The Outage Analysis of Inter-relay Coded Cooperation over Nakagami-m Fading Channels

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Abstract. In this paper we examine the outage behavior of a cooperative system, which utilizes the inter-relay coded cooperation. The outage analysis is done for independent, flat Nakagami-m fading channels. A closed form solution for outage probability is derived constrained on instantaneous received power which follows the Gamma distribution.

Keywords: Cooperative communication, outage probability, Nakagami-m fading

1 Introduction

Transmit diversity can be achieved in a wireless system by the cooperation among single antenna users to form a virtual array of antennas (VAA) [1,2]. Various cooperative protocols and their performance analysis were presented in [3] - [5]. In [5], an alternative mechanism, known as *coded cooperation*, was proposed in which concept of channel coding was associated with cooperative signalling. In coded cooperation each code word is essentially partitioned into two blocks, each of which is transmitted by one of the cooperative partners. Therefore the partners transmit some incremental redundancy instead of just repeating the received bits as in amplify and forward or decode and forward schemes.

An inter-relay cooperation scheme was presented in [6] for decode and forward relaying. This scheme allows the exchange of information among the relays (forming the VAA) to mitigate the propagation errors. Our contributions through this paper are (i) we present an inter-relay coded cooperation (IRCC) scheme, which improves the resource utilization within the VAA, and (ii) we have reached to a closed form expression of outage probability for the presented IRCC scheme. We consider Nakagami-m fading which is more generalized form of fading, as it can be used to model different fading channels through distinct values of parameter m [7]. The outage probability was defined in [8] in terms of instantaneous received power as the probability that the received power falls

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below a certain specified threshold. The present outage analysis is constrained on the instantaneous received power [9], however, the system model of present analysis is entirely different to that of [9].

The rest of this paper is outlined as follows. The system model is described in section 2. The closed form expression for the outage probability is derived in section 3. Numerical results are given in section 4. Section 5 concludes the paper with a brief note on the future scope of this work.

2 System Model

We have taken a simple model which consists of a source (s), destination (d) and two relays (r_1 and r_2) as shown in Fig. 1. As a practical assumption for low cost systems, we consider half duplex transmission, where a node cannot transmit and receive simultaneously. The inter-relay link is assumed to be reciprocal. To avoid multiple access interference links are considered to be orthogonal. Convolutional coding is been employed to convert N_0 bits of each source block into a codeword of length N.The coding mechanism is known to the source s, both the relays r_1 and r_2, and destination d.

The transmission is taking place in three phases. In first frame, source (s) broadcasts N_1 of N bits, which are received by each relay and destination. We define here the reliable relay as the relay which has correctly received the first phase broadcast. Further, If a relay is not reliable, it is unreliable. In second phase source (s) and reliable relay (or relays) will broadcast N_2 bits and destination along with unreliable relay, if any, will listen to it. N_1 and N_2 bits are obtained from length N codeword by partitioning it with rate R_1 and R_2 using a rate compatible punctured convolutional (RCPC) code [10]. Hence $N_1 + N_2 = N$. We define the cooperation ratio (α) as

$$\alpha = \frac{N_1}{N}. \tag{1}$$

The correctness of reception at relays will be validated on the basis of their cyclic redundancy checks (CRCs). The relay at which CRCs are not validated, will remain silent in the second frame. The quality of reception at each relay is obviously a function of the statistics of its channel with source. In the third phase the unreliable relay, will transmit the N_2 bits, if it received these bits correctly from the source and reliable node's broadcast in second phase. It may be noted that, although the link from source to unreliable relay was not good enough to receive correctly in the first phase as the power was not sufficient enough to label the threshold, this insufficient power will add up to the power received by the unreliable relay over relay to relay (inter-relay) link in second phase. In third phase, while making the decision at unreliable relay, we will consider the sum of two powers for comparison with threshold. As the reception is very energy economic than transmission, IRCC requires no additional energy while enhancing the resource utilization. However, the gains of IRCC don't come for free, it requires an extra time slot.

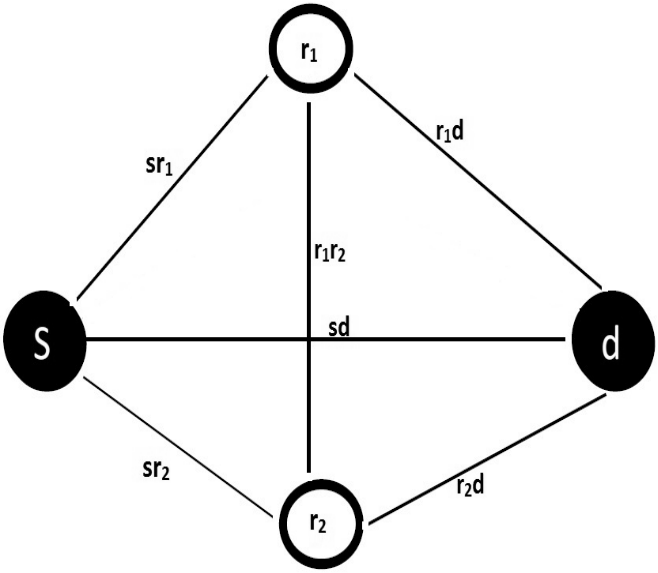

Fig. 1. System Model

We define P_n to be the threshold power so that if the instantaneous received power over any link falls below this threshold, the link is said to be in outage and corresponding outage event is $Pr\{P < P_n\}$. Thus for a link, corresponding outage event can be defined as,

$$P_{out} = Pr\{P < P_n\} = \int_0^{P_n} f(p)dp, \tag{2}$$

where p is the instantaneous received power and $f(p)$ is the probability density function (p.d.f) of p.

3 Outage Analysis

For Nakagami-m distribution, instantaneous received power (p) has a Gamma distributed p.d.f., hence outage probability as given in (2) for Nakagami-m fading can be written as,

$$P_{out} = \int_0^{P_n} \frac{1}{\Gamma(m)} \left(\frac{m}{\tilde{P}}\right)^m P^{m-1} \exp\left(\frac{-mP}{\tilde{P}}\right) dP = 1 - \frac{\Gamma\left(m, \frac{mP_n}{\tilde{P}}\right)}{\Gamma(m)}, \tag{3}$$

where $\tilde{P}$ denotes the average value of received power over the fading and shadowing effects. The function $\Gamma(m)$ is the Gamma function defined as $\Gamma(m) = \int_0^{\infty} x^{m-1} \exp(-x)dx$, and $\Gamma(m, \tau)$ is the upper incomplete Gamma function defined as $\Gamma(m,\tau) = \int_{\tau}^{\infty} x^{m-1}\exp(-x)dx$. We now define the outage events corresponding to each of four cooperative cases possible for the given system model described in section 2, as

- Case 1: When r_1, and r_2 both decode correctly i.e. if $P_{sr_1} > P_n/\alpha$, and $P_{sr_2} > P_n/\alpha$, the outage event is,
  $[\{\alpha P_{sd} + (1-\alpha)\left[P_{r_1d} + P_{r_2d}\right]\} < P_n]$
- Case 2: When r_1, and r_2 both don't decode correctly i.e. if $P_{sr_1} < P_n/\alpha$, and $P_{sr_2} < P_n/\alpha$
  the outage event is, $[\alpha P_{sd} < P_n]$
- Case 3: When only r_1 decodes correctly, and r_2 doesn't i.e. if $P_{sr_1} > P_n/\alpha$, and $P_{sr_2} < P_n/\alpha$
  then there are two further possibilities,
- Case 3(a): If $\{P_{sr_2} + P_{r_1r_2}\} > P_n/\alpha$,
  the outage event is,
  $[\{\alpha P_{sd} + (1-\alpha)\left[P_{r_1d} + P_{r_2d}\right]\} < P_n]$, or
- Case 3(b): If $\{P_{sr_2} + P_{r_1r_2}\} < P_n/\alpha$,
  the outage event is,
  $[\{\alpha P_{sd} + (1-\alpha)P_{r_1d}\} < P_n]$
- Case 4: When only r_2 decodes correctly, and r_1 doesn't i.e. if $P_{sr_1} < P_n/\alpha$, and $P_{sr_2} > P_n/\alpha$
  then there are two further possibilities,
- Case 4(a): If $\{P_{sr_1} + P_{r_1r_2}\} > P_n/\alpha$,
  the outage event is,
  $[\{\alpha P_{sd} + (1-\alpha)\left[P_{r_1d} + P_{r_2d}\right]\} < P_n]$, or
- Case 4(b): If $\{P_{sr_1} + P_{r_1r_2}\} < P_n/\alpha$,
  the outage event is,
  $[\{\alpha P_{sd} + (1-\alpha)P_{r_2d}\} < P_n]$

Assuming all the links to be mutually independent, the overall outage probability for these four disjoint cases, can be written as given in (4).

$$\begin{aligned}
P_{out} = &Pr\{P_{sr_1} > P_n/\alpha\}Pr\{P_{sr_2} > P_n/\alpha\}Pr\{\alpha P_{sd} + (1-\alpha)\left[P_{r_1d} + P_{r_2d}\right] < P_n\}\\
&+ Pr\{P_{sr_1} < P_n/\alpha\}Pr\{P_{sr_2} < P_n/\alpha\}Pr\{\alpha P_{sd} < P_n\}\\
&+ Pr\{P_{sr_1} > P_n/\alpha\}Pr\{P_{sr_2} < P_n/\alpha\}Pr\{\{P_{sr_2} + P_{r_1r_2}\} > P_n/\alpha\}\\
&\times Pr\{\alpha P_{sd} + (1-\alpha)[P_{r_1d} + P_{r_2d}] < P_n\}\\
&+ Pr\{P_{sr_1} > P_n/\alpha\}Pr\{P_{sr_2} < P_n/\alpha\}Pr\{\{P_{sr_2} + P_{r_1r_2}\} < P_n/\alpha\}\\
&\times Pr\{\alpha P_{sd} + (1-\alpha)P_{r_1d} < P_n\}\\
&+ Pr\{P_{sr_1} < P_n/\alpha\}Pr\{P_{sr_2} > P_n/\alpha\}Pr\{\{P_{sr_1} + P_{r_1r_2}\} > P_n/\alpha\}\\
&\times Pr\{\alpha P_{sd} + (1-\alpha)[P_{r_1d} + P_{r_2d}] < P_n\}\\
&+ Pr\{P_{sr_1} < P_n/\alpha\}Pr\{P_{sr_2} > P_n/\alpha\} \times Pr\{\{P_{sr_1} + P_{r_1r_2}\} < P_n/\alpha\}\\
&\times Pr\{\alpha P_{sd} + (1-\alpha)P_{r_2d} < P_n\} \qquad (4)
\end{aligned}$$

We have further assumed the average link powers of all the relay to destination links to be same, i.e.

$$\tilde{P}_{r_1d} = \tilde{P}_{r_2d} = \tilde{P}_{av}$$

We also consider the average power of inter-relay link to be equal to that of sorce to destination link. Now, using (3), the overall outage probability can be written as given in (5), (details of (5) are given in appendix).

$$\begin{aligned}
P_{out}=&\left[\frac{\Gamma\left(m,\frac{mP_n/\alpha}{\tilde{P}_{sr_1}}\right)}{\Gamma(m)}\right]\left[\frac{\Gamma\left(m,\frac{mP_n/\alpha}{\tilde{P}_{sr_2}}\right)}{\Gamma(m)}\right]\left[A_1-A_2\sum_{k=0}^{2m-1}A_3C\beta(k+1,m)_1F_1\left[m;k+m+1;\frac{BP_n}{\alpha}\right]\right]\\
&+\left[1-\frac{\Gamma\left(m,\frac{mP_n/\alpha}{\tilde{P}_{sr_1}}\right)}{\Gamma(m)}\right]\left[1-\frac{\Gamma\left(m,\frac{mP_n/\alpha}{\tilde{P}_{sr_2}}\right)}{\Gamma(m)}\right]\left[1-\frac{\Gamma\left(m,\frac{mP_n/\alpha}{\tilde{P}_{sd}}\right)}{\Gamma(m)}\right]+\left[\frac{\Gamma\left(m,\frac{mP_n/\alpha}{\tilde{P}_{sr_1}}\right)}{\Gamma(m)}\right]\\
&\times\left[1-\frac{\Gamma\left(m,\frac{mP_n/\alpha}{\tilde{P}_{sr_2}}\right)}{\Gamma(m)}\right]\left[\frac{\Gamma\left(2m,\frac{mP_n/\alpha}{\tilde{P}_{sd}}\right)}{\Gamma(m)}\right]\left[A_1-A_2\sum_{k=0}^{2m-1}A_3C\beta(k+1,m)_1F_1\left[m;k+m+1;\frac{BP_n}{\alpha}\right]\right]\\
&+\left[\frac{\Gamma\left(m,\frac{mP_n/\alpha}{\tilde{P}_{sr_1}}\right)}{\Gamma(m)}\right]\left[1-\frac{\Gamma\left(m,\frac{mP_n/\alpha}{\tilde{P}_{sr_2}}\right)}{\Gamma(m)}\right]\left[1-\frac{\Gamma\left(2m,\frac{mP_n/\alpha}{\tilde{P}_{sd}}\right)}{\Gamma(m)}\right]\\
&\times\left[A_1-A_2\sum_{k=0}^{m-1}A_3C\beta(k+1,m)_1F_1\left[m;k+m+1;\frac{BP_n}{\alpha}\right]\right]+\left[1-\frac{\Gamma\left(m,\frac{mP_n/\alpha}{\tilde{P}_{sr_1}}\right)}{\Gamma(m)}\right]\left[\frac{\Gamma\left(m,\frac{mP_n/\alpha}{\tilde{P}_{sr_2}}\right)}{\Gamma(m)}\right]\\
&\times\left[\frac{\Gamma\left(2m,\frac{mP_n/\alpha}{\tilde{P}_{sd}}\right)}{\Gamma(m)}\right]\left[A_1-A_2\sum_{k=0}^{2m-1}A_3C\beta(k+1,m)_1F_1\left[m;k+m+1;\frac{BP_n}{\alpha}\right]\right]\\
&+\left[1-\frac{\Gamma\left(m,\frac{mP_n/\alpha}{\tilde{P}_{sr_1}}\right)}{\Gamma(m)}\right]\left[\frac{\Gamma\left(m,\frac{mP_n/\alpha}{\tilde{P}_{sr_2}}\right)}{\Gamma(m)}\right]\left[1-\frac{\Gamma\left(2m,\frac{mP_n/\alpha}{\tilde{P}_{sd}}\right)}{\Gamma(m)}\right]\\
&\times\left[A_1-A_2\sum_{k=0}^{m-1}A_3C\beta(k+1,m)_1F_1\left[m;k+m+1;\frac{BP_n}{\alpha}\right]\right]
\end{aligned}\tag{5}$$

where $\beta(u,v)$ is beta function defined as $\beta(u,v)=\int_0^1 x^{u-1}(1-x)^{v-1}dx$, and ${}_1F_1[a;b;z]$ is the confluent hypergeometric function [11], and

$$\begin{aligned}
A_1&=[1-\varphi],\\
A_2&=\left[\frac{1}{\Gamma(m)}\left(\frac{m}{\tilde{P}_{sd}}\right)^m\exp\left(-\frac{mP_n}{(1-\alpha)\tilde{P}_{av}}\right)\right],\\
A_3&=\left[\frac{1}{k!}\left(\frac{m\alpha}{(1-\alpha)\tilde{P}_{av}}\right)^k\right],\\
B&=\left[\frac{m\alpha}{(1-\alpha)\tilde{P}_{av}}-\frac{m}{\tilde{P}_{sd}}\right],\\
C&=\left[\frac{P_n}{\alpha}\right]^{k+m}
\end{aligned}\tag{6}$$

We, in (5), have found analytically the closed form expression of the outage probability P_{out} for generalized Nakagami-m fading.

4 Numerical Results

For numerical analysis we assume that the average power of all the links is equal. Fig. 2 shows the effect of cooperation ratio over the outage behavior for different

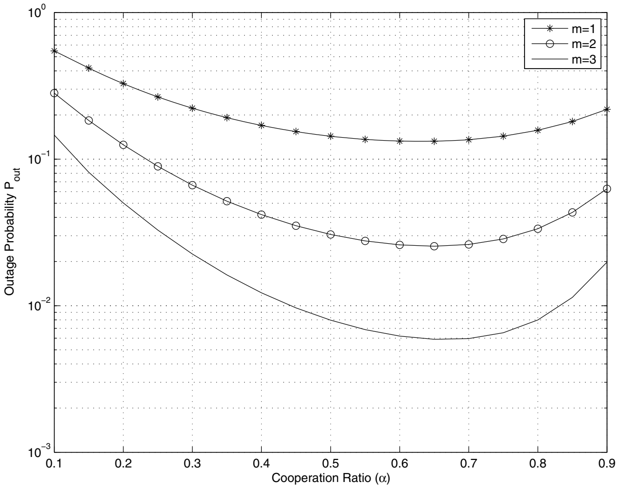

Fig. 2. Outage Probability vs Cooperation Ratio(α) for different values of m

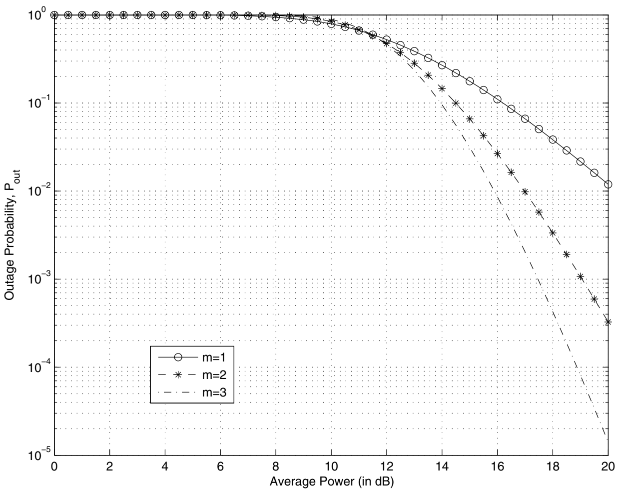

Fig. 3. Outage Probability vs Average Power(P_{av}) for different values of m

values of parameter m, assuming the threshold power and the average power both to be equal to 10 dB. Fig. 3 shows the plots of the outage probability given in (5), w. r. t. average power for different values of Nakagami parameter m. Here we consider threshold power $P_n = 5$ dB, and cooperation ratio $\alpha = 0.5$. The observations from this figure are very intuitive in sense that the performance of the system is increasing (i.e. outage probability is decreasing) on improving the channel conditions (i.e. increasing the value of parameter m).

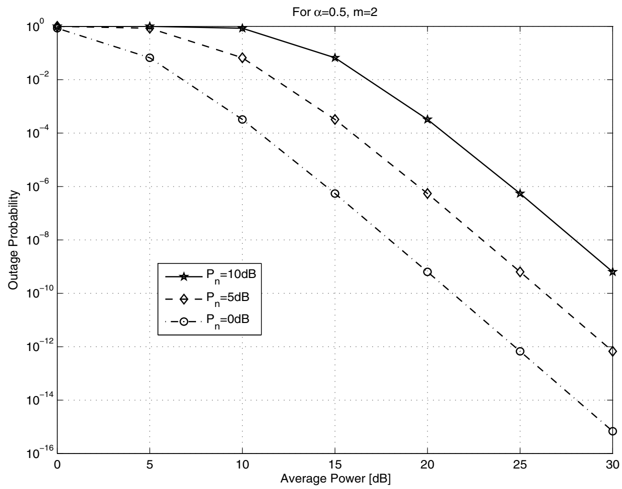

Fig. 4. Outage Probability vs Average Power(P_{av}) for different values of P_n

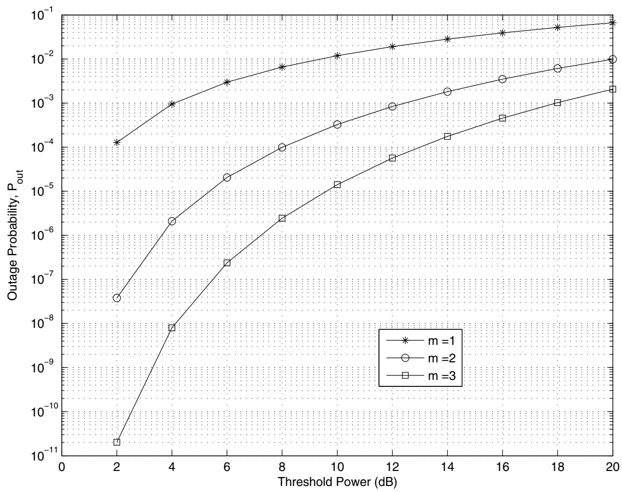

Fig. 5. Outage Probability vs Threshold Power (P_n) for different values of m

Fig. 4 plots the outage probability with respect to average power for different values of threshold power with $m = 2$ and $\alpha = 0.5$. We, from this figure, can observe that the system remains in the outage as long as the average power is below the threshold value. The variation of outage probability with threshold power for different values of parameter m, assuming average power $\tilde{P} = 20$ dB and cooperation ratio $\alpha = 0.5$, can be observed in Fig. 5.

5 Conclusion

This paper, using instantaneous power based approach, investigates the outage behavior of the given cooperative communication system with inter relay coded cooperation. The exact closed form expression for outage probability is obtained under Nakagami-m fading environment for the same. This work can be extended for multiple relays based system.

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Appendix

The terms in (4), containing the probability $Pr\{\alpha P_{sd} + (1-\alpha)\sum_{i\in\{\Theta_1\}} P_{xy} < \mathcal{P}\}$ where $\{\ l \in 1, 2\ \}$ are number of relays cooperating, and P_{xy} is the power over xy link, is given as

$$Pr\{\alpha P_{sd} + (1-\alpha) \sum_{\substack{i=1 \\ \forall l = \{1,2\}}}^{l} P_{xy} < \mathcal{P}\} = \int_0^{\frac{\mathcal{P}}{\alpha}} \int_0^{a} f(P_{sd}) f(z) dP_{sd} dz \tag{7}$$

where $z = \sum_{i=1}^{l} P_{xy} \sim \text{Gamma}\left[lm, \frac{\tilde{P}_{av_1}}{m}\right]$, and $a = \frac{\mathcal{P} - \alpha P_{sd}}{1-\alpha}$. Using (2) and (3), the integral $\int_0^a f(z)dz$ can be written as

$$\int_0^a f(z)dz = 1 - \frac{\Gamma\left(lm, \frac{ma}{\tilde{P}_{av_1}}\right)}{\Gamma(lm)}, \tag{8}$$

therefore

$$Pr(\alpha P_{sd} + (1-\alpha) \sum_{i=1}^{n} P_{xy} < \mathcal{P}) = \int_0^{\frac{\mathcal{P}}{\alpha}} f(P_{sd}) \left[1 - \frac{\Gamma\left(lm, \frac{ma}{\tilde{P}_{av_1}}\right)}{\Gamma(lm)}\right] dP_{sd} \tag{9}$$

Further, using [12, Eq. (8.352.2)], $\Gamma\left(lm, \frac{ma}{\tilde{P}_{av_1}}\right)$ can be written as

$$\Gamma\left(lm, \frac{ma}{\tilde{P}_{av_1}}\right) = (lm-1)! \exp(-\frac{ma}{\tilde{P}_{av_1}}) \sum_{k=0}^{lm-1} \frac{1}{k!} \left(\frac{ma}{\tilde{P}_{av_1}}\right)^k, \tag{10}$$

we get,

$$\begin{aligned} Pr(\alpha P_{sd} + (1-\alpha) \sum_{i=1}^{n} P_{xy} < \mathcal{P}) = \\ A_1 - A_2 \sum_{k=0}^{lm-1} A_3 \int_0^{\frac{\mathcal{P}}{\alpha}} \left(\frac{\mathcal{P}}{\alpha} - P_{sd}\right)^k {P_{sd}}^{m-1} \exp(BP_{sd}) dP_{sd} \\ = A_1 - A_2 \sum_{k=0}^{lm-1} A_3 C \beta(k+1, m)\, {}_1F_1\left[m; k+m+1; \frac{B\mathcal{P}}{\alpha}\right]. \end{aligned} \tag{11}$$

This simplification is achieved using [12, Eq. (3.383)]. Here, terms A_1, A_2, A_3, B and C can be written as

$$
\begin{aligned}
A_1 &= \left[1 - \frac{\Gamma\left(m, \frac{m\mathcal{P}/\alpha}{\tilde{P}_{sd}}\right)}{\Gamma(m)}\right], \\
A_2 &= \left[\frac{1}{\Gamma(m)}\left(\frac{m}{\tilde{P}_{sd}}\right)^m \exp\left(-\frac{m\mathcal{P}}{(1-\alpha)\tilde{P}_{av_1}}\right)\right], \\
A_3 &= \left[\frac{1}{k!}\left(\frac{m\alpha}{(1-\alpha)\tilde{P}_{av_1}}\right)^k\right], \\
B &= \left[\frac{m\alpha}{(1-\alpha)\tilde{P}_{av_1}} - \frac{m}{\tilde{P}_{sd}}\right], \\
C &= \left[\frac{\mathcal{P}}{\alpha}\right]^{k+m},
\end{aligned}
\tag{12}
$$

where $\beta(u,v)$ is beta function defined as $\beta(u,v) = \int_0^1 x^{u-1}1 - x^{v-1}dx$, and ${}_1F_1[a;b;z]$ is the confluent hypergeometric function [11].