Diameter Restricted Fault Tolerant Network Design

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Abstract. Low transmission delay, high fault tolerance and low design cost are the three main properties of any network which are best described by its topology. Transmission delay can be decreased by restricting the diameter of the network. Very few methods in literature have considered the importance of the diameter of the network to decrease the transmission delay. Fault tolerance in the network depends on the number of disjoint paths between a node pair. Designing a k-connected fault tolerant network subject to connectivity and diameter constraint at minimal cost is a NP hard problem. In this paper, an efficient constructive heuristic algorithm is proposed for designing a k-connected network while optimizing the cost of the network subject to the connectivity and diameter constraints. Diameter of resultant network would be of two links regardless of network size to get the speed comparable to complete connected network at low cost. Effectiveness of the proposed approach is also evaluated using different examples.

Keywords: Diameter, k-connected Network, Fault Tolerant, Eccentricity.

1 Introduction

Nowadays, communication network is used in every field. All enterprises have become dependent upon network or networked computation applications [1]. Speed and fault tolerance are two main parameters which should be high for a network to meet the high demand of availability. Transmission delay between the node pair not only depends on the data rate of the links in the path but also depends on the network topology. To route the data between two nodes, shortest route (length is measured as number of nodes traversed) is found between the two nodes for a speed up delivery. So diameter plays a very important role in the performance of the network as number of nodes along a shortest path for each node pair is best described by network diameter. Furthermore, fault tolerance of the network also depends on the topology. If there are multiple disjoint paths between the nodes, alternating path may be used in case of failure in the network.

Topological design of a k-connected network is a well known optimization problem. The risk of combinatorial explosion is obvious [2]. Heuristics methods [3] based on Genetic Algorithm [4], [5], [6] and Simulated Annealing [7] are not

V.V. Das and J. Stephen (Eds.): CNC 2012, LNICST 108, pp. 127–135, 2012.

computational effective as these require feasibility and optimality check for each intermediate topology until a sub optimal topology is found and these are not suitable for ad-hoc networks where we need the resultant topology quickly.

Various constructive heuristic approximation algorithms have also been used for designing k-connected network topology. These approximation algorithms generate topology from scratch by adding low cost links until a topology satisfying the desired constraints is found. Number of links and design cost are the parameters which are used for measuring the efficiency of the constructive algorithms in this paper. In [8] minimum number of links are much larger than the optimal number of links and there is also no restriction on diameter of the resultant network. In another approach [9], minimum number of links required are k*(n-k) which are near to optimal only when k>(n/2). But this is hardly required in any network [7]. It is also analyzed that when k is greater than n/2, then the resultant network using this approach would not be a k-connected network. In [10], cost of the resultant network is very much dependent on numbering of the nodes and may be less for the same network , numbered differently and there is no restriction on the diameter of the resultant network. Node degree is used here for connectivity which is not a sufficient condition [11]. Wassim El-Hajj et.al. [12] proposed an approach for designing a network of diameter of two links only regardless of network size. But design cost is not so much optimized in this approach.

In this paper, an efficient constructive heuristic algorithm is proposed for designing the topology subject to fault tolerance and diameter constraint while optimizing the cost. Diameter of the network is set to two regardless of network size as we want to design a fast network whose speed would be comparable to complete connected network at low cost. The resultant network would be a $\sqrt{n}$ connected network, where n is number of nodes. The paper is organized as follows: proposed constructive heuristic algorithm is explained in Section 2. In Section 3, we provide some examples of network design problem illustrated using our proposed approach. Comparative analysis of our proposed approach and other existing approaches is given in section 4. We end the paper with the concluding remarks in section 5.

2 Proposed Constructive Heuristic Algorithm

The proposed approach searches a subset of low cost links to design a k-connectivity network having two links diameter. k is here equal to $\sqrt{n}$ as this is the minimum connectivity requirement to design a network having two link diameter. The starting node from where the design starts significantly affects the cost of the resultant network. Keeping this in mind, we have proposed to start the topology design with the node whose eccentricity is the highest among all nodes in the network. Eccentricity of a node is the distance/cost from a node to its farthest node. Using this way we can appreciably reduce network cost because no two distant nodes would ever be connected as the chosen farthest node is always connected to its k nearest nodes.

First step is finding the node having highest eccentricity. For example, node s has the highest eccentricity. Node s is first connected to its k nearest nodes directly to satisfy the connectivity constraint. Once the connectivity requirement of node s is satisfied, next step is to connect it to all other remaining nodes in the network via its directly connecting k nodes to satisfy the two link diameter constraint. Now node s satisfies both constraints i.e. k connectivity and two links diameter. Next, we again find out the next highest eccentricity node i.e q in the set of the nodes in the given network excluding node s. Connect node q to k-degree(q) number of its nearest node to satisfy the connectivity condition. Now we check the length of the shortest path to all nodes from node q. If any node is unreachable to node q or having the shortest path of length more than two links then connect these nodes to their nearest directly connected nodes of node q. This procedure is repeated for all nodes. Resultant network would be a low cost k-connected network having two links diameter as at every step in our proposed approach we start with a highest eccentricity node and connect it to its nearest node so that no two farthest nodes are ever connected directly. Step by step details of proposed algorithm is given below.

1. Start with the highest eccentricity node s in set V. Set V = V- s
2. Find (k - degree(s)) number of nearest nodes from s and store them in set C.
3. Connect s to all its nearest nodes in set C either directly or via some other node depending upon the degree of the nodes in set C.
4. Set P = V- {C}
5. Remove the farthest node d of node s from the set P.
6. Connect node d to the nearest direct connecting node v of node s depending upon the degree of node s, node d, direct connecting nodes of s and direct connecting nodes of d.
7. Repeat steps 5 and 6 until set P is empty.
8. Repeat all above steps 1- 7 until the set V is empty.
9. Resultant graph would be a k-connected network having two links diameter.

Step 3 and Step 6 is about to connect highest eccentricity node to all other nodes of network either directly or via an intermediate node. Degree of the nodes plays a very important role in determining whether two nodes would be connected directly or through an intermediate node. In step 3, suppose node u is the nearest node in set C from node s. If the degree of node u is less than k then the node s and node u are connected directly. Otherwise if u has degree equal or greater than k, then node s would be connected to one of the direct connecting nodes of node u which is nearest to node s. And if all direct connecting nodes of u also have degree equal or greater than k then find out nearest node among node u and its direct connecting nodes to node s. So using this way, we directly connect only two nearest nodes. Step 6 is responsible for restricting the network diameter of two links only. So once the connectivity of a chosen node at step 3 is satisfied, then it is connected to remaining nodes via its direct connecting nodes so that it can reach to every other nodes either directly or using only one intermediate node. So step 6 are used for constructing path of maximum two link diameter from the currently chosen highest eccentricity node to all other nodes in the network. Working of step 3 and step 6 is best described by algorithm Create_ Link as given in fig.1.

```
Algorithm Create_Link(s,d)
Begin
1.  S= direct_connecting_nodes(s)
2.  D= direct_connecting_nodes(d)
3.  If ((Degree(s) < k)) and (Degree(d) <k)) then
          connect(s,d)
    Else if ((Degree(s<k)) and (Degree(d≥k))) then
          If (deg(D < k) then
                    m= find_nearest( s, D)
                    Connect(s,m)
          Else
                    m= find(s, d, D)
                    Connect(s,m)
    Else If ((Degree(s) ≥ k) and (Degree (d) <k)) then
          If (deg (S) < k)
                    m= find_nearest (d, S)
                    Connect (d, m)
          Else
                    m = find (d, s, S)
                    Connect (d, m)
    Else if ((Degree(s) ≥k) and (Degree (d) ≥k))
          If ((deg(S) < k) and (deg(D)< k))
                    m= find_nearest (d, S)
                    m1=find_nearest (s, D)
                    If (m< m1) then
                              Connect (d,m)
                    Else
                              Connect(s,m1)
          Else if ((deg (S) < k) and (deg (D) ≥ k))
                    m= find_nearest ( d, S)
                    Connect (d, m)
          Else if ((deg (S) ≥k) && deg (D <k))
                    m= find_nearest (s, D)
                    Connect(s, m)
          Else if ((deg (S) ≥ k) and (deg (D) ≥ k))
                    m= find (d, s, S)
                    m1=find (s, d, D)
                    If (m< m1) then
                              Connect(d,m)
                    Else
                              Connect(s, m1)
End
```

Direct_connecting_node(s) : This function return an array containing all direct connecting nodes of a node s .

Deg(S) :This function return the degree of a node who has lowest degree among all all nodes stored in S.

Fig. 1. Algorithm Create_Link

3 Design Examples

In this section we will illustrate our proposed approach to design two 7 nodes and 13 nodes networks.

Example 1

Number of nodes n=7
Connectivity $k = \sqrt{n} = 3$

Let us consider 7 nodes network as shown in the fig. 2. Table1 gives the cost matrix constructed out of the cost (which is a function of distance) associated with the pair of nodes. Cost and distance between two nodes are considered same here.

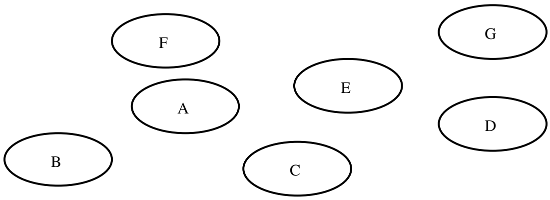

Fig. 2. Given network

Table 1. Cost associated with every pair of nodes

	A	B	C	D	E	F	G
A	0	5	3	7	4	2	9
B	5	0	5	10	8	6	12
C	3	5	0	6	4	5	10
D	7	10	6	0	4	8	5
E	4	8	4	4	0	5	5
F	2	6	5	8	5	0	8
G	9	12	10	5	5	8	0

From the table 1, it is clear that node B, node G are the two farthest node in this network. Choose either of two. Suppose, B is chosen. Degree of node B is 0. So, connect B to its k nearest nodes i.e. A, C and F directly. Rest of the nodes (D,E,G) in the network would be connected to node B via either of its direct connecting nodes (A,C,F) because degree of node B is now k. Next, choose the farthest node to B from the node set {D, E, G}. Node G is the farthest node from node B. Node G is connected to its nearest node F which is the nearest node among nodes A, C and F. Again choose the farthest node to B from the node set {A, C}. Node A is chosen this time and it is connected to its nearest node i.e. node E from nodes set {D, E, G}. Finally, the last remaining node D is connected to its nearest node i.e C from the node set {A, C, F}. Resultant network having two links diameter for node B is shown in fig. 3 which shows the network in which node B is connected to every other nodes either directly or using one intermediate node only.

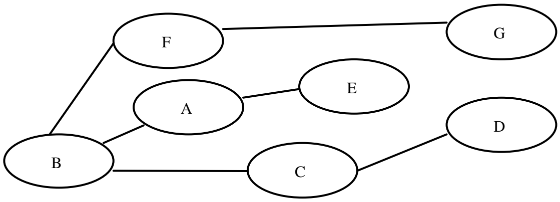

Fig. 3. Network having two link diameter for node B

As diameter requirement for node B is satisfied, So node B does not participate in finding the next node having the highest eccentricity. Node C and node F has highest eccentricity now. Choose any of the nodes. Suppose, node C is chosen. Degree of node C is two. It has deficiency of one link to make its degree k. Connect node C to its nearest node in the network excluding node B. Node C is connected to node E as it is the most nearest node among the nodes A,F,G and E. Nodes A, F and G would now be connected to node C via some intermediate nodes. Now farthest node to node C from the set {A, F, G} is node G. Connect node G to its nearest node among the direct connecting node of C. Connect Node G to node D as it is the nearest node to G. It is found that node A and node F in network are already reachable to node C via node B. Fig. 4 shows the resultant network after applying the same procedure to node C. Now node B and node C have diameter of two links.

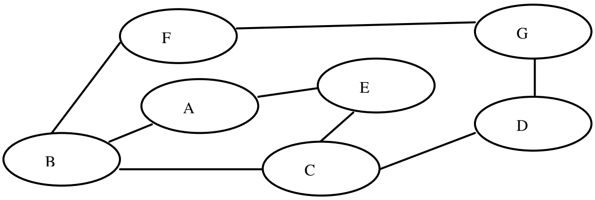

Fig. 4. Network having diameter of two links for node B and node C

Same procedure is applied to every node until the all nodes's diameter becomes of two links. So the final network is shown in fig. 5 having two links diameter of network.

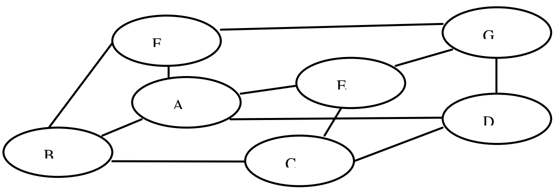

Fig. 5. Resultant 3-connected network having diameter of two links

Example: 2

Number of nodes n = 13
Connectivity k = √n = 4

Let us consider 13 nodes network as shown in the figure 6. Table 2 gives the cost (distance) matrix constructed out of the cost associated with the pair of nodes.

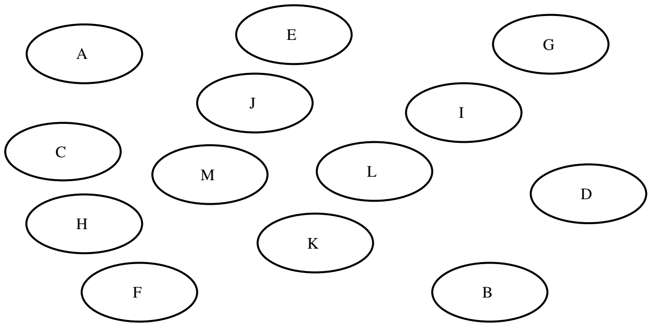

Fig. 6. Given network

Table 2. Cost associated with every pair of nodes

	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**	**L**	**M**
A	0	15	5	10	4	9	8	7	10	6	8	10	6
B	15	0	10	8	12	6	10	8	8	9	7	8	8
C	5	10	0	12	6	8	7	6	9	5	4	6	4
D	10	8	12	0	8	8	6	7	5	6	6	5	7
E	4	12	6	8	0	8	4	6	5	4	6	6	5
F	9	6	8	8	8	0	7	4	6	5	4	5	4
G	8	10	7	6	4	7	0	6	6	5	4	5	3
H	7	8	6	7	6	4	6	0	6	5	3	5	3
I	10	8	9	5	5	6	6	5	0	4	5	3	4
J	6	9	5	6	4	5	5	5	4	0	3	4	2
K	8	7	4	6	6	4	4	3	5	3	0	4	2
L	10	8	6	5	6	5	5	5	3	4	3	0	3
M	6	8	4	7	5	4	3	3	4	2	2	3	0

Resultant network after applying our proposed approach on network in fig. 6 is shown in fig. 7.

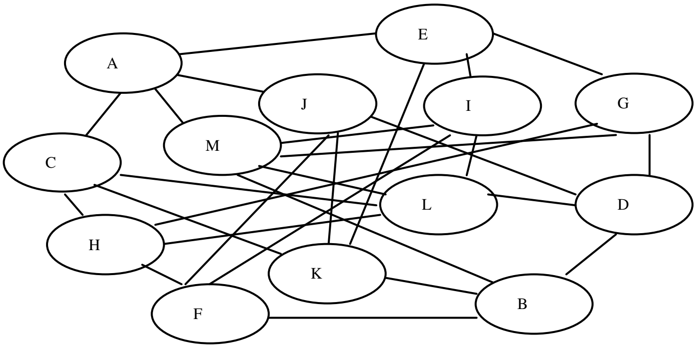

Fig. 7. Resultant network having diameter of two links

4 Comparative Results

In this section, our proposed approach is compared with the existing approaches [9], [12] as these are the only approaches which design a network of two links diameter. Network design cost and number of links used to design a network are considered as two parameters for comparison. Comparative analysis for link optimization and design cost of two networks designed in previous section is given in table 3. It is found that our proposed approach always results into less number of links and low design cost than the existing approaches.

Table 3. Comparative analysis for link optimization and design cost for two networks

Approach	n=7		n=13	
	Number of links required	Resultant design cost	Number of links required	Resultant design cost
Proposed approach in this paper	11	58	27	140
Approach in [12]	14	80	39	193
Approach in [9] *	12	71	36	245
Does not results into k-connected network when (k>(n/2))				

5 Conclusion and Future Work

In this paper, a constructive heuristic approach is proposed for designing a cost effective and √n-connected network having diameter of two links only. The resultant network is resilient to √n-1 link failures in the network at a very low design cost. It is analyzed that the proposed approach is very efficient than the existing approaches for designing a fault tolerant network of two link diameter. In future, we extend this work for designing a network of any given diameter.

Acknowledgement. Prof. P. C. Saxena highly acknowledges UGC for supporting this research.

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