On Generalized Nega–Hadamard Transform

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Abstract. In this paper, we consider generalized Boolean functions from $\mathbb{Z}_2^n$ to $\mathbb{Z}_q$ ($q \geq 2$, a positive integer). Here, we present some of the properties of generalized nega–Hadamard transform which are analogous to nega–Hadamard transform. Further, it is shown that if we represent a generalized Boolean function in terms of Boolean functions then there is a relation between their nega–Hadamard transforms.

Keywords: Generalized Boolean functions, nega–Hadamard transform, generalized nega–Hadamard transform.

1 Introduction

In the recent years, several generalizations of Boolean functions have been proposed and the effect of Walsh–Hadamard transform on them has been studied by various authors [3,9,14,15,11]. The nega-Hadamard transforms and negabent functions have been discussed [6,7,8,10,12,13]. Like the Boolean case, in the generalized setup, the functions which have flat spectra with respect to nega–Hadamard transform are said to be generalized negabent functions.

Let us list the notations:
$\mathbb{Z}$, $\mathbb{R}$ and $\mathbb{C}$ denote the set of integers, real numbers and complex numbers respectively;
q and n are positive integers;
'$+$' denotes the addition modulo q;
'$\oplus$' denotes the addition modulo 2;
$\mathbb{Z}_2$ is the prime field of order 2;
$\mathbb{Z}_2^n$ is the n-dimensional vector space over field $\mathbb{Z}_2$;
$\mathbb{Z}_q$ is the ring of integers modulo q;
$\mathbf{x} = (x_1, x_2, \cdots, x_n)$ is an element of $\mathbb{Z}_2^n$;
$< \mathbf{x}, \mathbf{y} >= x_1y_1 \oplus x_2y_2 \oplus \cdots \oplus x_ny_n$ is the inner product of vectors;
$\mathbf{x} * \mathbf{y} = (x_1y_1, \ldots, x_ny_n)$ is the intersection of two vectors.

The cardinality of the set S is denoted by $|S|$. If $z = a + b\imath \in \mathbb{C}$, then $|z| = \sqrt{a^2 + b^2}$ denotes the absolute value of z, and $\overline{z} = a - b\imath$ denotes the complex

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conjugate of z, where $\imath^2 = -1$, and $a, b \in \mathbb{R}$. The conjugate of a bit b will also be denoted by $\bar{b}$.

A function from $\mathbb{Z}_2^n$ to $\mathbb{Z}_2$ is said to be a Boolean function on n variables and the set of all such functions is denoted by $\mathcal{B}_n$. For more details on Boolean functions one may refer to [1,2,4]. A function from $\mathbb{Z}_2^n$ to $\mathbb{Z}_q$ ($q \geq 2$, a positive integer) is said to be a *generalized Boolean function* on n variables. In this paper, we are considering the generalization of Schmidt [9]. We denote the set of all such functions by $\mathcal{B}_n^q$.
Some basic definitions are given below:

Definition 1. *The (generalized)* Walsh–Hadamard transform *of $f \in \mathcal{B}_n^q$ at any point $\mathbf{u} \in \mathbb{Z}_2^n$ is the complex valued function defined by*

$$\mathcal{H}_f(\mathbf{u}) = 2^{-\frac{n}{2}} \sum_{\mathbf{x} \in \mathbb{Z}_2^n} \zeta^{f(\mathbf{x})} (-1)^{\mathbf{u} \cdot \mathbf{x}},$$

where $\zeta = e^{\frac{2\pi\imath}{q}}$ is the q-th primitive root of unity. A function $f \in \mathcal{B}_n^q$ is said to be generalized bent *if and only if $|\mathcal{H}_f(\mathbf{u})| = 1$ for all $\mathbf{u} \in \mathbb{Z}_2^n$.*

Definition 2. *The* nega–Hadamard transform *of $f \in \mathcal{B}_n$ at any vector $\mathbf{u} \in \mathbb{Z}_2^n$ is the complex valued function*

$$\mathcal{N}_f(\mathbf{u}) = 2^{-\frac{n}{2}} \sum_{\mathbf{x} \in \mathbb{Z}_2^n} (-1)^{f(\mathbf{x}) \oplus \mathbf{u} \cdot \mathbf{x}} \imath^{wt(\mathbf{x})}.$$

A function $f \in \mathcal{B}_n$ is said to be negabent *if and only if $|\mathcal{N}_f(\mathbf{u})| = 1$ for all $\mathbf{u} \in \mathbb{Z}_2^n$.*

We define generalized nega–Hadamard transform and generalized negabent function in the following manner:

Definition 3. *The* generalized nega–Hadamard transform *of $f \in \mathcal{B}_n^q$ at any point $\mathbf{u} \in \mathbb{Z}_2^n$ is defined by*

$$\mathcal{N}_f^q(\mathbf{u}) = 2^{-\frac{n}{2}} \sum_{\mathbf{x} \in \mathbb{Z}_2^n} \zeta^{f(\mathbf{x})} (-1)^{\mathbf{u} \cdot \mathbf{x}} \imath^{wt(\mathbf{x})}.$$

Definition 4. *A function $f \in \mathcal{B}_n^q$ is a* generalized negabent function *if $|\mathcal{N}_f^q(\mathbf{u})| = 1$ for all $\mathbf{u} \in \mathbb{Z}_2^n$.*

We recall the following result:

Lemma 1 ([10], Lemma 1). *For any $\mathbf{u} \in \mathbb{F}_2^n$, we have*

$$\sum_{\mathbf{x} \in \mathbb{F}_2^n} (-1)^{\mathbf{u} \cdot \mathbf{x}} \imath^{wt(\mathbf{x})} = 2^{\frac{n}{2}} \omega^n \imath^{-wt(\mathbf{u})}, \tag{1}$$

where $\omega = \frac{1+\imath}{\sqrt{2}}$ is a primitive 8th root of unity.

We shall also use the following well-known identities

$$wt(\mathbf{x} \oplus \mathbf{y}) = wt(\mathbf{x}) + wt(\mathbf{y}) - 2wt(\mathbf{x} * \mathbf{y}) \qquad \text{(given in [5])} \tag{2}$$

and

$$\sum_{\mathbf{x}} (-1)^{\mathbf{v}\cdot\mathbf{x}} = \begin{cases} 2^n & \text{if } \mathbf{v} = \mathbf{0} \\ 0 & \text{if } \mathbf{v} \neq \mathbf{0}, \end{cases} \qquad \text{(see [2, p.8])} \tag{3}$$

In this paper, we prove various results in Section 2 on the behavior of the generalized nega–Hadamard transform on affine functions and sums of functions.

In Section 3, we show that if we represent a generalized Boolean function in terms of Boolean functions then there is a relation between their nega–Hadamard transform.

2 Properties of Generalized Nega–Hadamard Transform

Stănică et al. [12,13] investigated various properties of nega–Hadamard transform.
The following theorem gives the generalized nega–Hadamard transform of various combinations of generalized Boolean functions.

Theorem 1. *Let f, g, h be in $\mathcal{B}_n^q$. The following statements are true:*

(*a*) *For any affine function $\ell_{\mathbf{a},c}(\mathbf{x}) = \left(\frac{q}{2}\right) \mathbf{a}\cdot\mathbf{x} + c$ and $f \in \mathcal{B}_n^q$, $\mathcal{N}^q_{f+\ell_{\mathbf{a},c}}(\mathbf{u}) = \zeta^c \mathcal{N}^q_f(\mathbf{a}\oplus\mathbf{u})$. Further, $\mathcal{N}^q_{\ell_{\mathbf{a},c}}(\mathbf{u}) = \zeta^c\,\omega^n\,\imath^{-wt(\mathbf{a}\oplus\mathbf{u})}$.*

(*b*) *If $h(\mathbf{x}) = f(\mathbf{x}) + g(\mathbf{x})$ on $\mathbb{Z}_2^n$, then for $\mathbf{u} \in \mathbb{Z}_2^n$,*

$$\mathcal{N}^q_h(\mathbf{u}) = 2^{-\frac{n}{2}} \sum_{\mathbf{v}\in\mathbb{Z}_2^n} \mathcal{N}^q_f(\mathbf{v})\mathcal{H}_g(\mathbf{u}\oplus\mathbf{v}) = 2^{-\frac{n}{2}} \sum_{\mathbf{v}\in\mathbb{Z}_2^n} \mathcal{H}_f(\mathbf{v})\mathcal{N}^q_g(\mathbf{u}\oplus\mathbf{v}).$$

(*c*) *If $h(\mathbf{x},\mathbf{y}) = f(\mathbf{x}) + g(\mathbf{y})$, $\mathbf{x},\mathbf{y} \in \mathbb{Z}_2^n$, then $\mathcal{N}^q_{f+g}(\mathbf{u},\mathbf{v}) = \mathcal{N}^q_f(\mathbf{u})\mathcal{N}^q_g(\mathbf{v})$.*

(*d*) *If $h(\mathbf{x}) = f(A\mathbf{x}\oplus\mathbf{a})$, then $\mathcal{N}^q_h(\mathbf{u}) = (-1)^{\mathbf{a}\cdot(A\mathbf{u})}\,\imath^{wt(\mathbf{a})}\,\mathcal{N}^q_f(A\mathbf{u}\oplus\mathbf{a})$, where A is an $n\times n$ orthogonal matrix over $\mathbb{Z}_2$ (and so, $A^TA = I_n$).*

Proof. (*a*) We obtain

$$\begin{aligned}
\mathcal{N}^q_{f+\ell_{\mathbf{a},c}}(\mathbf{u}) &= 2^{-\frac{n}{2}} \sum_{\mathbf{x}\in\mathbb{Z}_2^n} \zeta^{(f+\ell_{\mathbf{a},c})(\mathbf{x})}(-1)^{\mathbf{u}\cdot\mathbf{x}}\,\imath^{wt(\mathbf{x})} \\
&= 2^{-\frac{n}{2}} \sum_{\mathbf{x}\in\mathbb{Z}_2^n} \zeta^{f(\mathbf{x})+\left(\frac{q}{2}\right)\mathbf{a}\cdot\mathbf{x}+c}\,(-1)^{\mathbf{u}\cdot\mathbf{x}}\,\imath^{wt(\mathbf{x})} \\
&= 2^{-\frac{n}{2}}\,\zeta^c \sum_{\mathbf{x}\in\mathbb{Z}_2^n} \zeta^{f(\mathbf{x})}(-1)^{(\mathbf{a}\oplus\mathbf{u})\cdot\mathbf{x}}\,\imath^{wt(\mathbf{x})} \\
&= \zeta^c\,\mathcal{N}^q_f(\mathbf{a}\oplus\mathbf{u}).
\end{aligned}$$

Further,

$$\mathcal{N}^q_{\ell_{\mathbf{a},c}}(\mathbf{u}) = 2^{-\frac{n}{2}} \sum_{\mathbf{x}\in\mathbb{Z}_2^n} \zeta^{\ell_{\mathbf{a},c}(\mathbf{x})}(-1)^{\mathbf{u}\cdot\mathbf{x}}\,\imath^{wt(\mathbf{x})}$$
$$= 2^{-\frac{n}{2}} \sum_{\mathbf{x}\in\mathbb{Z}_2^n} \zeta^{\frac{q}{2}(\mathbf{a}\cdot\mathbf{x})+c}\,(-1)^{\mathbf{u}\cdot\mathbf{x}}\,\imath^{wt(\mathbf{x})}$$
$$\mathcal{N}^q_{\ell_{\mathbf{a},c}}(\mathbf{u}) = 2^{-\frac{n}{2}}\,\zeta^c \sum_{\mathbf{x}\in\mathbb{Z}_2^n} (-1)^{(\mathbf{a}\oplus\mathbf{u})\cdot\mathbf{x}}\,\imath^{wt(\mathbf{x})}$$
$$= \zeta^c\,\omega^n\,\imath^{-wt(\mathbf{a}\oplus\mathbf{u})} \qquad \text{(using (1))}.$$

Here we show the first identity of (b). Since

$$\mathcal{N}^q_f(\mathbf{v}) = 2^{-\frac{n}{2}} \sum_{\mathbf{x}\in\mathbb{Z}_2^n} \zeta^{f(\mathbf{x})}\,(-1)^{\mathbf{v}\cdot\mathbf{x}}\,\imath^{wt(\mathbf{x})}$$
$$\mathcal{H}_g(\mathbf{u}\oplus\mathbf{v}) = 2^{-\frac{n}{2}} \sum_{\mathbf{y}\in\mathbb{Z}_2^n} \zeta^{g(\mathbf{y})}\,(-1)^{\mathbf{y}\cdot(\mathbf{u}\oplus\mathbf{v})}$$

we obtain (sums are over $\mathbb{Z}_2^n$)

$$\sum_{\mathbf{v}} \mathcal{N}^q_f(\mathbf{v})\mathcal{H}_g(\mathbf{u}\oplus\mathbf{v}) = 2^{-n} \sum_{\mathbf{v},\mathbf{x},\mathbf{y}} \zeta^{f(\mathbf{x})+g(\mathbf{y})}\,(-1)^{\mathbf{v}\cdot\mathbf{x}\oplus\mathbf{y}\cdot(\mathbf{u}\oplus\mathbf{v})}\,\imath^{wt(\mathbf{x})}$$
$$= 2^{-n} \sum_{\mathbf{x},\mathbf{y}} \zeta^{f(\mathbf{x})+g(\mathbf{y})}\,(-1)^{\mathbf{u}\cdot\mathbf{y}}\,\imath^{wt(\mathbf{x})} \sum_{\mathbf{v}} (-1)^{\mathbf{v}\cdot(\mathbf{x}\oplus\mathbf{y})}$$
$$= \sum_{\mathbf{x}} \zeta^{f(\mathbf{x})+g(\mathbf{x})}\,(-1)^{\mathbf{u}\cdot\mathbf{x}}\,\imath^{wt(\mathbf{x})} \qquad \text{(using (3))}$$
$$= 2^{\frac{n}{2}} \mathcal{N}^q_{f+g}(\mathbf{u}).$$

Similarly we can prove second identity.

(c) If $h(\mathbf{x},\mathbf{y}) = f(\mathbf{x}) + g(\mathbf{y})$, where $\mathbf{x}$, $\mathbf{y} \in \mathbb{F}_2^n$.
We obtain (sums are over $\mathbb{Z}_2^n$)

$$\mathcal{N}^q_h(\mathbf{u},\mathbf{v}) = 2^{-n} \sum_{\mathbf{x},\mathbf{y}} \zeta^{h(\mathbf{x},\mathbf{y})}(-1)^{\mathbf{u}\cdot\mathbf{x}\oplus\mathbf{v}\cdot\mathbf{y}}\,\imath^{wt(\mathbf{x})+wt(\mathbf{y})}$$
$$= 2^{-\frac{n}{2}} \sum_{\mathbf{x}} \zeta^{f(\mathbf{x})}(-1)^{\mathbf{u}\cdot\mathbf{x}}\,\imath^{wt(\mathbf{x})}\, 2^{-\frac{n}{2}} \sum_{\mathbf{y}} \zeta^{g(\mathbf{y})}(-1)^{\mathbf{v}\cdot\mathbf{y}}\,\imath^{wt(\mathbf{y})}$$
$$= \mathcal{N}^q_f(\mathbf{u})\,\mathcal{N}^q_g(\mathbf{v}).$$

To show (d), we compute, for $h(\mathbf{x}) = f(A\mathbf{x}\oplus\mathbf{a})$, where A is an $n\times n$ orthogonal matrix over $\mathbb{Z}_2$.
We compute (sums are over $\mathbb{Z}_2^n$)

$$\mathcal{N}^q_h(\mathbf{u}) = 2^{-\frac{n}{2}} \sum_{\mathbf{x}} \zeta^{h(\mathbf{x})}\,(-1)^{\mathbf{u}\cdot\mathbf{x}}\,\imath^{wt(\mathbf{x})}$$
$$= 2^{-\frac{n}{2}} \sum_{\mathbf{x}} \zeta^{f(A\mathbf{x}\oplus\mathbf{a})}\,(-1)^{\mathbf{u}\cdot\mathbf{x}}\,\imath^{wt(\mathbf{x})}$$

$$= 2^{-\frac{n}{2}} \sum_{\mathbf{y}} \zeta^{f(\mathbf{y})} (-1)^{\mathbf{u}\cdot A^T(\mathbf{y}\oplus\mathbf{a})} \imath^{wt(A^T(\mathbf{y}\oplus\mathbf{a}))}$$

$$= 2^{-\frac{n}{2}} \sum_{\mathbf{y}} \zeta^{f(\mathbf{y})} (-1)^{A\mathbf{u}\cdot\mathbf{y}+A\mathbf{u}\cdot\mathbf{a}} \imath^{wt(\mathbf{y})+wt(\mathbf{a})-2wt(\mathbf{y}*\mathbf{a})} \quad \text{(using (2))}$$

$$\mathcal{N}_h^q(\mathbf{u}) = 2^{-\frac{n}{2}} (-1)^{A\mathbf{u}\cdot\mathbf{a}} \imath^{wt(\mathbf{a})} \sum_{\mathbf{y}} \zeta^{f(\mathbf{y})} (-1)^{(A\mathbf{u}\oplus\mathbf{a})\cdot\mathbf{y}} \imath^{wt(\mathbf{y})}$$

$$= (-1)^{A\mathbf{u}\cdot\mathbf{a}} \imath^{wt(\mathbf{a})} \mathcal{N}_f^q(A\mathbf{u}\oplus\mathbf{a}).$$

Since $\imath^{2wt(\mathbf{y}*\mathbf{a})} = (-1)^{\mathbf{y}\cdot\mathbf{a}}$ and $wt(A^T(\mathbf{y}\oplus\mathbf{a})) = wt(\mathbf{y}\oplus\mathbf{a})$ (here we needed the orthogonality of A, since it preserves vectors lengths).

3 Relation between Generalized Nega–Hadamard Transform and Nega–Hadamard Transform

In this section, the generalized Boolean function $f : \mathbb{Z}_2^{2n} \to \mathbb{Z}_4$ is considered.

Generalized Boolean function can be represented by combination of Boolean functions. In the following theorem, it is shown that the generalized nega–Hadamard transform can be derived from the nega–Hadamard transform of Boolean functions.

Theorem 2. *Let $f : \mathbb{Z}_2^{2n} \to \mathbb{Z}_4$ be any generalized Boolean function. Represented it as $f(\mathbf{x},\mathbf{y}) = a(\mathbf{x},\mathbf{y}) + 2b(\mathbf{x},\mathbf{y})$; for any $\mathbf{x},\mathbf{y} \in \mathbb{Z}_2^n$, where $a, b : \mathbb{Z}_2^{2n} \to \mathbb{Z}_2$ are Boolean functions. Between nega–Hadamard transforms of $f, a+b, b$ there is the relation*

$$\mathcal{N}_f^4 = \frac{1}{2}\left[\mathcal{N}_b(\mathbf{u},\mathbf{v}) + \mathcal{N}_{a+b}(\mathbf{u},\mathbf{v})\right] + \frac{i}{2}\left[\mathcal{N}_b(\mathbf{u},\mathbf{v}) - \mathcal{N}_{a+b}(\mathbf{u},\mathbf{v})\right]$$

and

$$|\mathcal{N}_f^4(u,v)|^2 = \frac{1}{2}|\mathcal{N}_b(\mathbf{u},\mathbf{v}) - \imath\mathcal{N}_{a+b}(\mathbf{u},\mathbf{v})|^2$$

Proof.

$$\mathcal{N}_f^4(u,v) = 2^{-n} \sum_{\mathbf{x},\mathbf{y}\in\mathbb{Z}_2^n} \imath^{a(\mathbf{x},\mathbf{y})+2b(\mathbf{x},\mathbf{y})} (-1)^{\mathbf{u}\cdot\mathbf{x}+\mathbf{v}\cdot\mathbf{y}} \imath^{wt(\mathbf{x},\mathbf{y})}$$

$$= 2^{-n} \sum_{\mathbf{x},\mathbf{y}\in\mathbb{Z}_2^n} \imath^{a(\mathbf{x},\mathbf{y})} (-1)^{\mathbf{u}\cdot\mathbf{x}+\mathbf{v}\cdot\mathbf{y}+b(\mathbf{x},\mathbf{y})} \imath^{wt(\mathbf{x},\mathbf{y})}$$

Applying the formula $\imath^s = \frac{1+(-1)^s}{2} + \left(\frac{1-(-1)^s}{2}\right)\imath$,

for $s = a(\mathbf{x},\mathbf{y})$, We have

$$\mathcal{N}_f^4(u,v) = \frac{2^{-n}}{2} \sum_{\mathbf{x},\mathbf{y}\in\mathbb{Z}_2^n} \left[1+(-1)^{a(\mathbf{x},\mathbf{y})} + \imath\left(1-(-1)^{a(\mathbf{x},\mathbf{y})}\right)\right] (-1)^{\mathbf{u}\cdot\mathbf{x}+\mathbf{v}\cdot\mathbf{y}+b(\mathbf{x},\mathbf{y})} \imath^{wt(\mathbf{x},\mathbf{y})}$$

$$
\begin{aligned}
&= 2^{-(n+1)} \sum_{\mathbf{x},\mathbf{y}\in\mathbb{Z}_2^n} (-1)^{b(\mathbf{x},\mathbf{y})+\mathbf{u}\cdot\mathbf{x}+\mathbf{v}\cdot\mathbf{y}}\, \imath^{wt(\mathbf{x},\mathbf{y})} \\
&\quad +2^{-(n+1)} \sum_{\mathbf{x},\mathbf{y}\in\mathbb{Z}_2^n} (-1)^{a(\mathbf{x},\mathbf{y})+b(\mathbf{x},\mathbf{y})+\mathbf{u}\cdot\mathbf{x}+\mathbf{v}\cdot\mathbf{y}}\, \imath^{wt(\mathbf{x},\mathbf{y})} \\
&\quad +\imath\, 2^{-(n+1)} \sum_{\mathbf{x},\mathbf{y}\in\mathbb{Z}_2^n} (-1)^{b(\mathbf{x},\mathbf{y})+\mathbf{u}\cdot\mathbf{x}+\mathbf{v}\cdot\mathbf{y}}\, \imath^{wt(\mathbf{x},\mathbf{y})} \\
&\quad -\imath\, 2^{-(n+1)} \sum_{\mathbf{x},\mathbf{y}\in\mathbb{Z}_2^n} (-1)^{a(\mathbf{x},\mathbf{y})+b(\mathbf{x},\mathbf{y})+\mathbf{u}\cdot\mathbf{x}+\mathbf{v}\cdot\mathbf{y}}\, \imath^{wt(\mathbf{x},\mathbf{y})} \\
\mathcal{N}_f^4(u,v) &= \frac{1}{2}\left[\mathcal{N}_b(\mathbf{u},\mathbf{v}) + \mathcal{N}_{a+b}(\mathbf{u},\mathbf{v})\right] + \frac{\imath}{2}\left[\mathcal{N}_b(\mathbf{u},\mathbf{v}) - \mathcal{N}_{a+b}(\mathbf{u},\mathbf{v})\right] \\
|\mathcal{N}_f^4(u,v)|^2 &= \frac{1}{2}\left(|\mathcal{N}_b(\mathbf{u},\mathbf{v})|^2 + |\mathcal{N}_{a+b}(\mathbf{u},\mathbf{v})|^2\right) \\
&\quad + \frac{\imath}{2}\left(\mathcal{N}_b(\mathbf{u},\mathbf{v})\overline{\mathcal{N}_{a+b}(\mathbf{u},\mathbf{v})} - \mathcal{N}_{a+b}(\mathbf{u},\mathbf{v})\overline{\mathcal{N}_b(\mathbf{u},\mathbf{v})}\right) \\
|\mathcal{N}_f^4(u,v)|^2 &= \frac{1}{2}|\mathcal{N}_b(\mathbf{u},\mathbf{v}) - \imath\mathcal{N}_{a+b}(\mathbf{u},\mathbf{v})|^2
\end{aligned}
$$

4 Conclusion

In this paper, we have investigated some properties of generalized nega–Hadamard transfom. Moreover, it is shown that if a generalized Boolean function is represented by the combination of Boolean functions, there is the relation between their nega–Hadamard transform.

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