 500 evolutions two or three states emerge with the largest number of nodes adopting that state. Here we show selected graphs (Figure 2 (a) – (d)) for states 7 – 10 as representative of all states, to demonstrate changing node loyalty and resultant states of adopted nodes at the final evolution number.



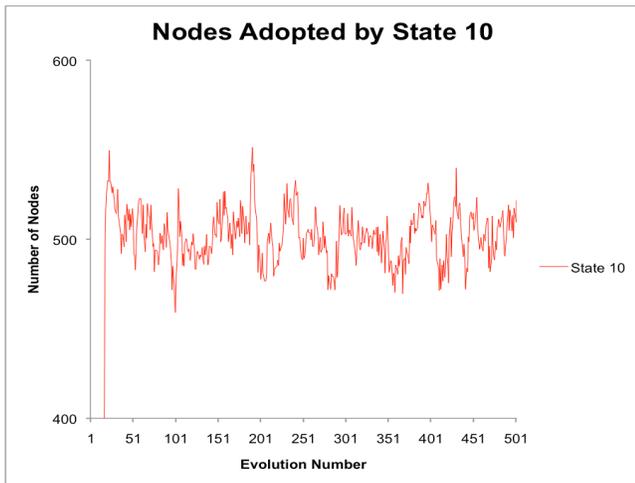
**(a)**



(b)



(c)



(d)

**Figure 2.** The number of nodes adopting specific states: (a) state 7; (b) state 8; (c) state 9; and (d) state 10, shows variation in node loyalty as nodes interact.

Graphs of the number of evolutions versus the number of nodes for selected states 7 to 10, show changing node loyalty as

interaction between nodes occurs for specific evolution numbers. In experimental runs where nodes have the “choice” of 3 – 10 states, one to three states emerge as adopting the largest number of nodes with the remaining states adopting smaller numbers (Table 1).

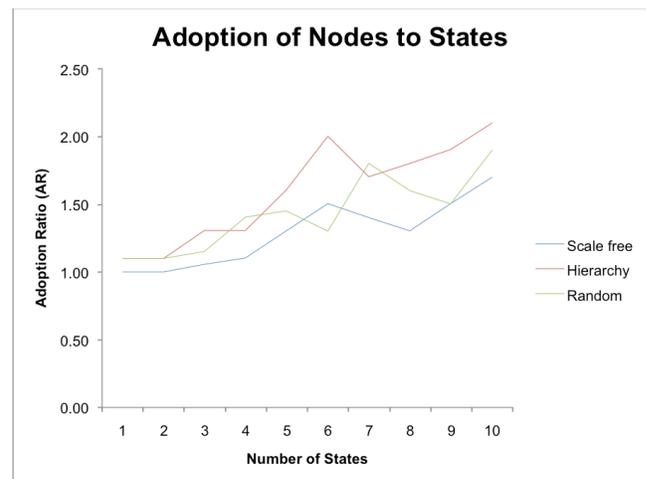
**Table 1. Evolution Number v Nodes per State**

Evolution v. State	100	200	300	400	500
3	525	511	496	486	497
4	521	501	487	479	509
5	533	469	487	458	524
6	483	530	506	498	482
7	484	521	528	539	526
8	477	506	511	484	482
9	509	506	499	509	461
10	459	477	502	508	521

The Adoption Ratio  $AR$  (Equation 2) measures the relationship between the expected number of nodes adopting an idea for a given network population and the changes to that adoption over time as the nodes interact.

When we consider previous work<sup>[37]</sup> done with static hierarchy, random, and scale free networks, the network structure is prescribed (see <sup>[34,35]</sup> for algorithms that generate the respective topologies) and both population size and links remain the same for the duration of each experimental run. Thus, the expected number of nodes adopting a state at initialisation depends on the number of states, an intuitive result.

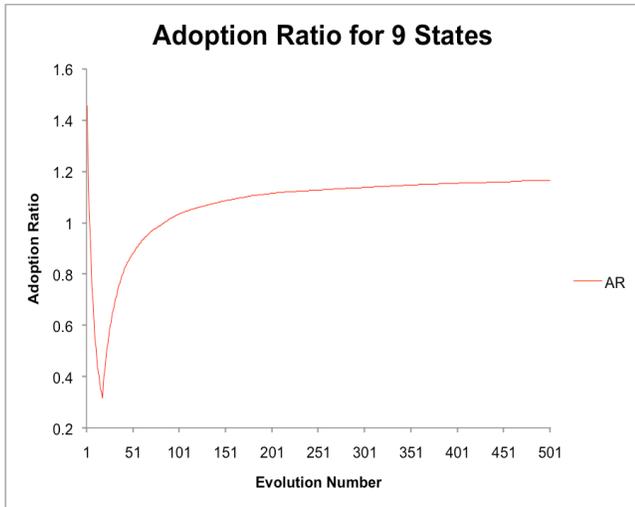
However, it is also evident that counter-intuitive behaviour is occurring (Figure 3). When connectivity is held constant, the  $AR$  increases with the number of states available for each of the three network structures. In the hierarchy network (Figure 3 – red), the  $AR$  is consistently greater than 1.1 and varies between 1.1 and 2.2. In the random network (Figure 3 – yellow), the  $AR$  increases from 1.1 – 1.8 and in the scale free network, the  $AR$  shows a steady increase from 1.0 – 1.7 (Figure 3 – blue).



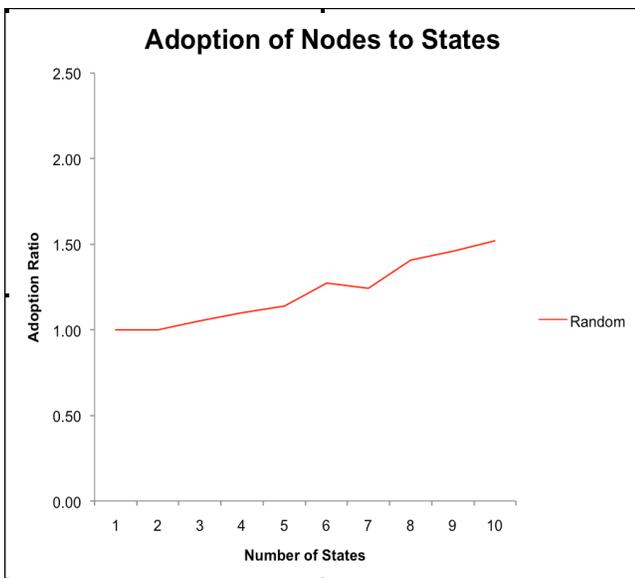
**Figure 3.** Graph showing the increase of  $AR$  with increasing number of states available for three types of static network structures – scale free, hierarchy and random (after<sup>[37]</sup>).

Similarity between the behaviour of the three network structures demonstrates universally consistent movement away from the initialised state of the model. It shows that as the number of states available to the system increases, the more nodes will adopt fewer states. This supports the notion that group opinion will converge on a few “majority-held” views, and that a few small “fringe” groups will also emerge. That the graph shows peaks around the same number of states suggests that there might be criticality with respect to those values.

When we consider dynamic random networks, as the population rapidly grows and stabilises at the maximum population size,  $AR$  also stabilises around 1.2 regardless of variation in the simulation parameters (Figure 4).



**Figure 4. Adoption Ratio  $AR$  for connectivity of 0.3 in a dynamic random network structure with 9 choices of state, where growth is 0.4 and decay is 0.1.**



**Figure 5. Graph showing the increase of  $AR$  with increasing number of states available for dynamic random network structures.**

However, there is similarity between the behaviour of dynamic and static random network structures also shown by consistent movement away from the initialised state of the model (Figure 5). As the number of states available to the system increases, “preferential” adoption of nodes by a small number of states occurs as a general principle and is dependent on the number of states available.

#### 4. DISCUSSION and CONCLUSIONS

Members of society are involved in work, social organisations, church communities, sporting groups, and so on. As individuals, they are committed to these groups as part of the network of members that make up the structures of those groups. They share attitudes, beliefs, ideas, innovation and other resources within the membership of a single group. But because of their membership of other groups of varying size<sup>[12,13,41]</sup>, resource information is also transferred between groups. Special interest groups thus form from the interaction of members of diverse social groupings based on common ideals, beliefs, attitudes, etc.

As an extension of previous work this simulation also supports that in random networks, the degree of connectivity between members will have an impact on the formation of public opinion in social groups that share common ideals, attitudes or opinions, particularly at levels where criticality in static networks occurs.

In this work with dynamic random social structures, results demonstrate that as social group population develops exponentially, the individual members’ opinions will (approximately) equally share a range of possible ideas.

However, once the population size stabilises, individual node loyalty to singular ideas from the range varies. Interaction between nodes allows for influence and susceptibility to determine opinion change in each node, resulting in fluctuating numbers of nodes adopting each idea. Groups that do share the same opinions or states emerge as a result of a choice of those states, although once population stability is reached the maximum number of nodes that adopt each specific state reduces as the number of available states increases. Thus the more choices there are the more difficult it is to make a choice.

However, the Adoption Ratio  $AR$  demonstrated counter-intuitive behaviour. Once the network stabilises, regardless of changes in connectivity, growth and decay of the network, and an increasing number of states available, the Adoption Ratio also increases to a stable level of around 1.2, demonstrating a universality or general principle that applies to the formation of sub-groups within dynamic populations. Simply stated, individual members of a society will aggregate to form dynamic networks of two to three major groupings around common ideologies or opinions, with several small “fringe” groups, often proposing radical (sometimes anti-social) viewpoints.

There are implications from this research for the management of change in social systems and the survival of groups whose members will be connected to sources of different ideas, attitudes and opinions. This is a rich area of research. Future projects might include: investigating the formation of aberrant groups in social systems; crowd behaviour in high-density social gatherings; informal and ad-hoc communication networks; large-scale strategy and decision-making; defence and security management in democratic societies; and the like.

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