A Scalable Feedback-Based Approach to Distributed Nullforming

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Abstract. We present a novel approach to the problem of distributed nullforming where a set of transmitters cooperatively transmit a common message signal in such a way that their individual transmissions precisely cancel each other at a designated receiver. Under our approach, each transmitter iteratively makes an adjustment to the phase of its transmitted RF signal, by effectively implementing a gradient descent algorithm to reduce the amplitude of the overall received signal to zero. We show that this gradient search can be implemented in a purely distributed fashion at each transmitter assuming only that each transmitter has an estimate of its own channel gain to the receiver. This is an important advantage of our approach and assures its scalability; in contrast any non-iterative approach to the nullforming problem requires centralized knowledge of the channel gain of every transmitter. We prove analytically that the gradient search algorithm converges to a null at the designated receiver. We also present numerical simulations to illustrate the robustness of this approach.

Keywords: distributed nullforming, cooperative transmission, virtual antenna arrays.

1 Introduction

We consider the problem of distributed nullforming where a set of transmitters in a wireless network cooperatively transmit a common message signal in such a way that their individual transmissions cancel each other at a designated receiver. In effect the transmitters form a *virtual antenna array* and shape the array's antenna pattern to create a null at the desired location. The technique of distributed nullforming has many potential applications including interference avoidance for increased spatial spectrum reuse [1], cognitive radio [2], physical-layer security [3] and so on.

Distributed nullforming requires precise control of the amplitude and phase of the radio-frequency signal transmitted by each cooperating transmitter to ensure

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that they cancel each other. This is an extremely challenging problem because each transmitter usually obtains its RF signal from a separate local oscillator (LO), and signals obtained from different LOs invariably have Brownian motion driven phase drifts due to manufacturing tolerances and temperature variations. The nullforming algorithm must estimate, track and compensate for the effect of these drifts.

While the idea of cooperative communication has been studied for decades [4], the early work in this area neglected the RF synchronization issues that are crucial for the practical implementation of these ideas. Recently, however, there has been a significant amount of research activity on distributed transmit beamforming [5], [6], [7], including implementation on commodity hardware [8, 9].

While the synchronization techniques developed for distributed beamforming can be adapted for nullforming, there are two important differences that make nullforming significantly more challenging: (a) While beamforming gains are highly robust and insensitive to small phase errors (upto about 30 degrees [5]), nullforming is substantially more sensitive [13] to even modest errors. (b) One implication of this sensitivity to small phase errors is that the simple 1-bit feedback algorithm [10] that has proved to be effective for beamforming does not work for nullforming. However, we show in this paper that a gradient descent algorithm using multi-bit feedback similar to [11] works very well for nullforming. (c) For beamforming, each transmitter only needs the knowledge of the phase of its own transmitted signal at the receiver. In contrast for nullforming, the amplitude and phase of the transmitted signal at each node cannot be chosen independently of the amplitudes and phases of other nodes [13]. Nullforming essentially depends on a node's transmitted signal cancelling the signals from all other transmitters. Therefore state-of-the-art distributed nullforming algorithms, [12] and [13] assume that each transmitter knows *every transmitter's* complex channel gain to the receiver. This requirement poses a severe challenge for scalability.

In contrast to previous work on distributed nullforming [12, 13], in this paper we assume that each transmitter knows *only its own channel gain* to the null location. Using this in Section 2 we formulate our gradient descent based algorithm, in which each node adjusts its transmitted phase knowing only its channel gain, and a common feedback signal from the receiver at which the null is desired. This feedback signal is simply the complex baseband signal received by the receiver. Section 3 presents an analysis of the stability and convergence properties of the algorithm under simplifying assumptions. Section 4 provides simulations, that include the effect of channel phase offsets and oscillator drift. Section 5 concludes.

2 Scalable Algorithm for Nullforming

We now describe a scalable gradient descent algorithm for distributed nullforming in a node. As noted in the introduction, we assume that at the beginning of a nullforming epoch, each transmitter has access to its own complex channel gain to the receiver, using which it equalizes its channel to the receiver. This is in sharp contrast to [12] and [13] where each transmitter knows the Channel State Information (CSI) for every transmitter. We assume there are N transmitter nodes that have been synchronized in frequency, using the techniques of [12], [13] and [11].

Assume at time slot k, the i-th node transmits the baseband signal $e^{j\theta_i[k]}$. The total baseband signal at the receiver is thus:

$$s[k] = R[k] + jI[k], \tag{1}$$

where

$$R[k] = \sum_{i=1}^{N} r_i \cos\left(\theta_i[k] + \phi_i[k]\right), \tag{2}$$

$$I[k] = \sum_{i=1}^{N} r_i \sin\left(\theta_i[k] + \phi_i[k]\right), \tag{3}$$

r_i is the equalized channel gain from the i-th transmitter and $\phi_i[k]$ is a small uncompensated channel phase from the i-th transmitter. The receiver feeds back at each time slot the signal $s[k]$. Consequently, at each time slot the i-th transmitter has access to $R[k]$, $I[k]$, r_i and $\theta_i[k]$; $\phi_i[k]$ is not available to any one. Define, $\theta[k] = [\theta_1[k], \cdots, \theta_N[k]]^\top$. The total received power in the k-th time slot is:

$$J(\theta[k]) = I^2[k] + R^2[k]. \tag{4}$$

Throughout we make the following standing assumption:

Assumption 2.1. *The r_i are such that there is a choice of θ_i for which $J(\theta) = 0$.*

Since the r_i are equalized gains, each receiver can always choose its r_i to equal 1, ensuring the existence of a choice of θ_i that achieve the null mandated by Assumption 2.1. For a suitably small $\mu > 0$, in our algorithm the i-th transmitter updates its phase according to:

$$\theta_i[k+1] = \theta_i[k] + \mu r_i \left(\sin\left(\theta_i[k]\right) R[k] - \cos\left(\theta_i[k]\right) I[k]\right). \tag{5}$$

Few features are of note. The algorithm is totally distributed, as each node only needs the common feedback signal $s[k]$ and r_i and $\theta_i[k]$, to implement it. This contrasts with [12], [13] where much more information is needed. Second, suppose in vector form the algorithm were expressed as:

$$\theta[k+1] = \theta[k] - f[k]. \tag{6}$$

Then, when the phase offsets $\phi_i[k]$ are all zero, the $f[k]$ corresponding to (5) is simply:

$$f[k] = \mu \left.\frac{\partial J(\theta)}{\partial \theta}\right|_{\theta=\theta[k]}. \tag{7}$$

In other words the algorithm attempts the gradient descent minimization of the received power. Finally, the fact that the algorithm works from a common feedback signal supplied by the receiver, makes it *totally scalable* as the feedback overhead does not grow with the size of the transmitter array.

3 Stability

Our stability analysis will be conducted under the idealized assumption of no noise and zero $\phi_i[k]$. The underlying philosophy is driven by total stability theory, [14], that states in essence that should the algorithm uniformly converge to desired stationary points in the idealized (zero noise, zero ϕ_i) case, uniformity being with respect to the initial time, then it will exhibit robustness to noise and small ϕ_i. Indeed we will demonstrate the *practical* uniform convergence of (5) under the following assumption:

Assumption 3.1. *In (2) and (3) for all* $i \in \{1, \cdots, N\}$ *and all* k, $\phi_i[k] = 0$.

Let us clarify what we mean by *practical* uniform convergence. As will be evident from the sequel, under Assumption 3.1 the algorithm in (5) has entire manifolds of stationary points at least to one of which the algorithm converges uniformly. Some stationary correspond to nulls. The rest, which we dub as being *spurious*, do not. We will show that the latter are locally unstable. Thus they are rarely attained, and even if attained not practically maintained as the slightest noise would drive the phase trajectories away from them. Thus, by showing the local stability of the stationary points corresponding to nulls, we would have demonstrated the practical uniform converence of the algorithm to a null.

We relax Assumption 3.1 to permit non-zero *but constant* ϕ_i. Under these conditions from (5) we obtain that the stationary points fall into the following categories. (A) $R[k] = I[k] = 0$. (B) If $R[k] \neq 0$, then for all i, $\tan\theta_i[k] = \frac{I[k]}{R[k]}$. (C) If $I[k] \neq 0$, then for all i, $\cot\theta_i[k] = \frac{R[k]}{I[k]}$. Clearly [A] corresponds to stationary points reflecting nulls. Both [B] and [C] reflect the condition that for all i, l, $\tan\theta_i = \tan\theta_l$. Some of these may still correspond to nulls. The rest are spurious.

We will now invoke Assumption 3.1. We have the following Theorem.

Theorem 3.1. *Under Assumption 3.1, (2), (3), (5) and (4), there exists a* $\mu^* > 0$, *such that for all* $0 < \mu < \mu^*$, $\theta[k]$ *converges uniformly to one of the stationary points in (A-C) above.*

Standard theory shows that the local instability of the algorithm in (5) is assured if the algorithm linearized around that stationary point has poles outside the unit circle. Under 3.1 this in turn is assured if the Hessian of $J(\cdot)$ evaluated at such a stationary point has a negative eigenvalue. As under (B,C) all off diagonal elents of Hessian are ± 1, this is in turn assured by the Hessian evaluated at such a stationary point having a nonpositive diagonal element. The (i, l)-th element of such an Hessian obeys:

$$[H(\theta)]_{il} = \begin{cases} -2\sum_{m\neq i}^{N} \cos(\theta_i - \theta_m) & i = l \\ 2\cos(\theta_i - \theta_l) & i \neq l \end{cases}$$

It is readily seen that for arbitrary $N \geq 2$ at least one diagonal element is negative or zero.

Thus practical uniform convergence is guaranteed by showing that all stationary points corresponding to a true null are locally stable. To this end we must examine the Hessian at these points corresponding nulls. Assumption 2.1 guarantees the existing of stationary points. Under Assumption 3.1 at a stationary point corresponding to a null, i.e. when $R = I = 0$, there holds:

$$[H(\theta)]_{il} = \begin{cases} 2 & i = l \\ 2\cos(\theta_i - \theta_l) & i \neq l \end{cases}$$

It is readily seen that at such a stationary point, with $c = \left[\cos\theta_1 \cdots \cos\theta_N\right]^\top$ and $s = \left[\sin\theta_1 \cdots \sin\theta_N\right]^T$ the Hessian is $2cc^\top + 2ss^\top$. Thus the Hessian evaluated at a null is postive semidefinite, but with rank at most 2. There are several zero eigenvalues of the Hessian. Using as we did in [15], center manifold theory, one can nonetheless show that these stationary points are indeed locally stable. The proof being complicated is omitted. This thus proves the practical uniform convergence of (5) to a null is guaranteed under Assumptions 2.1 and 3.1.

4 Simulations

We now provide simulations that attest to the efficacy of the algorithm. All simulations involve 10 transmitters. In the following discussion, SNR is defined as the ratio of the per-node received power to the noise power.

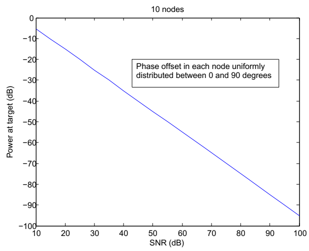

Fig. 1. Power at null target vs. SNR

Fig.1 shows a simulation plot of time-averaged total power at null target as a function of SNR when there are no phase drifts at the oscillators, but each of the ten transmitters sees a phase offset ϕ_i, that is uniformly distributed between 0 and $\pi/2$. The SNR limits the accuracy of the individual phase estimate and this in turn leads to fluctuations in the estimated gradient and therefore the overall received signal strength at the null target. As expected the power at the null target decreases monotonically with increase in SNR.

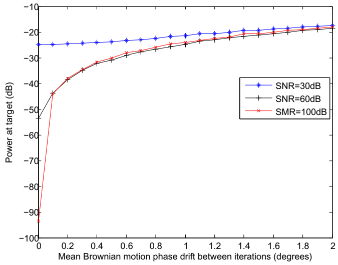

Fig. 2. Power at null target vs. phase drift for equal channel gains

Fig. 2 shows the variation of time-averaged total power at null target as a function of the Brownian motion phase drift for different SNRs. It can be seen that for very small Brownian motion drifts, the null power is determined by the SNR. However once drift increases to about a tenth of a degree between two iterations of the gradient descent, the null is largely limited by the drift and is more or less independent of the SNR. Observe that the highest phase drift of two degrees between phase updates corresponds to the very low feedback rate of 5 Hz, for even the cheapest of oscillators.

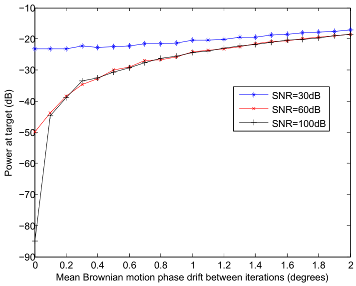

Fig. 3. Power at null target vs. phase drift for unequal channel gains

Fig. 3 is very similar to Fig. 2, except that unlike Fig. 2, that involves a setting where all gains are 1, in Fig. 3 the actual gains are obtained from a Rayleigh distribution and then equalized to one. As can be seen Fig. 3, the resulting potential noise amplification, has virtually no effect on the performance of the gradient descent nullforming algorithm.

5 Conclusion

We have provided a new gradient descent based distributed nullforming algorithm that requires far less feedback than all its predecessors, in that each transmitter is required by this algorithm to only know its channel state information to the receiver. In constrast, previous algorithms required that channel state information to the receiver from each transmitter be known to each other transmitter in the virtual array. This coupled with the fact that it requires an additional *common* signal fed back by to all transmitters by the receiver, ensures its scalability. We have proved practical uniform convergence of the algorithm to a null. This ensures robustness to noise and channel phase estimation errors., verified by simulations, that involve nontrivial channel phase estimation errors compounded by Brownian motion driven oscillator drift.

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