

Compare Effective Fuzzy Associative Memories for Grey-Scale Image Recognition

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Abstract. Pattern recognition (PR) is the most important field of image processing that is widely developed by many scientists. Reason is that PR provides complete information of objects from noisy inputs. Several types of approach have been proposed to solve this problem, such as recognition base on key features, recognition base on the distribution of histogram. In studies about recognition using key features, Fuzzy Associative Memory (FAM) is an artificial neural network that solve effectively for PR. Advantages of FAM consist of compressing data and recalling from noisy inputs (noise tolerance). Therefore, FAM stores many patterns and retrieves stored patterns. In this paper, we present designs of effective FAM models and experiments to compare the ability of recall with nine types of noise. From results of experiments, we propose useful comments to choose an effective FAM model for pattern recognition applications.

Keywords: Fuzzy Associative Memory, Pattern Recognition, Associative Memory.

1 Introduction

Pattern recognition, a sub-branch of image processing, have attracted many scientists because of many applications in today life such as face recognition, reading bar-code, detecting mistakes on products, action recognition. The most important advantage that a application of PR must possess is correctly recognizing objects from noisy inputs. FAM is a artificial neural network that solve effectively for PR. FAM has two key advantages, including learning capacity of neural network, recalling from noisy inputs by using operators of Fuzzy logic and mathematical morphology (MM).

Thus, studies of FAM are various and can be divided into two categories, including developing the design of models and using models for applications. In the first category, operators of fuzzy set and MM are widely used in designing models [12,2,5,7]. Scientists also use operators of math for computing of models to suite for specific problems [17,15]. In the second category, FAMs are applied for many fields, including pattern recognition [13,12,14,17,15], control [10,18], estimation [16,1], prediction [19,11] and inference [6,3]. Previous studies show FAMs are widely apply for pattern recognition.

In this paper, we present designs of effective FAM models and experiments in grey-scale image recognition with nine types of noise in two working modes, including auto-association and hetero-association. These models possess three important advantages, including (i) learning and recalling process in an iteration, (ii) unlimited storage capacity, and (iii) high noise tolerance. The design of each model is presented the easy way to understand and compare. Experiments are conducted with nine types of noise of grey-scale images to compare the ability of recall among models. Then, we sum up useful comments to choose an effective model for an application of PR.

The rest of the paper is organized as follows. Back grounds of fuzzy set and MM are presented in Section 2. In section 3, we present the design of effective FAMs for PR. Section 4 shows experiments to compare noise tolerance among models. Then, we propose meaningful comments for choosing a effective model for applications. Conclusions and developing this study are written in Section 5.

2 Back Ground

2.1 Fuzzy Set [8]

Definition

Let X be a non-empty set. A fuzzy set A in X is characterized by its membership function $\mu_A : X \rightarrow [0, 1]$ and $\mu_A(x)$ is interpreted as the degree of membership of element x in fuzzy set A for each $x \in X$.

Operators of Fuzzy Logic

Let A and B are fuzzy subsets of a non-empty set X .

The *intersection* of A and B is defined as $A \wedge B(x) = \min\{\mu_A(x), \mu_B(x)\}$ for all $x \in X$

The *union* of A and B is defined as $A \vee B(x) = \max\{\mu_A(x), \mu_B(x)\}$ for all $x \in X$

The *complement* of fuzzy set A is defined as $\neg A(x) = 1 - \mu_A(x)$ for all $x \in X$

The *fuzzy conjunction* of A and B is an increasing mapping $C : [0, 1] \times [0, 1] \rightarrow [0, 1]$ that satisfies $C(0, 0) = C(0, 1) = C(1, 0) = 0$ and $C(1, 1) = 1$. For example, the minimum operator and the product are typical examples.

A fuzzy conjunction $T : [0, 1] \times [0, 1] \rightarrow [0, 1]$ that satisfies $T(x, 1) = x$ for $x \in [0, 1]$ is called *triangular norm* or *t-norm*. The fuzzy conjunctions C_M, C_P , and C_L are examples of t-norms.

$$C_M(x, y) = x \wedge y \tag{1}$$

$$C_P(x, y) = x \cdot y \tag{2}$$

$$C_L(x, y) = 0 \vee (x + y - 1) \tag{3}$$

A *fuzzy disjunction* is an increasing mapping $D : [0, 1] \times [0, 1] \rightarrow [0, 1]$ that satisfies $D(0, 0) = 0$ and $D(0, 1) = D(1, 0) = D(1, 1) = 1$.

A fuzzy disjunction $S : [0, 1] \times [0, 1] \rightarrow [0, 1]$ that satisfies $S(1, x) = x$ for every $x \in [0, 1]$ is called *triangular co-norm* or *short s-norm*. The following operators represent s-norms:

$$D_M(x, y) = x \vee y \tag{4}$$

$$D_P(x, y) = x + y - x.y \tag{5}$$

$$D_L(x, y) = 1 \wedge (x + y) \tag{6}$$

An operator $I : [0, 1] \times [0, 1] \rightarrow [0, 1]$ is called a *fuzzy implication* if I extends the usual crisp implication on $[0, 1] \times [0, 1]$ with $I(0, 0) = I(0, 1) = I(1, 1) = 1$ and $I(1, 0) = 0$. Some particular fuzzy implications:

$$I_M(x, y) = \begin{cases} 1, & x \leq y \\ y, & x > y \end{cases} \tag{7}$$

$$I_P(x, y) = \begin{cases} 1, & x \leq y \\ y/x, & x > y \end{cases} \tag{8}$$

$$I_L(x, y) = 1 \wedge (y - x + 1) \tag{9}$$

2.2 The Complete Lattice Framework of MM [4]

A Complete Lattice of MM

Mathematical morphology is a theory which is concerned with the processing and analysis of objects by using operators and functions based on topological and geometrical concepts. The most general mathematical framework of MM is given by complete lattices.

A complete lattice is defined as a partially ordered set L in which every (finite or infinite) subset has an infimum and a supremum in L . For any $Y \subseteq L$, the infimum of Y is denoted by the $\bigwedge Y$ and the supremum is denoted by the $\bigvee Y$. The class of fuzzy sets inherits the complete lattice structure of the unit interval $[0, 1]$.

Basic Operators of MM

Erosion and dilation are two basic operators of mathematical morphology. An *erosion* is a mapping ε from a complete lattice L to a complete lattice M that satisfies the following equation:

$$\varepsilon(\bigwedge Y) = \bigwedge_{y \in Y} \varepsilon(y) \tag{10}$$

Similarly, an operator $\delta : L \rightarrow M$ is called *dilation* if it satisfies the following equation:

$$\delta(\bigvee Y) = \bigvee_{y \in Y} \delta(y) \tag{11}$$

3 Effective FAMs for PR

Assuming that FAM stores p fuzzy pattern pairs that are noted (A^k, B^k) where $A_k = (A_1^k, \dots, A_1^m)$ and $B_k = (B_1^k, \dots, B_1^n)$.

3.1 Kosko’s Model [7]

The associative matrix W^k of the pattern pair (A^k, B^k) are computed by the following equation:

$$W_{ij}^k = \min\{A_i^k, B_j^k\} \tag{12}$$

and the general weight matrix W that stores all pattern pairs is formulated by below equation:

$$W_{ij} = \bigvee_{k=1}^p W_{ij}^k \tag{13}$$

With an input X , the output Y is calculated by the output function:

$$Y_j = \bigvee_{i=1}^m \min\{X_i, W_{ij}\} \tag{14}$$

3.2 The Model of Junbo et al. [5]

The associative matrix W_k of the pattern pair (A_k, B_k) are computed by the following equation:

$$W_{ij}^k = I_M(A_i^k, B_j^k) \tag{15}$$

and the general weight matrix W that stores all pattern pairs is formulated by below equation:

$$W_{ij} = \bigwedge_{k=1}^p W_{ij}^k \tag{16}$$

With an input X , the output Y is calculated by the output function:

$$Y_j = \bigvee_{i=1}^m \min\{X_i, W_{ij}\} \tag{17}$$

3.3 Models of Fulai and Tong [2]

The associative matrix W_k of the pattern pair (A_k, B_k) are computed by the following equation:

$$W_{ij}^k = I_M(A_i^k, B_j^k) \tag{18}$$

and the general weight matrix W that stores all pattern pairs is formulated by below equation:

$$W_{ij} = \bigwedge_{k=1}^p W_{ij}^k \tag{19}$$

With an input X , the output Y is calculated by the output function:

$$Y_j = \bigwedge_{k=1}^p \varphi(X_i, W_{ij}^k) \tag{20}$$

where φ is a t-norm.

3.4 Models of Ping Xiao et al. [17]

The associative matrix W^k of the pattern pair (A^k, B^k) are computed by the following equation:

$$W_{ij}^k = \varphi(A_i^k, B_j^k) \tag{21}$$

where φ is formulated as

$$\varphi(\mathbf{x}, \mathbf{y}) = \begin{cases} 1, & x = y \\ \frac{\min(A_i^k, B_j^k)}{\max(A_i^k, B_j^k)}, & x \neq y \end{cases} \tag{22}$$

and the general weight matrix W that stores all pattern pairs is formulated by below equation:

$$W_{ij} = \bigwedge_{k=1}^p W_{ij}^k \tag{23}$$

With an input X , the output Y is calculated by the output function:

$$Y_j = \bigvee_{i=1}^m X_i \cdot W_{ij} \tag{24}$$

3.5 Models of S.T.Wang et al. [15]

The associative matrix W_k of the pattern pair (A_k, B_k) are computed by the following equation:

$$W_{ij}^k = B_i^k / A_j^k \tag{25}$$

and the general weight matrix W that stores all pattern pairs is formulated by below equation:

$$W_{ij} = \bigwedge_{k=1}^p W_{ij}^k \tag{26}$$

OR

$$W_{ij} = \bigvee_{k=1}^p W_{ij}^k \tag{27}$$

With an input X , the output Y is calculated by the output function:

$$Y_j = \bigvee_{i=1}^m (X_j + W_{ij}) \tag{28}$$

OR

$$Y_j = \bigwedge_{i=1}^m (X_j + W_{ij}) \tag{29}$$

3.6 Models of Sussner and Valle [11]

The associative matrix W_k of the pattern pair (A_k, B_k) are computed by the following equation:

$$W_{ij}^k = \varphi(B_i^k, A_j^k) \tag{30}$$

where φ be I_M or I_P or I_L

and the general weight matrix W that stores all pattern pairs is formulated by below equation:

$$W_{ij} = \bigwedge_{k=1}^p W_{ij}^k \tag{31}$$

OR

$$W_{ij} = \bigvee_{k=1}^p W_{ij}^k \tag{32}$$

With an input X , the output Y is calculated by the output function:

$$Y_j = \varphi(X_j, W_{ij}) \vee \theta_j \quad (33)$$

where φ be s-norm.

θ is computed by:

$$\theta_j = \bigwedge_{k=1}^p B_j \quad (34)$$

4 Experiments and Summarise Useful Comments

We make experiments with grey-scale images to test noise tolerance of models. In each experiment, models of FAM are test with two working modes, including auto-association and hetero-association. Moreover, nine types of noise are applied for training inputs to make noisy inputs. We use the peak signal-to-noise ratio (PSNR) to measure quality between the training and a output image [9]. The higher the PSNR, the better the quality of the output image.

We choose 4 images from the grey-scale image database of tests in Matlab, including Cameraman, Pepper2, Zelda, Lena. Normal images are training patterns and nine noisy images are inputs for experiments. Noisy inputs are made from the training images by using Matlab's function. Nine types of noisy inputs, including on and off pixels, Gaussian white noise, dilate image with structure DISK, erode image with structure DISK, morphological open image with structure DISK, morphological close image with structure DISK, erode image with structure BALL, erode image with structure LINE, and dilate image with structure LINE.

Six models of FAM are similar in both learning and recalling process. Therefore, we only interest noise tolerance of models. Studies of Sussner and Valle, Fulai and Tong, and S.T.Wang et al. proposed a family of FAM but we only show results of best models.

4.1 Experiments

Experiment 1: FAMs in Auto-association Mode

In auto-association mode, $B_k = A_k$ with all pattern pairs. Thus, this experiment suits to applications such as face recognition, bar-code recognition. Table 1 presents PSNR of models with nine types of noisy inputs.

Bold numbers in Table 1 show models that are the best for each type of noise. In summary, two models of Wang are the best models for four types of noise (respectively 1, 2, 3, 9) and models of Sussner are the best models for types of noise from 4 to 8 in auto-association mode.

Table 1. PSNR of models with nine noisy types in Experiment 1

Noise	Type1	Type2	Type3	Type4	Type5	Type6	Type7	Type8	Type9
Fulai1	8.9	18.7	18.7	32.2	33.2	23.4	29.1	31.6	18.5
Fulai2	14.9	28.6	25.9	51.9	62.5	36.9	1000.0	52.7	24.8
Junbo	14.9	28.6	25.9	51.9	62.5	36.9	1000.0	52.7	24.8
Kosko	23.1	23.4	23.5	25.0	24.6	23.8	23.8	24.6	23.8
Ping	16.6	28.9	29.0	49.5	59.6	39.0	656.2	51.3	28.8
Sussner1	11.7	24.1	24.1	59.8	68.9	34.8	651.2	61.7	22.5
Sussner2	14.9	29.6	26.3	45.0	58.5	40.5	1000.0	47.2	25.0
Sussner3	14.9	28.6	25.9	51.9	62.5	36.9	1000.0	52.7	24.8
Wang1	30.1	30.1	30.1	30.1	30.1	30.1	30.1	30.1	30.1
Wang2	30.1	30.1	30.1	30.1	30.1	30.1	30.1	30.1	30.1

Experiment 2: FAMs in Hetero-association Mode

In hetero-association mode, $B_k \neq A_k$ with all pattern pairs. Thus, this experiment suits to applications that provide patterns associated with inputs. For example, proposing the face image of a person from a noisy input of fingerprint image. Table 2 presents PSNR of models with nine types of noisy inputs.

Table 2. PSNR of models with nine noisy types in Experiment 2

Noise	Type1	Type2	Type3	Type4	Type5	Type6	Type7	Type8	Type9
Fulai1	9.0	18.5	18.4	25.4	26.4	21.0	25.3	25.9	18.8
Fulai2	14.8	27.8	25.8	35.2	36.0	31.1	37.8	35.5	25.3
Junbo	14.8	27.8	25.8	35.2	36.0	31.1	37.8	35.5	25.3
Kosko	24.2	24.7	24.7	27.9	26.7	25.4	25.4	26.9	25.4
Ping	21.4	31.3	30.9	35.7	38.2	35.3	41.1	37.3	30.9
Sussner1	12.7	25.2	25.0	42.5	45.4	31.9	47.6	43.1	24.4
Sussner2	14.8	27.9	25.6	31.6	33.7	31.3	35.7	32.5	25.1
Sussner3	14.8	27.8	25.8	35.2	36.0	31.1	37.8	35.5	25.3
Wang1	13.5	13.5	78.8	17.4	27.8	87.4	366.8	17.7	212.3
Wang2	30.1	30.1	30.1	30.1	30.1	30.1	30.1	30.1	30.1

Bold numbers in Table 2 show models that are the best for each type of noise. To sum up, models Wang are the best models for types of noise (respectively 1, 3,6, 7, 9), the first model of Sussner for three types (respectively 4, 5, 8), and the model of Ping for Type 2 in hetero-association mode. However, the model of Ping is good for all types of noise.

4.2 Summarise Useful Comments

Each application features by data, algorithm. Moreover, the most important advantage of PR is providing complete information of objects from noisy inputs. Therefore, each application should choose a best model base on noisy types of inputs and working mode. We propose following useful comments:

1. In auto-association mode with each type of noise, we sort models of authors by the descending order of the ability of recall in each column of Table 3. When PSNR of the best model of each type of noise is low, use the second model because testing data are abnormal.

Table 3. Choose a suitable model for each type of noise in auto-association mode

Type1	Type2	Type3	Type4	Type5	Type6	Type7	Type8	Type9
Wang Kosko	Wang Ping	Wang Ping	Sussner Junbo Fulai	Sussner Junbo Fulai	Sussner Ping	Sussner Junbo Fulai	Sussner Junbo Fulai	Wang Ping

2. Similarly, we list comments to choose an effective model in hetero-association mode in Table 4. Data show the model of Ping is the best models for all types of noise.

Table 4. Choose a suitable model for each type of noise in hetero-association mode

Type1	Type2	Type3	Type4	Type5	Type6	Type7	Type8	Type9
Wang Kosko Ping	Ping Wang Junbo	Wang Ping	Sussner Ping	Sussner Ping	Wang Ping	Wang Sussner Ping	Sussner Ping	Wang Ping

3. Select models of Sussner or Wang in auto-association mode and select the model of Ping in hetero-association for applications that have to work with complex or unknown types of noise.

5 Conclusion

In this paper, we summarise studies about the design of effective FAMs for PR in the simple way to understand and compare by describing equations in a clear way and using uniform signs. With each model, we propose equations to compute the associative weight matrix of each pattern pair, the general weight matrix of all pattern pair, and outputs of the given input. Moreover, we conduct many sub-experiments to measure the ability of recall of models with two working modes. In each working mode, nine types of noise are used to test. From results of experiment, we propose three meaningful comments to choose a effective model for an application of PR.

FAMs also apply for medicine, economy, and machine because FAMs work well with uncertain data for tasks such as estimation, prediction and inference. Moreover, on-line learning rule is an idea method that allows FAMs meet the need of real-time application. We will investigate to develop an on-line learning rule of FAM for estimation and prediction in the future.

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