On the Wavelet Families for OFDM System - Comparisons over AWGN and Rayleigh Channels

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Abstract. In the study of OFDM systems, discrete wavelet transforms have been reported to perform better than Fast Fourier Transform in multicarrier systems (MCS) - in terms of spectral efficiency because they can operate without a cyclic prefix, have reduced side-lobes and improved BER. However all of the wavelet families do not perform alike. This study has investigated various wavelet families such as Daubechies, Symlet, Haar (or db1), biorthogonal, reverse-biorthogonal and Coiflets for OFDM system design over an AWGN and multipath channels. Results show that Daubechies, Symlet, Haar and Coiflet wavelet families perform considerably better than other families considered, thus these families could be better in OFDM.

Keywords: DWT, Family, OFDM.

1 Introduction

While wavelet transforms are seriously being considered for OFDM systems, it is pertinent to heuristically select the best among the numerous available members of the wavelet families. The criteria upon which any selection should be made must offer the best possible trade-off in comparison to its other family members. Beside other prominent applications, OFDM is used in Digital Video Broadcasting, DVB [1], which is of the form DVB-C for cable, DVB-S for satellite and DVB-T for terrestrial communications respectively. In [2], the discrete wavelet transform, DWT, has been applauded for its application advantages in video compression, Internet communication compression, object recognition, numerical analysis and signal processing. DWT does not require a cyclic prefix [3] and due to its longer basis functions and reduced side-lobes, bit error ratio (BER) in wavelet based OFDM is improved [4]. These advantages have been possible since the wavelet transform smartly eludes the limitation of non-simultaneous representation of signals in frequency and time. Recall that the Heisinberg uncertainty principle posits that it is impossible to represent a signal as a single point in time and frequency. However, wavelets use a multiresolution, time-frequency representation of a signal called time-scale (or translation-scaling) representation. To do that, a signal, $s(t)$ can be observed

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using this translation-scaling relation such that the signal is scaled and translated (shifted) periodically [5][6]. However, there are limits to the advantages of the wavelets transform in multicarrier system design. Continuous wavelet transforms, S_{CWT} has been defined as [6][7],

$$S_{CWT}(k,\tau) = \int s(t)\psi^{*}_{k,\tau}(t)dt \tag{1}$$

where * is a complex conjugate. $\psi_{k,\tau}(t)$ is the mother wavelet from which all other basis functions- daughter wavelets used in transformation are derived, through scaling (dilation or compression) and translation (shifting) [7]. It is given by [6], that

$$\psi_{k,\tau}(t) = \frac{1}{\sqrt{k}}\psi\left(\frac{t-\tau}{k}\right) \tag{2}$$

k is a scaling factor and τ is a translation factor. In its continuous form, the expressions are redundant infinite in number and have no analytical (closed-form) solution. As such, they are unsuitable for practical application in MCSs. We can solve this problem by making the wavelets discrete, i.e. by modifying (2) to be of the form [6];

$$\psi_{j,n}(t) = \frac{1}{\sqrt{k_0^j}}\psi\left(\frac{t-n\tau_0 k_0{}^j}{k_0^j}\right) \tag{3}$$

And its basis function given by;

$$\int \psi_{j,n}(t)\psi^{*}_{w,u}dt = \begin{cases} 1 & if\ j = w\ and\ n = u \\ 0 & otherwise \end{cases} \tag{4}$$

Where j, n, w and u are integers and $k_0 > 1$is a fixed dilation step on which the translation factor, τ_0 depends. Precisely, $k_0$is usually set to 2 and τ_0 as 1 to satisfy the dyadic property. [8] explained that the dilation problem can be solved by decomposing both the low-pass and high-pass filtered signals alike.

In the following sections, we introduce the DWT in OFDM. Then, we discuss the simulation environment in which this investigation was carried out. Graphs depicting results from this study are then presented, showing how the different wavelet families of different order perform. Using these comparisons, a selection from the wavelet family was then used to compare FFT-OFDM and DWT-OFDM. In the final section, we conclude the study and a list of references follows.

2 DWT for OFDM Systems

2.1 The DWT Scheme

For use in MCS such as OFDM, DWT is seen [2][6] as consisting of a quadrature mirror filter (QMF) bank with low-h and high-g pass filters that convolve with the signal according to the following scheme; $s_{low}[n] = h[n] * s[n]$ and $s_{high}[n] = g[n] * s[n]$, where $s[n]$ represents the observed signal. The low-pass filter is related to the high-pass filter as follows, $h[n] = (-1)^n g[1-n]$. $s[n]$ is analyzed and reconstructed according to the Fig. 1;

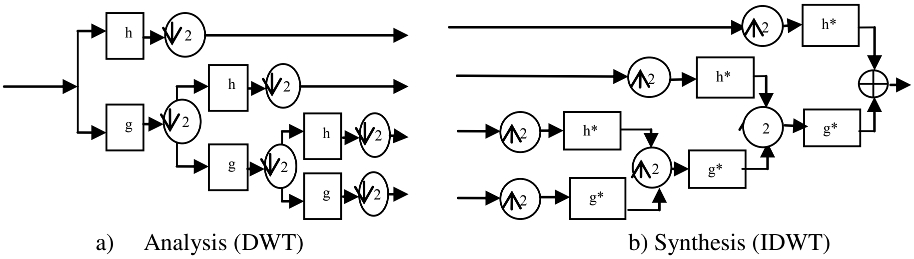


Fig. 1. The DWT/IDWT Processes

In Fig. 1a, the signal is decomposed, filtered and then downsampled. The reverse of this process is done in the receiver wherein the resultant parallel QAM or PSK-modulated samples are first upsampled and then filtered accordingly. This is called reconstruction. In OFDM, however, IDWT and DWT are recursively applied to the parallely aligned signals as long as decomposition and reconstruction length (the leaves of the tree and level) of the signal permits, as in the following block diagram;

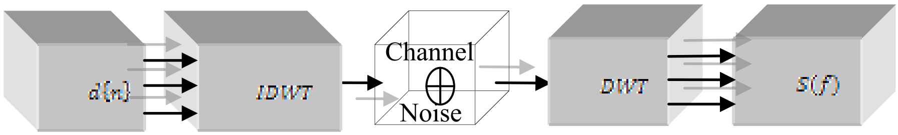


Fig. 2. DWT-OFDM

The randomly generated data are recursively up-sampled and filtered by g* and h* (reconstructed) respectively and passed through the channel with additive white Guassian noise (AWGN). The resultant signal is then decomposed and down-sampled (DWT) and then recovered according to the designer's mapping scheme of choice.

2.2 Decomposition and Reconstruction Conditions

Fig.1 shows a 3-level schematic decomposition and reconstruction structure of a typical DWT for multicarrier systems. Decomposed wavelets can be reconstructed [9] if the energy of the wavelets satisfies this property;

$$\alpha \| s[n] \|^2 \leq \sum_{j,k} | \langle s[n], \psi_{j,k} \rangle |^2 \leq \beta \| s[n] \|^2 \qquad (5)$$

Where α and β are the lower and upper energy limits independent of $s[n]$. Eq. 5 suggests that the signal energy$||s[n]||^2$ must be in the positive bound but not infinite which is the basic characteristic of natural signals. The signal, $s[n]$ under study can be perfectly reconstructed according to Fig. 1b and the following [6];

$$h(z)h^*(z^{-1}) + g(z)g^*(z^{-1}) = 2 \qquad (6a)$$

$$h(z)h^*(-z^{-1}) + g(z)g^*(-z^{-1}) = 0 \qquad (6b)$$

h(z) and $g(z)$ are the lowpass and highpass synthesis filters respectively, $h^*(z^{-1})$ and $g^*(z^{-1})$ are the analysis lowpass and highpass filters respectively. With Fig. 1, sub-carrier frequency increase with increase in the decomposition level, thus there would be more resolution and low frequency components.

3 Simulation Environment and Results

To succinctly distinguish among the DWT member families, BPSK has been used to modulate the data transmitted over Rayleigh fading channel with AWGN. Results below show the performance of different discrete wavelet families when compared for BER and SNR. Meanwhile, the following parameters have been used;

Table 1. Simulation Parameters

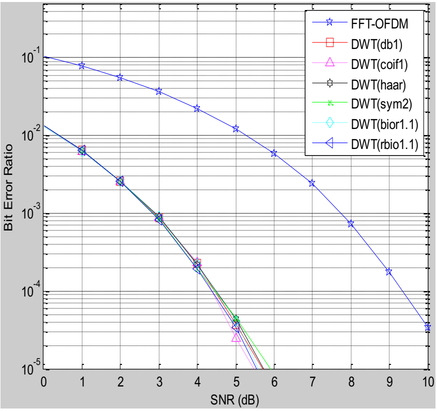
	DWT	**FFT**
Modulation Scheme	BPSK	BPSK
FFT Size	Nil	64
Cyclic Prefix	Nil	25%
DWT Families	db1, db2, db3, db5, db44, haar, coif1, coif2, coif3, coif4, sym2, sym3, sym4, sym5, bior1.1, bior1.3, bior1.5, bior2.2, rbio1.1, rbio1.3, rbio1.5, rbio2.2	Nil
Decomposition level	$k = log2\ (N)$, N = 64.	Nil
Symbol length	2*10^4	2*10^4

In DWT, transmitted data must be in order of $M = 2^k$ to ensure possible decomposition and reconstruction. This has influenced our choice of $N = 64$.

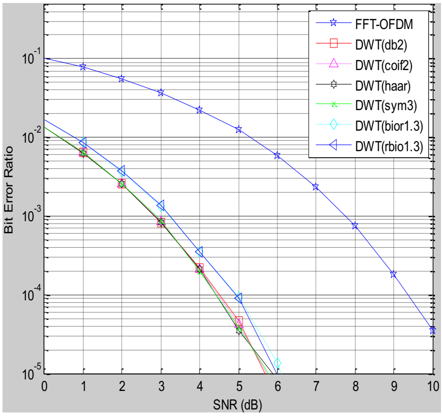
3.1 AWGN Channel Only

In FFT-OFDM, if the bandwidth length, N is extended by L-cyclic prefix, it costs an effective bandwidth of $N/(L+N)$ and signal power proportionate to $N/(L+N)$

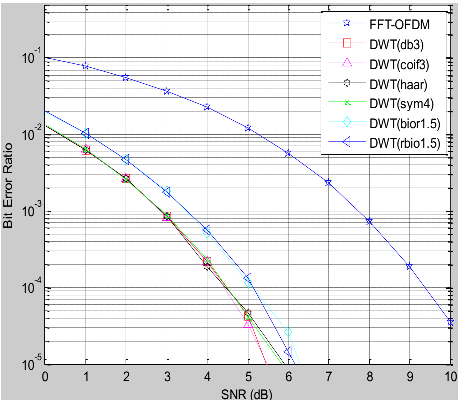
[10]. We compensate for this power cost with an equivalence of $sqrt(N/L+N)$ and append same to the transmitted signal. This has been cared for in the simulation with Rayleigh channel where cyclic prefix is required for channel immunity. This cost and scaling is not necessary in the DWT-OFDM since the scheme requires no cyclic prefix. The graphs below depict the DWT families and FFT-OFDM compared over an AWGN channel;



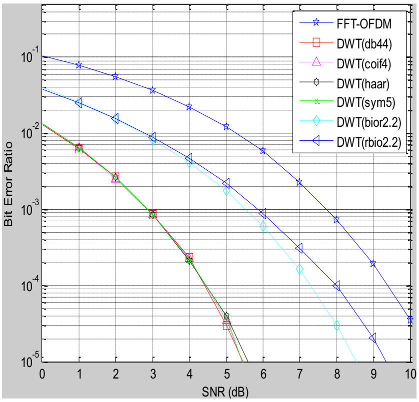
a) db1, haar, coif1, sym2, bior1.1 and rbio1.1 DWT over AWGN



b) db2, haar, coif2, sym3, bior1.3 and rbio1.3 DWT over AWGN



c) db3, haar, coif3, sym4, bior1.5 and rbio1.5 DWT over AWGN



d) db44, haar, coif4, sym5, bior2.2 and rbio2.2 DWT over AWGN

Fig. 3. Performance of DWT families and FFT-OFDM over AWGN channel

Using the enumerated parameters, the signal is reconstructed for $log2(N)$ or 4-levels in our case, with $log2(N)+1$ leaves of the wavelet trees. In the receiver, the received signal is again decomposed up to 4-levels with $log2(N)+1$ leaves. These leaves form the subchannels of the MCS and these subchannels modulate the data over the transmitter and the receiver. Over AWGN channel, results show that different wavelet families perform differently with Daubechies family performing best, while the biorthogonal and reverse-biorthogonal families perform worst. Since different families posses different filters, thus different wavelet families must perform differently. However, other family members, such as Haar, Coiflet, and Symlet perform likely equally with the Daubechies family members.

3.2 Rayleigh Fading Channel

In [11], a Rayleigh fading channel is shown to consist of several multipath channels. It is characterized by a number of attenuated, time delayed copies of the original transmitted signal. In the baseband, its impulse response can be modeled as [12];

$$h(t,\tau) = \sum_{n=1}^{N} a_n(t,\tau) e^{-j\theta_n} \delta(t - \tau_n(t)) \qquad (6)$$

$a_n(t,\tau)$ and τ_n are the amplitude and time delay respectively with a phase shift of $\theta_n = 2\pi f_c \tau_n(t)$ for the n^{th} multipath at a prevailing time, t. N is the possible number of multipath and $\delta(\bullet)$ is the Dirac delta. Assuming no line of sight [13][14], Eq.6 correctly models a Rayleigh distribution for a time varying multipath channel. In the frequency domain, the channel transfer function, H, is obtained as a Fourier transform pair of the channel impulse response.

3.3 The Channel Estimation Scheme

In [7], Zero-forcing, ZF channel estimation method in time domain of wavelet-OFDM for feedforward systems has been shown for In-home Power Line Communication, PLC channel. We extend this ZF channel estimation in time domain of a DWT-OFDM system to a Rayleigh fading channel with AWGN noise.

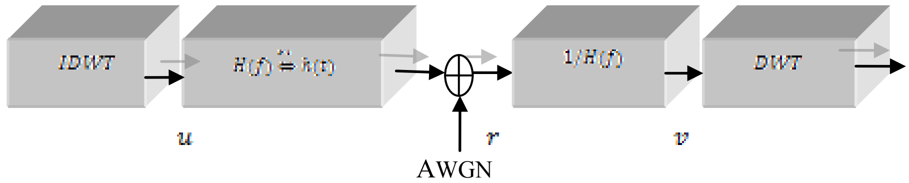
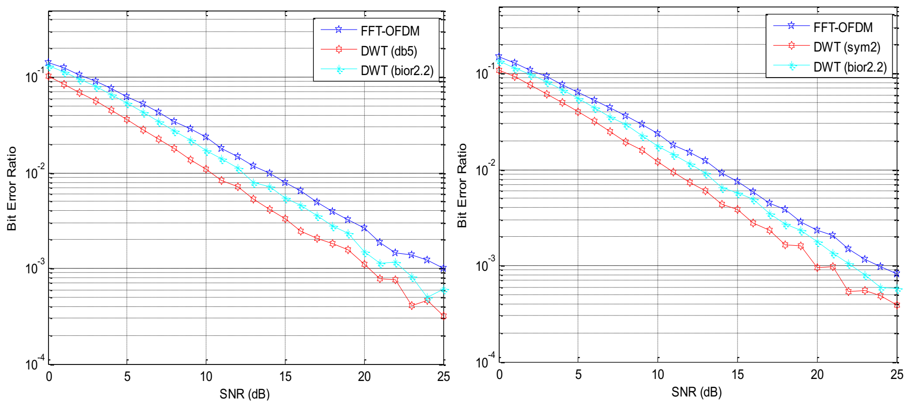


Fig. 4. The ZF channel estimation scheme

Given that r is the received signal, expressed as; $r = Hs + w$, where w is the AWGN. If we define the channel impulse response of the systems as h, its transfer function is a Fourier transform of h and is given by H. Since the DWT offers a long basis function thus promising a total ISI and ICI combat, no CP is thus required so that we equalize the channel just before the DWT operation into frequency domain according to the following; $v = r.(1/H)$. By the above scheme, the result below shows the performance of db5, bior2.2 and FFT-OFDM in a Rayleigh fading channel.



a) (db5), biorthogonal (bior2.2) DWT and FFT-OFDM

b) Symlet (sym2), biorthogonal (bior2.2) DWT and FFT-OFDM

Fig. 5. Performance comparison of Daubechies (db5), Symlet (sym2), biorthogonal (bior2.2) DWT and FFT-OFDM over Rayleigh fading channel with AWGN

Comparing the db5 and sym2 with the FFT for instance, there is nearly 5.0dB gain (SNR) and with the bior2.2, there is up to 3.0dB gain (SNR). Thus, we see that the db5 outperforms all the family members considered in this respect. It then follows that Daubechies DWT family members perform better than other DWT family members.

Conclusion

An in-depth study into the performance behaviour of discrete wavelet transform for multicarrier systems, such as OFDM, has been presented. Results show that DWT of the Daubechies families champion in performance when observed in terms of BER and SNR. Haar DWT which is on the other the db1 sub-family can be considered alongside the Daubechies families. Except for biorthogonal and the reverse birothogonal, all other wavelet families studied perform alike. Within the Daubechies DWT families, lower order sub-families which have lower number of filters can be considered for OFDM systems in terms of improved BER and run-time at the expense of the higher order members.

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