A Global Convexity Analysis on the MAX-SAT Domain

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ABSTRACT

To this day, there is little theoretical understanding on the mechanics of local search algorithms and metaheuristics. Therefore, this work studies the solution space of Random MAX-SAT, which is a SAT variant. This analysis is focused on the study of global convexity, which may be exploited by some metaheuristics, as already observed in the literature. Understanding the structure of the solution space will give more insight on metaheuristics effectiveness for MAX-SAT. This paper presents experimental results suggesting that Random MAX-SAT is not globally convex, except for some overconstrained instances. Consequently, for the latter case, it is possible to predict that a population-based metaheuristic would be able to defeat an efficient local search algorithm, as experimentally proven in this work.

Keywords

SAT, Local Search, Metaheuristics, Global Convexity

1. INTRODUCTION

The satisfiability problem in propositional logic or SAT is the problem of finding an assignment of values to Boolean variables for a given propositional formula to make it true. SAT was the first problem proved to be $\mathscr{NP}$-complete [4], and as such is among the most important problems in computer science.

There exist several algorithms to solve SAT. These algorithms can be divided in two classes: *Complete* and *Incomplete*. Complete algorithms state whether a given formula is true or not and return an assignment of values to variables. Incomplete algorithms try to find an assignment that approximates the satisfiability of the formula.

A new approach based on metaheuristics [11] is becoming very popular. These algorithms are incomplete and include methods such as Simulated Annealing, Evolutionary Algorithms, Ant Colony Optimization to name a few. To this day, SAT solvers based on metaheuristics cannot compete against state-of-the-art local search algorithms [11].

Research in the field of metaheuristics has evolved on the basis of trial and error [2], often motivated by the competition for improving the best known solutions for given problems and not by identifying the reasons of success and failure of these algorithms. Some researchers suggested a new approach for the design of algorithms [2, 13], where the problem is studied first and then an appropriate algorithm is applied according to the characteristics of the search space.

Very little work has been done in the study of the SAT assignments space. Dimitriou and Spirakis [6] gave evidence that it is possible to tell in advance what neighborhood structure will give rise to a good search algorithm analyzing the properties of the search space. Zhang [14] studied the fitness-distance correlation in configuration landscapes, where local optima form large clusters, wich could be exploited by local search algorithms.

This work presents a topological analysis of Random-SAT assignments space. This analysis is focused on the study of global convexity, which was observed on the traveling salesman problem [2, 1, 13] and other problems, and it may be exploited by a class of ant colony algorithm [7]. The goal is to provide a framework from which to explain the behavior of metaheuristics in random SAT instances, and the design efficient algorithms as suggested in ref. [2, 13]. This research is complementary to the works of Zhang [14], who studied only one aspect of the global convexity analysis.

The remainder of this work is organized as follows. Section 2 explains the SAT problem. Section 3 presents global convexity. Section 4 introduces the theoretical framework. Section 5 and 6 present the experimental analysis. Finally, conclusions and future work are left for section 7.

2. THE SAT PROBLEM

SAT is defined by a set of boolean variables $\{x_1, \ldots, x_n\}$ and a boolean formula or sentence $\phi : \{0,1\}^n \to \{0,1\}$ where n represents the number of variables. The objective is to find a variable assignment $m = \langle x_1, \ldots, x_n \rangle \in \{0,1\}^n$ (i.e., a model) where $\phi(m) = 1$. The formula is called *satisfiable* if such a model exists, and *unsatisfiable* otherwise. A literal is a variable (also called atom or symbol) or its negation. A clause is a disjunction of literals, i.e., literals connected by an $\vee$ operator. The formula ϕ is in conjunctive normal form (CNF) if $\phi(m) = c_1(m) \wedge \cdots \wedge c_p(m)$, where each c_i is a clause and p is the number of clauses. The family of k-CNF sentences has exactly k literals per clause. SAT can be assumed having CNF formulae without loss of generality, and k-SAT only contains sentences in k-CNF.

MAX-SAT is an optimization variant of SAT. Given a

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set of clauses, MAX-SAT is the problem of finding an assignment m that maximizes the number of satisfied clauses $f(m)$. In *weighted* MAX-SAT each clause c_i has a weight w_i assigned to it, while *unweighted* MAX-SAT uses $w_i = 1$ for all i. There are only some differences between particular cases of SAT and MAX-SAT.

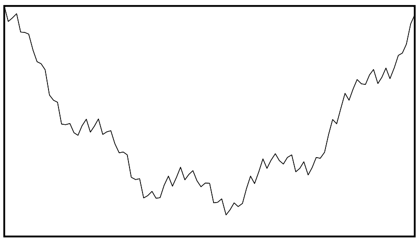
In 3-SAT, rapid transitions in solvability can be observed [3]. This phenomenon is called *phase transition*. If the problem has a very few number of clauses it is said to be *underconstrained* and it is easy to find a model. If there is a large number of clauses it is said to be *overconstrained* and a good heuristic will be able to quickly cut-off most of the search tree. Phase Transition occurs when going through instances that are underconstrained to instances that are overconstrained [5]. Hard instances are between these two kinds of instances, when 50% of them are satisfiable with approximately $\frac{p}{n} \approx 4.3$ [5]. This point is known as the *crossover point* [5].

This work deals with unweighted Random MAX-3-SAT, which from now will be referred simply as RMS.

3. SEARCH SPACE PROPERTIES

Previous works found the existence of a topological structure for the traveling salesman problem known as Global Convexity [2, 1, 13]. Global convexity is not convexity in the strict sense [2], but may be used to denote the empirical observation that the best local optima are gathered in a small part of the solution space, which hopefully includes the global optimum.

In a minimization context, Boese [1] suggested an analogy with a big valley structure, in which the set of local minima appears convex with one central global minimum. Even though there is no formal definition of global convexity, figure 1 gives an intuitive picture.



(a) in a minimization context

Figure 1: Intuitive picture of a big valley or globally convex solution space structure.

Any assessment of global convexity only makes sense once a topology (distance between solutions and surface cost) has been established in the solution space [2]. Hence, a quality metric and a neighborhood structure need to be defined on the solution space.

The global convexity hypothesis is based on two assumptions [2]:

1. *Convexity*: Local optima are gathered in a relatively small region of the solution space.

2. *Centrality*: The best local optima are located centrally with respect to these local optima.

If both assumptions are valid, one should also expect that local optima are gathered in a small region close to the best local optimum [2].

A global convexity analysis for RMS with two different topologies is presented in this work. These topologies use the same objective function (number of satisfied clauses) and differ only in the neighborhood structure, which are [6]:

1. **GREEDY**: two assignments are neighbors if and only if one results from the other by flipping the truth value of any single variable. Thus, any assignment has exactly n neighbors.

2. **WALKSAT**: two assignments are neighbors if and only if one results from the other by flipping the truth value of a variable that belongs to an unsatisfied clause. Note that the number of neighbors may be smaller than n.

The motivation for studying these neighborhood structures is due to their popularity and to see how the solution space is affected when using one or the other.

4. MEASURING THE SOLUTION SPACE

One important criterion for local search algorithms (and metaheuristics) is that the neighborhood structure does not have a tendency to create deceptive cost surfaces. This is created if the global optimum is located far from local optima [2]. Hence, a distance metric is needed to understand the distribution of assignments in the search space, which is defined as the Hamming distance between assignments m_i and m_j, and denoted as $\delta(m_i, m_j)$.

In globally convex problems, good solutions are usually found in the central region of the solution space [7]. Several classes of population-based metaheuristics keep the best solutions found during the optimization process to guide the search towards the central region of the solution space [7]. Generally, in one way or another, most metaheuristics implement some kind of elitism. Therefore, it is important to know the distribution of assignments with respect to the best solutions. Thus, two other metrics are introduced.

Definition 1. Let M^* be the set of best solutions for a sample set M (i.e., $M^* \subset M$), $m \in M$ an assignment, and $|\cdot|$ denotes cardinality. The mean distance of an assignment to the set of best solutions in the sample set is defined as:

$$\delta(m, M^*) = \frac{1}{|M^*|} \sum_{m_i \in M^*} \delta(m, m_i)$$

Definition 2. Let M^* be the set of best solutions for a sample set M and $m \in M$ an assignment. The minimum distance of an assignment to the set of best solutions in the sample set is defined as [9]:

$$\Delta(m, M^*) = \min_{m_i \in M^*} \{\delta(m, m_i)\}$$

The following definition gives an idea of the convergence degree for the best assignments in a sample set.

Definition 3. Let M^* be the set of best solutions for a sample set M. The mean distance of M^* is defined as:

$$\delta(M^*) = \frac{2}{|M^*|^2} \sum_{i=1}^{|M^*|-1} \sum_{j=i+1}^{|M^*|} \delta(m_i, m_j); \quad m_i, m_j \in M^*$$

Note that, the closer are the assignments in M^*, the smaller is $\delta(M^*)$.

5. EXPERIMENTAL ANALYSIS

5.1 Studied Instances

Two classes of random instances from SATLIB [8] are used: Satisfiable and Unsatisfiable. Table 1 shows all instances with their corresponding number of variables and clauses.

Table 1: Studied instances from SATLIB.

	sat	unsat	n	p	0.5p	1.5p
1	uf50-01	uuf50-01	50	218	109	327
2	uf100-01	uuf100-01	100	430	215	645
3	uf125-01	uuf125-01	125	538	269	807
4	uf150-01	uuf150-01	150	645	323	968
5	uf175-01	uuf175-01	175	753	377	1130
6	uf200-01	uuf200-01	200	860	430	1290
7	uf225-01	uuf225-01	225	960	480	1440
8	uf250-01	uuf250-01	250	1065	533	1598

All instances of table 1 are in the phase transition region. It is also interesting to consider the underconstrained and overconstrained cases. Hence, to transform each instance to an underconstrained SAT problem, 50% of the clauses are randomly deleted; and to transform each instance to an overconstrained SAT problem, 50% more clauses are added, following the rules given in [8]. Table 1 also shows the number of clauses for the underconstrained and overconstrained cases considered in this paper.

5.2 Process Detail

For each of the 48 ($8 \times 2 \times 3 = 48$) instances in table 1 a sample set was created with $|M| = n^2$ randomly generated assignments without repetition. Then, each assignment for all sample sets was locally optimized using the neighborhood structures GREEDY and WALKSAT, having this way 96 ($48 \times 2 = 96$) new sample sets.

The following variables were measured: 1) number of satisfied clauses $f(m)$; 2) mean distance to sample set $\delta(m, M)$; 3) mean distance to the best solutions $\delta(m, M^*)$ (def. 1); 4) minimum distance to the best solutions $\Delta(m, M^*)$ (def. 2). Then, correlations between the above variables were calculated for each sample set considering: 1) number of satisfied clauses versus mean distance to the sample set, denoted as $\rho_{f,M}$; 2) mean distance to the set of best solutions versus mean distance to the sample set, denoted as $\rho_{B,M}$; 3) minimum distance to the set of best solutions versus mean distance to the sample set, denoted as $\rho_{\Delta,M}$; 4) minimum distance to the set of best solutions versus mean distance to the set of best solutions, denoted as $\rho_{\Delta,B}$. These correlations are meant to study to what extend the assignments space (for GREEDY and WALKSAT neighborhoods) is globally convex.

5.3 Results

A summary of some experimental results are in table 2. The column $\rho_{f,M}$ shows the concentration of assignments in a region of the search space. Note that the closer is $\rho_{f,M}$ to -1, the closer are the good assignments to each other, i.e., more globally convex is the search space. For both neighborhoods, the underconstrained instances present small values towards 0, indicating that assignments are disperse in the search space. Hence, no global convexity property can be seen on these instances. When looking at the overconstrained instances, the values for $\rho_{f,M}$ are more negative (towards -1) when compared to the underconstrained instances. Although some cases exhibit low correlations, others are larger. These results suggest that those overconstrained instances are more globally convex. The instances at the phase transition region present correlations between the underconstrained and overconstrained cases, i.e., they are neither globally convex nor non-globally convex.

The fact that the best local optima are more close to each other for the overconstrained instances is confirmed with columns $|M^*|$ and $\delta(M^*)$. When the problem is underconstrained there are a lot of global optima with large values of $\delta(M^*)$. When going from underconstrained to an overconstrained instances, correlations become larger, suggesting a concentration of good local optima in a central region of the solution space for overconstrained instances.

An underconstrained instance presents the formation of clusters in the set M^*, i.e., there exists local optima that cannot be reached from other local optima by means of a variable flip[1]. This fact shows the absence of global convexity for an underconstrained instance. Particularly, for the underconstrained cases, the values of $\rho_{B,M}$ are close to 1, because there exist a lot of assignments close to each cluster, i.e., $\delta(m, M)$ and $\delta(m, M^*)$ have similar values; whereas the values of $\rho_{\Delta,M}$ are close to 0, because there exists only one cluster that is close to each assignment.

When going from underconstrained, through phase transition, to an overconstrained instance, the values of $\rho_{B,M}$ gets smaller; meanwhile, $\rho_{\Delta,M}$ gets larger. This is because the number of clusters tends towards one.

The studies presented in [14] only examined the correlations between $f(m)$ and $\Delta(m, M^*)$, and not the distribution of assignments with respect to each other. Therefore, all these results suggest that RMS is not globally convex, except for some overconstrained instances. As a consequence, population-based metaheuristics are not completely successful solving SAT when compared to local search algorithms. However, according the experimental results of table 2, it is possible to postulate that population-based metaheuristics may be effective for overconstrained RMS given their global convexity property.

As a final remark, there is not too much difference between GREEDY and WALKSAT neighborhoods since correlations were very similar for both. Hence, it is also expected that population-based metaheuristics would not be able to find better assignments using one or the other. This looks like a contradiction, since algorithms based on WALKSAT neighborhood proved to be a lot better than the ones based on GREEDY [11]. However, previous works [6] suggested that WALKSAT neighborhood is more suited for search with algorithms that handle only one solution at a time. This may also explain the advantage of local search algorithms and single-solution metaheuristics, over population-based metaheuristics for RMS.

6. LOCAL SEARCH VERSUS METAHEURISTICS: FIRST ANALYSIS

The obtained correlations suggest that RMS is non-globally convex when it is underconstrained or at the phase

[1] this was confirmed by the authors who studied instances with eight variables not reported here.

Table 2: Results for instances optimized with GREEDY and WALKSAT neighborhoods.

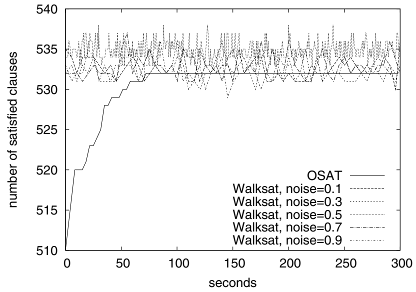
	GREEDY						WALKSAT					
	$\rho_{f,M}$	$\rho_{B,M}$	$\rho_{\Delta,M}$	$\rho_{\Delta,B}$	$\|M^*\|$	$\delta(M^*)$	$\rho_{f,M}$	$\rho_{B,M}$	$\rho_{\Delta,M}$	$\rho_{\Delta,B}$	$\|M^*\|$	$\delta(M^*)$
	Underconstrained						Underconstrained					
uf50-01	-0.37	0.94	0.362	0.453	875	22.522	-0.361	0.943	0.331	0.423	879	22.493
uf100-01	-0.412	0.931	0.325	0.406	1265	46.011	-0.408	0.933	0.322	0.401	1278	45.895
uf125-01	-0.386	0.955	0.322	0.365	857	57.191	-0.369	0.945	0.317	0.365	896	57.066
uf150-01	-0.348	0.908	0.326	0.395	549	68.136	-0.343	0.93	0.325	0.376	543	68.141
uf175-01	-0.347	0.948	0.333	0.377	527	80.249	-0.345	0.948	0.32	0.364	551	80.043
uf200-01	-0.333	0.909	0.35	0.408	227	92.941	-0.333	0.902	0.376	0.438	245	92.72
uf225-01	-0.336	0.926	0.332	0.383	310	103.621	-0.315	0.953	0.306	0.338	571	102.376
uf250-01	-0.343	0.927	0.325	0.377	466	113.330	-0.338	0.922	0.36	0.411	249	113.963
Average	**-0.359**	**0.93**	**0.334**	**0.395**	**634.5**	**73**	**-0.351**	**0.935**	**0.332**	**0.389**	**651.5**	**72.837**
uuf50-01	-0.418	0.958	0.424	0.486	824	22.652	-0.462	0.961	0.446	0.498	821	22.529
uuf100-01	-0.398	0.935	0.347	0.418	873	45.108	-0.394	0.921	0.361	0.446	857	45.185
uuf125-01	-0.305	0.906	0.276	0.353	709	57.044	-0.305	0.923	0.276	0.339	681	57.039
uuf150-01	-0.39	0.95	0.319	0.358	667	69.46	-0.386	0.944	0.319	0.368	656	69.545
uuf175-01	-0.33	0.913	0.343	0.403	339	80.212	-0.328	0.918	0.337	0.391	344	80.476
uuf200-01	-0.307	0.842	0.382	0.478	93	93.21	-0.307	0.817	0.372	0.489	85	93.617
uuf225-01	-0.319	0.916	0.386	0.445	221	102.617	-0.323	0.926	0.383	0.434	234	102.969
uuf250-01	-0.334	0.894	0.422	0.491	106	115.352	-0.329	0.892	0.394	0.456	109	115.309
Average	**-0.35**	**0.914**	**0.362**	**0.429**	**479**	**73.207**	**-0.354**	**0.913**	**0.361**	**0.428**	**473.375**	**73.334**
	Phase Transition						Phase Transition					
uf50-01	-0.565	0.746	0.764	0.983	5	19.7	-0.567	0.644	0.644	1	2	22
uf100-01	-0.456	0.643	0.606	0.82	4	41.833	-0.453	0.314	0.314	1	1	0
uf125-01	-0.471	0.339	0.339	1	1	0	-0.484	0.51	0.51	1	1	0
uf150-01	-0.527	0.554	0.554	1	1	0	-0.524	0.749	0.558	0.746	7	62.048
uf175-01	-0.392	0.347	0.347	1	1	0	-0.385	0.349	0.349	1	1	0
uf200-01	-0.542	0.47	0.47	1	1	0	-0.537	0.545	0.545	1	1	0
uf225-01	-0.497	0.763	0.6	0.772	8	97.643	-0.491	0.504	0.452	0.93	2	90
uf250-01	-0.555	0.551	0.551	1	1	0	-0.555	0.587	0.49	0.881	3	104.333
Average	**-0.501**	**0.552**	**0.529**	**0.947**	**2.750**	**19.897**	**-0.5**	**0.525**	**0.483**	**0.945**	**2.25**	**34.798**
uuf50-01	-0.522	0.529	0.531	0.958	2	22	-0.541	0.614	0.577	0.946	4	19.167
uuf100-01	-0.539	0.502	0.502	1	1	0	-0.542	0.738	0.601	0.846	4	40.333
uuf125-01	-0.498	0.744	0.579	0.797	9	57.333	-0.516	0.688	0.598	0.883	5	55.4
uuf150-01	-0.533	0.615	0.602	0.971	2	67	-0.515	0.76	0.554	0.773	9	63.889
uuf175-01	-0.479	0.709	0.599	0.869	4	76.167	-0.485	0.512	0.462	0.929	2	94
uuf200-01	-0.421	0.519	0.519	1	1	0	-0.42	0.408	0.408	1	1	0
uuf225-01	-0.429	0.538	0.415	0.843	2	104	-0.43	0.371	0.371	1	1	0
uuf250-01	-0.477	0.504	0.504	1	1	0	-0.475	0.503	0.503	1	1	0
Average	**-0.487**	**0.582**	**0.531**	**0.93**	**2.75**	**40.813**	**-0.491**	**0.574**	**0.509**	**0.922**	**3.375**	**34.099**
	Overconstrained						Overconstrained					
uf50-01	-0.771	0.883	0.871	0.944	67	17.158	-0.763	0.9	0.876	0.948	57	17.081
uf100-01	-0.487	0.574	0.533	0.946	2	42	-0.474	0.528	0.528	1	1	0
uf125-01	-0.662	0.597	0.597	1	1	0	-0.661	0.615	0.599	1	3	44.667
uf150-01	-0.528	0.633	0.633	1	1	0	-0.522	0.778	0.75	0.968	2	50
uf175-01	-0.567	0.58	0.58	1	1	0	-0.555	0.51	0.452	1	2	57
uf200-01	-0.547	0.525	0.525	1	1	0	-0.553	0.628	0.628	1	1	0
uf225-01	-0.5	0.447	0.447	1	1	0	-0.559	0.515	0.515	1	1	0
uf250-01	-0.542	0.623	0.623	1	1	0	-0.566	0.599	0.599	1	1	0
Average	**-0.575**	**0.608**	**0.601**	**0.986**	**9.375**	**7.395**	**-0.582**	**0.634**	**0.618**	**0.972**	**8.5**	**21.093**
uuf50-01	-0.409	0.578	0.468	0.891	10	20.111	-0.427	0.61	0.502	0.899	8	21.25
uuf100-01	-0.475	0.543	0.543	1	1	0	-0.464	0.321	0.323	0.998	2	31
uuf125-01	-0.518	0.381	0.381	1	1	0	-0.534	0.543	0.5	0.953	2	50
uuf150-01	-0.553	0.525	0.476	0.899	4	65.667	-0.549	0.28	0.28	1	1	0
uuf175-01	-0.557	0.593	0.593	1	1	0	-0.556	0.595	0.595	1	1	0
uuf200-01	-0.481	0.58	0.58	1	1	0	-0.472	0.496	0.496	1	1	0
uuf225-01	-0.558	0.495	0.495	1	1	0	-0.558	0.494	0.494	1	1	0
uuf250-01	-0.57	0.596	0.596	1	1	0	-0.575	0.619	0.619	1	1	0
Average	**-0.515**	**0.536**	**0.517**	**0.974**	**2.5**	**10.722**	**-0.517**	**0.495**	**0.476**	**0.981**	**2.125**	**12.781**

transition region. When the problem is overconstrained it is more globally convex, and population-based metaheuristics may be competitive.

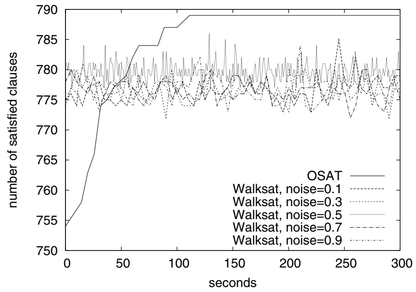
To prove this claim, two algorithms are compared: the well-known Walksat [10] and an Ant Colony Optimization (ACO) algorithm known as Omicron ACO [7], which was renamed as Omicron SAT (OSAT). The instance `uf125-01` was selected as benchmark given that correlations obtained for it were the most significant.

For the two solvers the execution time was limited to five minutes. Walksat (version v46) was run with a maximum of 10^6 flips per try, and different noise ratios. OSAT was experimentally tuned to the following parameters: 10 ants, Omicron=600, $\alpha = \beta = 1$, and a colony size of 15. Figure 2 shows a comparison between the solvers for the phase transition and overconstrained cases.

Walksat finds better assignments than OSAT at the phase transition region according to figure 2(a); meanwhile, OSAT beats Walksat at the overconstrained region according to figure 2(b). The best noise ratio for Walksat was 0.5. OSAT is more effective than Walksat at the overconstrained region because it is an elitist algorithm that exploits global convexity [7], and the selected instance presented good correlations. The test was repeated several times, always obtaining the same results. Therefore, efficient population-based metaheuristics may outperform local search algorithms at the



(a) phase transition



(b) overconstrained

Figure 2: Comparison between Walksat and OSAT on instance uf125-01.

overconstrained region where large correlations were found. The results for the underconstrained case showed that both algorithms are very efficient solving it, but Walksat was able to find a model faster than OSAT.

Note that OSAT and Walksat were selected to study to what extend the established hypotheses holds in a very simple context. It remains to be studied how better heuristics perform on larger overconstrained instances.

7. CONCLUSIONS AND FUTURE WORK

This work presented an experimental analysis of MAX-3-SAT solution space. Several Random-3-SAT instances from SATLIB were analyzed at the underconstrained, phase transition and overconstrained regions. Results suggest that underconstrained and phase transition instances are non-globally convex. However, some overconstrained instances exhibit the property of global convexity.

Global convexity is an important property that population-based metaheuristics can exploit in order to find good quality local optima [7]. Thus, the claim that population-based metaheuristics are effective compared to local search algorithms for overconstrained Random MAX-3-SAT was established. In that sense, a comparison between Walksat and OSAT (a population-based metaheuristic) showed that the previous hypothesis is true, at least experimentally in a very simple context[2]. Therefore, research on finding good population-based metaheuristics using the MAX-SAT objective function is, at first glance, discouraging for a general SAT problem. However, this paper presents good experimental results for overconstrained instances.

The generalization of these results to Constraint Satisfaction Problems will allow the design of new metaheuristics for this class of problems. Also, a global convexity analysis for structured instances will give more insight into the effectiveness of local search algorithms for SAT.

[2]A more systematic comparison was carried out in [12]

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