Reduced Complexity Pseudo-fractional Adaptive Algorithm with Variable Tap-Length Selection

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Abstract. The structural complexity and overall performance of the adaptive filter depend on its structure. The number of taps is one of the most important structural parameters of the liner adaptive filter. In practice the system length is not known a-priori and has to be estimated from the knowledge of the input and output signals. In a system identification framework the tap length estimation algorithm automatically adapts the filter order to the desired optimum value which makes the variable order adaptive filter a best identifier of the unknown plant. In this paper an improved pseudo-fractional tap-length selection algorithm has been proposed to find out the optimum tap-length which best balances the complexity and steady state performance. Simulation results reveal that the proposed algorithm results in reduced complexity and faster convergence in comparison to existing tap-length learning methods.

Keywords: Adaptive filter, tap-length, structure adaptation, least mean square (LMS), system identification, mean square error (MSE).

1 Introduction

Inherent stability and tapped delay line (TDL) feed forward structure makes finite impulse response (FIR) adaptive filter widely popular than its infinite impulse response (IIR) counterpart [1], [2]. IIR system is preferred due to its less computational complexity [1]. An FIR system has a finite impulse so an ideal tap-length can be selected to match the system order in an identification application. Whereas the infinity impulse response of an IIR system used in a system identification framework makes the tap-length of the adaptive filter critical one to best adjust the system performance. The performance of the TDL structure of the adaptive filter in which the weights/tap-coefficients are recursively updated by adaptive algorithm such as the least mean square (LMS), recursive least square (RLS) is highly affected by the filter order or in other words the tap-length selection [3]. The LMS algorithm has been extensively used in many applications because of its simplicity and robustness [1]. A too short order filter results in inefficient model of the system and increases the mean square error (MSE) [4], [5]. In principle minimum MSE (MMSE) is a monotonic non increasing function of the filter order but it is not advisable to have a too long order filter as it introduce adaptation noise and extra

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complexity due to more taps [6]-[9]. Therefore to balance the adaptive filter performance and complexity there should be an optimum order of the filter. More relevant work was proposed in [10] where the filter is partitioned into segments and order is adjusted by one segment either being added or removed from the filter according to the difference of the output errors from the last two segments. This algorithm suffers from the drawback of carefully selecting the segment parameters and use of absolute error rather than MSE i.e. to solve the problem of suitable tap-length estimation it creates another issue of selecting the proper length of the segment.

The fractional tap-length LMS (FT-LMS) algorithm was first proposed in [6]-[8] relaxing the constraint that the filter order must be an integer. This fractional order estimation procedure retains the advantage of both segmented filter and gradient decent algorithm and has less complexity than the previously proposed methods. But it suffers from noise level and parameter variation due to unconstrained and random use of the leaky factor and step size used for order adaptation [11], [12]. An improved variable tap-length variable step LMS (VT-VLMS) algorithm produces better convergence and steady state error performance than the FT-LMS algorithm [11], [13]. But it depends on a careful selection of leaky factor which controls the overall tap-length adaptation. This algorithm [11], [13] is found to be more suitable for the echo cancellation applications with the parameter guidelines it suggest. In this paper a new gradient search method based on the pseudo-fractional order estimation technique is proposed which finds the optimum filter order dynamically with a modified tap-length learning procedure. The filter order can be increased to and decreased from any value to achieve the desired tap-length for structure adaptation. There should be a trade-off between a suitable steady state tap-length and convergence rate. The steady state performance analysis of the proposed algorithm shows the importance of variable error width parameter. The proposed algorithm shows better performance both in convergence as well as MSE in comparison to the famous FT-LMS [6] and VT-VLMS algorithm [11]. It reduces the overall design complexity and hence proves to be a cost saving design.

The paper is organized as follows. The tap-length optimization with an improved pseudo-fractional tap-length selection algorithm has been proposed in Section-2. In Section-3 the computer simulation setup has been designed both for FIR and IIR system identification frameworks. The results and discussion are given in Section-4.

2 Pseudo-fractional Tap-Length Optimization

Selecting the tap-length for any system identification frame work is not a trivial task. The selection depends on the nature of the system to be identified, memory requirement, desired performance, computational complexity, noise level and parameter variation etc. For example, in a multiuser acoustic echo cancellation arrangement the optimum tap-length may vary with the variation in time as echo length keeps changing due to the users entering and leaving the room/system [9]. In most filter designs unfortunately the tap-length is fixed at some fixed value creating the problem of too short and too long filters.

In LMS based algorithms the stability condition needs to be checked each time the order changes. So it is advocated to use the normalized LMS (NLMS) algorithm for better convergence and constant level of misadjustment [2].

$$W(n+1) = W(n) + \frac{\overline{\mu}}{X^T(n)X(n)} X(n)e(n) \tag{1}$$

Here $\overline{\mu}$ is the step size for NLMS algorithm. NLMS converges to mean square for condition [1], $0 < \overline{\mu} < 2$.

In the proposed approach NLMS is used which provides inherent stability and robustness again the modification to it improves the convergence [11],

$$W_{P(n)}(n+1) = W_{P(n)}(n) + \frac{\mu'}{X^T_{P(n)}(n)X_{P(n)}(n)[2+P(n)]} X_{P(n)}(n)e^P_{P(n)}(n) \tag{2}$$

where μ' is a constant, $\sigma_X^2 = X^T(n)X(n)$ is the variance of input signal. $P(n)$ is the instantaneous variable adaptive tap-length obtained from the proposed fractional order estimation algorithm. $W_{P(n)}, X_{P(n)}$ are the weight and input vector pertaining to the order $P(n)$.

$$e^P_{P(n)}(n) = d(n) - W^T_{P(n)} X_{P(n)}(n) \tag{3}$$

$$d(n) = W^T_{P_{Opt}}(n) * X_{P_{opt}}(n) + t(n) \tag{4}$$

where $W_{P_{Opt}}(n), X_{P_{opt}}(n)$ are the weight and input vector pertaining to optimum tap-length P_{Opt} and $t(n)$ is the system noise.

$$\mu(n) = \frac{\mu'}{\sigma_X{}^2[2 + P(n)]} \tag{5}$$

Then (2) can be written as

$$W_{P(n)}(n+1) = W_{P(n)}(n) + \mu(n)X_{P(n)}(n)e^P_{P(n)}(n) \tag{6}$$

which forms a variable step NLMS (VNLMS) algorithm where the step size depends on the order estimation. If the difference of the MSE output of any two consecutive taps of the adaptive filter falls below a very small positive value, when the tap-length is increased, then it can be concluded that adding extra taps added to the present order do not reduce the MSE. Let us define $\Delta_P = J_{P-1}(\infty) - J_P(\infty)$ as the difference between the converged MSE when the filter order is increased from *P-1* to *P*. Now the optimum order can be defined as $\overline{P}$ that satisfies,

$$\Delta_P \le \delta \qquad \text{for all } P > \overline{P} \tag{7}$$

where δ is a very small positive number set pertaining to the system requirement. The cost function for tap-length selection can be defined as $\min\{P \mid J_{P-1} - J_P \le \delta\}$.The issue of false optimum tap-length sometimes creates confusion in the search of desired optimum filter length. These pseudo tap-lengths can be defined as follows. Let there exist a positive integer L that satisfies,

$$L<\overline{P} \text{ and } \Delta_L<\delta \tag{8}$$

where L is called the pseudo-optimum filter order . If the above condition is satisfied by a group of concatenated integer L, $L+1$,........ $L+S$-1 then $S+1$ is called the width of the pseudo-optimum filter order. These taps satisfies the optimality condition but cannot be treated as the optimal filter order as it under model the system. The issue of this pseudo-optimum tap-length can be removed by choosing a variable error width Δ which is shown later in this section.

The steady state MSE is not available usually and can be found out by exponential averaging,

$$J(n+1)=(1-H)e^2(n+1)+H\,J(n) \tag{9}$$

where H is the smoothing constant which control the effective memory of the iterative process. A smoothed estimated error can be obtained from,

$$\tilde{e}^{P}_{P(n)}(n)=(1-f)\sum_{i=0}^{n-1} f^{\,n-i} e^{P(i)}_{P_{Opt}(i)}(i)+t(i)*f^{\,i} \tag{10}$$

where n is the time index, P_{Opt} is the optimum suitable selection of tap-length. f is a forgetting smoothing factor which can be evaluated as,

$$f=\frac{\log_{10}\Delta_P(n)}{(\Delta_{\max}-\Delta_{\min})} \tag{11}$$

where $\Delta_P(n)$ is the variable error spacing parameter which has been evaluated in the next section, $(\Delta_{\max},\Delta_{\min})$ can be set according to the system requirements.

The MSE can be written as the sum of excess MSE (EMSE) and the system noise as,

$$E[(e^{P}_{P(n)}(n))^2]=E[J_{ex}{}^2(n)]+E[t^2(n)] \tag{12}$$

where $J_{ex}(n)$ is the EMSE which is used in updating the iteration parameter as it increases to large value in the early stage and later decreases to small value with the variation in tap-length. The performance of the squared smoothed estimated error can be simplified at the steady state as, [16]

$$E[\tilde{e}^{P}_{P(n)}(n)^2]\cong\frac{(1-f)}{(1+f)}\sigma_t^2 \tag{13}$$

where it is assumed that the error signal approximates to the system noise at the steady state. Now the algorithm for tap-length adaptation in a time varying environment can be defined as,

$$P_{nf}(n+1)=\left[P_{nf}(n)-K_n\right]+[(e^{P}_{P(n)}(n))^2-(e^{P}_{P(n)-\Delta_P(n)}(n))^2]\bar{K}_n \tag{14}$$

Finally the tap-length $P(n+1)$ in the adaptation of filter weights for next iteration can be formulated as follows, [11]

$$P(n+1) = \left\langle P_{nf}(n) \right\rangle \; if \left| P(n) - P_{nf}(n) \right| > \frac{K_n}{\bar{K}_n} \qquad (15)$$
$$P(n) \qquad otherwise$$

$P_{nf}(n)$ is the tap-length which can take fractional values. As the actual order of the adaptive filter cannot be a fractional value so $P_{nf}(n)$ is rounded to the nearest integer value to get the suitable optimum tap-length. In (14) the factor K_n is the leakage factor which prevents the order to be increased to an unexpectedly large value and $\bar{K}_n$ is the step size for filter order adaptation. In [11] the value of $(K_n, \bar{K}_n)$ was based on setting a random leaky factor which performed well for FIR systems especially for the issues of acoustic echo cancellation [13]. In this paper a unique method for setting these parameters has been defined which can find better performance in structure adaptation and hence decreases the overall design complexity.

$$K_n = \min(K_{n,\max}, K_n(i+1)) \qquad (16)$$

where
$$K_n(i+1) = \frac{\tilde{e}^2(i+1)}{\tilde{e}^2(i+1) + \Delta_{p_{ss}}(i)} \qquad (17)$$

$$\tilde{e}^P_{P(n)}(i+1) = f * \tilde{e}^P_{P(n)}(i) + (1-f) e^P_{P_{Opt}(n)}(i+1) \qquad (18)$$

$\Delta_{p_{ss}}$ defines the variable error spacing parameter at steady state tap-length P_{ss}. At the steady state, [16]

$$K_n \rightarrow \frac{(1-f)\sigma_t^2}{(1+f)\Delta_{p_{ss}}(\infty)} \qquad (19)$$

Similarly the adaptation step size depends on the bias between MSE values with a Δ_p difference. If the difference is more, then adaptation should be slow and vice versa.

$$\bar{K}_n = \min(\bar{K}_{n,\max}, \tau * \bar{K}_{n,\max}) \qquad (20)$$

where
$$\tau = \frac{[e^P_{P(n)}(n))^2 - (e^P_{P(n)-\Delta_P(n)}(n))^2]}{(e^P_{P_{Opt}(n)}(n))^2 + f * [e^P_{P(n)}(n))^2 - (e^P_{P(n)-\Delta_P(n)}(n))^2]} \qquad (21)$$

The variable error width parameter Δ_p decides the bias between the unknown optimum tap-length P_{opt} and the steady state tap-length in a system identification framework. It removes the suboptimum values and finds the optimum tap-length. A large value of Δ_p produces large error width and brings heavy computational complexity whereas a small Δ_p slow down the convergence and makes it difficult to overcome the suboptimum values. [14], [15]

The steady state tap-length is approximately equal to $L_{opt} + \Delta_p$.

In order to maintain the trade-off between convergence and steady state error [15]

$$\hat{e}^2 = \rho\hat{e}^2(n-1) + (1-\rho)\hat{e}^2(n) \tag{22}$$

$$\Delta_p(n) = \min(\Delta_{P,\max}, \upsilon * \hat{e}^2(n)) \tag{23}$$

where ρ is the smoothing parameter and υ is a constant which depends on the characteristics of the unknown plant.

Although different $\Delta_p(n)$ are needed for different applications, whereas for a certain application it can be easily decided in advance according to the noise conditions.

3 Simulation Setup

The simulation is performed for adaptive filter system modeling module as shown in Figure.1. MATLAB 7.7 platform has been chosen for simulation purpose. The input samples *x(n)* are from a white process having mean zeros and variance one. The proposed algorithm deals with a modified version of NLMS algorithm to avoid this slow convergence. $x(n)$ is fed to both unknown plant as well as LMS adaptive filter. The output of unknown filter is mixed with a white noise $t(n)$ such that the SNR remains as 40dB throughout the process. The unknown system is modeled as an Infinite Impulse Response (IIR) system. Because of infinite impulse response it is important to measure the optimum order in a system identification framework which can exactly replicate the performance of the IIR system. For simulations purpose $H_1(z) = \frac{1}{1-0.75z^{-1}+0.45z^{-2}}$ and has been fixed as the impulse response of the unknown system to be identified in the framework by an adaptive filter. The algorithm has to find out a filter with minimum co-efficient to completely match the impulse response of the plant in a time varying environment so that the structural design complexity can be minimized.

For comparison of the proposed analysis with present variable tap-length estimation algorithms like the FT-LMS, VT-VLMS has also been implemented along with the proposed algorithm under various noise conditions and parameter variations. The value of $\Delta_{P,\max}$ is kept fixed at 100, υ at 0.5 for the IIR system and δ at 1. For FT-LMS the step size is set at 0.005 and for VT-VLMS the leaky factor varies from 0 to 0.6 [11].

4 Results and Discussion

The adaptive filter response let's say $H(z)$ is matched to the unknown IIR plant in the system identification framework. Fig.1 depicts the performance of converged MSE with tap-length variation for the proposed as well as the FT-LMS [6]-[8] and VT-VLMS [11], [13] algorithms at SNR=40dB. For the proposed variable tap-length algorithm the MSE remains constant after the 20th tap whereas we get suboptimum order of 9 which has been removed by the use of variable error width $\Delta_p(n)$ and choice of parameter υ in the proposed algorithm as mentioned in (23). The optimum tap-length that adjusts the IIR system performance for the FT-LMS and VT-VLMS algorithm is obtained approximately at 48th and 35th tap respectively as shown in Fig. 1. The MSE and tap-length optimization performance of VT-VLMS algorithm is proved to be better than the FT-LMS algorithm but far short than the proposed variable tap-length algorithm. This clearly shows that the improved tap-length learning method analyzed in this paper reduces the structural complexity with 15 to 28 fewer taps. On the other hand it achieves the best MSE performance among all the simulated methods. The MSE performance with number of iterations has been shown in Fig. 2 keeping the SNR fixed at 40dB with averaging over 200 independent runs. The MSE decreases with increased number of iterations as per the general convention [1] but the proposed algorithm clearly outperforms its counterparts by achieving the best MSE performance and 5 to 8dB SNR improvement over 10000 iterations.

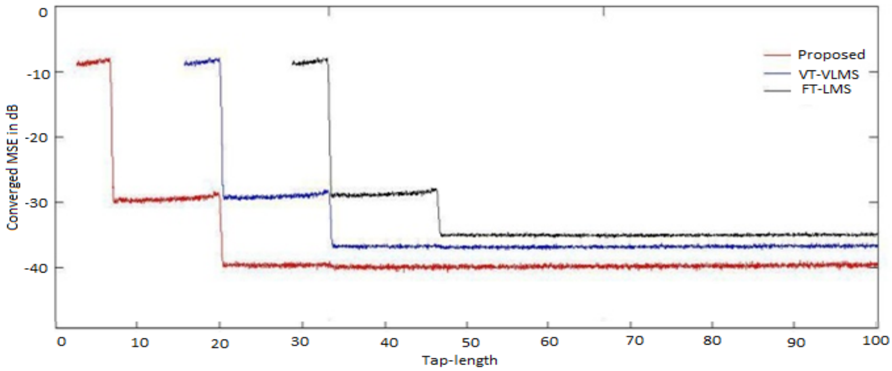

Fig. 1. Converged MSE Vs Tap-length at SNR=40dB

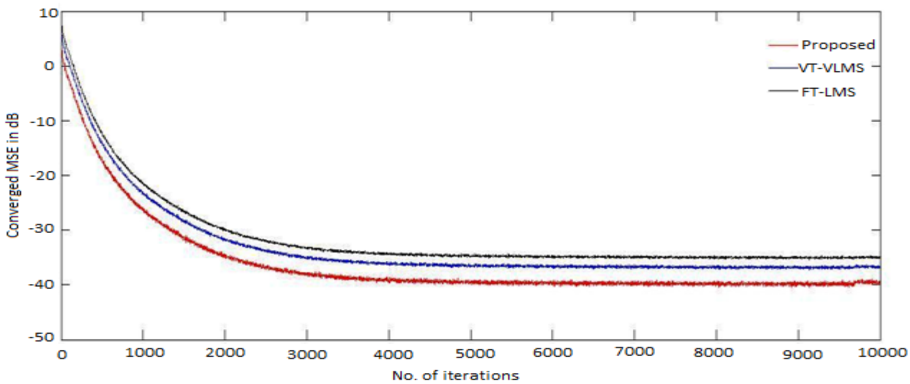

Fig. 2. Converged MSE Vs Iterations at SNR=40dB

The variation of error spacing parameter Δ_P with increased number of iterations, averaged over 200 Monte Carlo runs has been shown in Fig. 3. If Δ_P is being varied with respect to number of iterations then two transient points are noticed between 0 to 10 and 350 to 400 numbers of iterations. These transients are shown in Fig.4 and Fig.5 respectively. It depicts that after some initial transition Δ_P attains steady state value i.e. the optimum tap-length is achieved as the variation between consecutive converged MSE remains at a fixed value. It is discussed and mathematically analysed in Section 2.

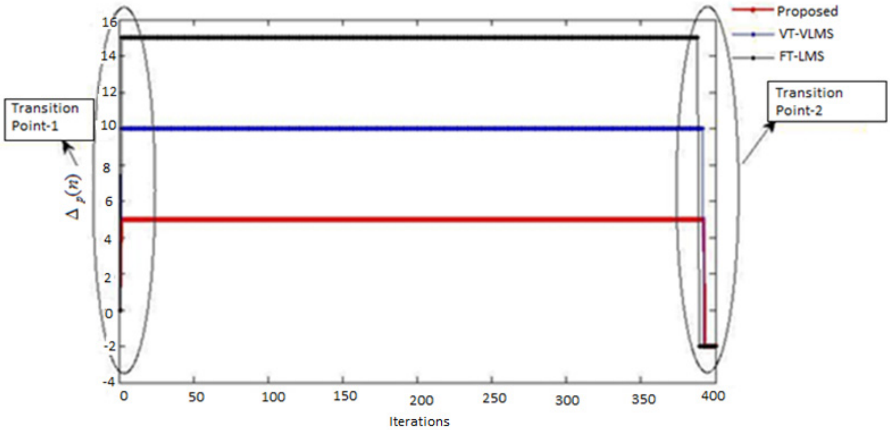

Fig. 3. Δ_P Vs Iterations

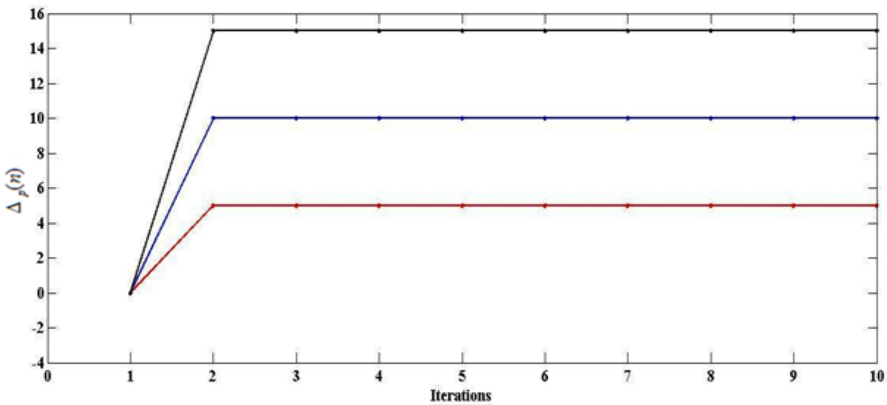

Fig. 4. Δ_P Vs Iterations (Transition point-1)

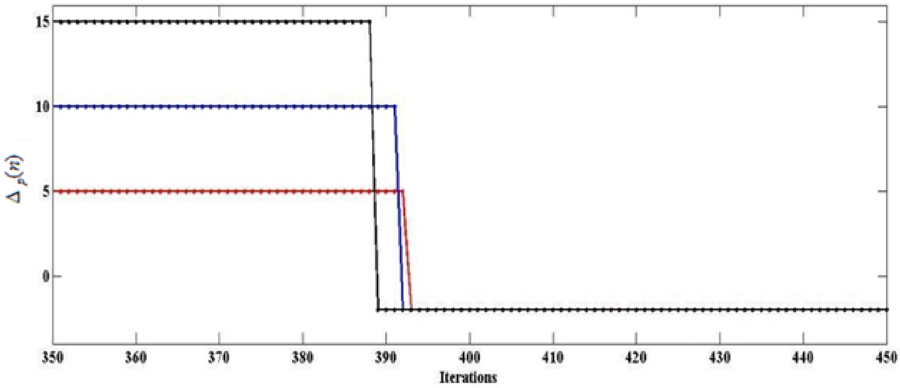

Fig. 5. Δ_P Vs Iterations (Transition point-2)

The variation for the proposed algorithm in comparison to its counterparts is between the minimum if we consider the absolute values. In transition point-1 the Δ_P goes from 0 to 4, 10 and 15 for the proposed, VT-VLMS and FT-LMS respectively in Fig. 5 which shows a steady increase in value before achieving the steady state up to 390 to 400 taps. Then it again decreases to -2 as shown in Fig.5 and attains that value till 5000 iterations which indicates that the desired optimum tap-length has been achieved. The proposed algorithm makes the best use of the variable error spacing parameter which affects the tap-length adaptation up to a large extent.

5 Conclusion

An improved pseudo-fractional tap-length selection for automatic structure adaptation in a dynamic time varying environment has been proposed. The key parameters were set according to the structure adaptation to best adjust the system performance and convergence in an identification framework. The proposed algorithm is compared with the existing tap-length learning algorithms and the improvements are addressed. The computer simulation and results are shown to verify the analysis.

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