A Game Theoretic Optimization of the Multi-channel ALOHA Protocol

Kobi Cohen, Amir Leshem, and Ephraim Zehavi

Faculty of Engineering, Bar-Ilan University,
Ramat-Gan, 52900, Israel
kobi.cohen10@gmail.com

Abstract. In this paper we consider the problem of distributed throughput maximization of networks with multi-channel ALOHA medium access protocol. In the multi-channel ALOHA protocol, each user tries to randomly access a channel using a probability vector defining the access probability to the various channels. First, we characterize the Nash Equilibrium Points (NEPs) of the network when users solve the unconstrained rate maximization. We show that in this case, for any NEP, each user's probability vector is a standard unit vector (i.e., each user tries to access a single channel with probability one and does not try to access other channels). Specifically, when the number of users, N, is equal to the number of channels there are $N!$ NEPs. However, when the number of users is much larger than the number of channels, most of the users get a zero utility (due to collisions). To overcome this problem we propose to limit each user's total access probability and solve the problem under a total probability constraint. We characterize the NEPs when user rates are subject to a total transmission probability constraint. We propose a simple best-response algorithm that solves the constrained rate maximization, where each user updates its strategy using its local channel state information (CSI) and by monitoring the channel utilization. We prove that the constrained rate maximization can be formulated as an exact potential game. This implies that convergence of the proposed algorithm is guaranteed. Finally, we provide numerical examples to demonstrate the algorithm's performance.

Keywords: Collision channels, multi-channel ALOHA, Nash equilibrium point, best response, potential games.

1 Introduction

In typical wireless communication networks, the bandwidth is shared by several users. Medium Access Control (MAC) schemes are used to manage the access of users to the shared channels. The slotted ALOHA access protocol is popular due to its simple implementation and random-access nature [1]. In each time-slot, a user may access a shared channel according to a specific transmission probability. Transmission is successful only if a single user tries to access a shared channel in a given time-slot. If more than one user transmits at the same time slot over

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the same channel a collision occurs. Here, we examine the ALOHA protocol with multi-channel systems, dubbed multi-channel ALOHA. In multi-channel systems, the bandwidth is divided into K orthogonal sub-bands using Orthogonal Frequency Division Multiple Access (OFDMA). Each sub-band can be a cluster of multiple carriers. A diversity of channel realizations is advantageous when users exploit local CSI to access good channels. Multi-channel systems are widely investigated recently in cognitive radio networks, where cognitive users share an unlicensed spectrum band, while avoiding interferences with licensed users. A related work on this subject can be found in [2–6].

In distributed optimization algorithms, users take autonomous decisions based on local information and coordination or massage passing between users are not required. Therefore, in wireless networks, distributed optimization algorithms are simple to implement and generally preferred over centralized solutions. A natural framework to analyze distributed optimization algorithms in wireless networks is non-cooperative game-theory. A related work on this subject can be found in [7–12].

In this paper we present a game theoretic approach to the problem of distributed rate maximization of multi-channel ALOHA networks. In the multi-channel ALOHA protocol, each user tries to randomly access a channel using a probability vector defining the access probability to the various channels. First, we characterize the Nash Equilibrium Points (NEPs) of the network when users solve the unconstrained rate maximization. We show that in this case, for any NEP, each user's probability vector is a standard unit vector (i.e., each user occupies a single channel with probability one and does not try to access other channels). When considering the unconstrained rate maximization, we are mainly interested in the case where the number of channels is greater or equal to the number of users, to avoid collisions. Specifically, in the case where the number of users, N, is equal to the number of channels there are $N!$ NEPs. However, when the number of users is much larger than the number of channels, most users get a zero utility (due to collisions). To overcome this problem we propose to limit each user's total access probability and solve the problem under a total probability constraint. We characterize the NEPs when user rates are subject to a total transmission probability constraint. We propose a simple best-response algorithm that solves the constrained rate maximization, where each user updates its strategy using its local CSI and by monitoring the channel utilization. We prove that the constrained rate maximization can be formulated as an exact potential game [13]. In potential games, the incentive of all players to change their strategy can be expressed in a one global function, the potential function. The existence of a bounded potential function corresponding to the constrained rate maximization problem implies that the convergence of the proposed algorithm is guaranteed. Furthermore, the convergence is in finite time, starting from any point and using any updating dynamics across users.

The rest of this paper is organized as follows. In section 2 we present the network model and game formulation. In section 3 and 4 we discuss the unconstrained and the constrained rate maximization problems, respectively. In section 5 we provide simulation results to demonstrate the algorithm performance.

2 Network Model and Game Formulation

In this paper we consider a wireless network containing N users who transmit over K orthogonal collision channels. The users transmit using the slotted ALOHA scheme. In each time slot each user is allowed to access a single channel. A transmission can be successful only if no other user tries to access the same channel simultaneously. In this paper we denote the collision-free achievable rate of user n at channel k by $u_n(k)$. Furthermore, we define a virtual zero-rate channel $u_n(0) = 0, \ , \forall n$, i.e., accessing a channel $k = 0$ refers to no-transmission.

The collision-free rate vector of user n in all $K+1$ channels is given by:

$$\mathbf{u}_n \triangleq \left[u_n(0) \; u_n(1) \; u_n(2) \cdots \; u_n(K) \right] \ , \tag{1}$$

and the collision-free rate matrix of all N users in all $K+1$ channels is given by:

$$\mathbf{U} \triangleq \begin{bmatrix} u_1(0) & u_1(1) & u_1(2) & \cdots & u_1(K) \\ u_2(0) & u_2(1) & u_2(2) & \cdots & u_2(K) \\ \vdots & & & & \\ u_N(0) & u_N(1) & u_N(2) & \cdots & u_N(K) \end{bmatrix} \ . \tag{2}$$

Let $p_n(k)$ be the probability that user n tries to access channel k. Let $\mathcal{P}_n$ be the the set of all probability vectors of user n in all $K+1$ channels. A probability vector $\mathbf{p}_n \in \mathcal{P}_n$ of user n is given by:

$$\mathbf{p}_n \triangleq \left[p_n(0) \; p_n(1) \; p_n(2) \cdots \; p_n(K) \right] \ , \tag{3}$$

Let $\mathcal{P}$ be the set of all probability matrices of all N users in all $K+1$ channels. The probability matrix $\mathbf{P} \in \mathcal{P}$ is given by:

$$\mathbf{P} \triangleq \begin{bmatrix} p_1(0) & p_1(1) & p_1(2) & \cdots & p_1(K) \\ p_2(0) & p_2(1) & p_2(2) & \cdots & p_2(K) \\ \vdots & & & & \\ p_N(0) & p_N(1) & p_N(2) & \cdots & p_N(K) \end{bmatrix} \ , \tag{4}$$

where $\sum_{k=0}^{K} p_n(k) = 1 \ \forall n$.

Let $\mathcal{P}_{-n}$ be the set of all probability matrices of all N users in all $K+1$ channels, except user n. The probability matrix $\mathbf{P}_{-n} \in \mathcal{P}_{-n}$ is given by:

$$\mathbf{P}_{-n} \triangleq \begin{bmatrix} p_1(0) & p_1(1) & p_1(2) & \cdots & p_1(K) \\ \vdots & & & & \\ p_{n-1}(0) & p_{n-1}(1) & p_{n-1}(2) & \cdots & p_{n-1}(K) \\ p_{n+1}(0) & p_{n+1}(1) & p_{n+1}(2) & \cdots & p_{n+1}(K) \\ \vdots & & & & \\ p_N(0) & p_N(1) & p_N(2) & \cdots & p_N(K) \end{bmatrix} \ , \tag{5}$$

We focus in this paper on stationary access strategies, where each user decides whether or not to access a channel based on the current utility matrix and all other users' strategy.

Definition 1: A stationary strategy for user n is a mapping from $\{\mathbf{P}_{-n}, \mathbf{u}_n\}$ to $\mathbf{p}_n \in \mathcal{P}_n$.

Remark 1: Note that $\mathbf{u}_n$ depends on the local CSI of user n, which can be obtained by a pilot signal in practical implementations. On the other hand, in the sequel we show that user n does not need the complete information on the matrix $\mathbf{P}_{-n}$ to update its strategy, but only to monitor the channel utilization by other users, defined by:

$$q_n(k) \triangleq 1 - \prod_{i=1\ ,i \neq n}^{N} (1 - p_i(k)) \ . \tag{6}$$

Remark 2: We refer the probability matrix $\mathbf{P}$ as the multi-strategy contained all users' strategy, and $\mathbf{P}_{-n}$ as the multi-strategy contained all users' strategy except the strategy of user n.

When user n perfectly monitors the k^{th} channel utilization, it observes:

$$v_n(k) \triangleq 1 - q_n(k) = \prod_{i=1\ ,i \neq n}^{N} (1 - p_i(k)) \ , \tag{7}$$

which is the probability that the k^{th} channel is available.

Since a collision occurs when more than one user tries to access the same channel, the achievable rate of user n in the k^{th} channel is given by:

$$r_n(k) \triangleq u_n(k) v_n(k) \ . \tag{8}$$

Hence, the achievable expected rate of user n is given by:

$$R_n \triangleq R_n(\mathbf{p}_n, \mathbf{P}_{-n}) = \sum_{k=1}^{K} p_n(k) r_n(k) \ . \tag{9}$$

In this paper, we consider a distributed rate maximization problem, where each user tries to maximize its own expected rate subject to a total transmission probability constraint:

$$\max_{\mathbf{p}_n} \quad R_n \quad \text{s.t.} \quad \sum_{k=1}^{K} p_n(k) \leq P_{max} \ . \tag{10}$$

We are interested in unconstrained (i.e., $P_{max} = 1$) and constrained (i.e., $P_{max} < 1$) NEP solutions of this game. A NEP for our model is a multi-strategy $\mathbf{P}$, given in (4), which is self-sustaining in the sense that non of the users can increase its utility by unilaterally modifying its strategy $\mathbf{p}_n$.

Definition 2: A multi-strategy $\mathbf{P}$ is a Nash Equilibrium Point (NEP) if

$$R_n(\mathbf{p}_n, \mathbf{P}_{-n}) \geq R_n(\tilde{\mathbf{p}}_n, \mathbf{P}_{-n}) \ \ \forall n \ , \forall \tilde{\mathbf{p}}_n \ . \tag{11}$$

Formally, we define the non-cooperative multi-channel ALOHA game in this paper as follows:

Definition 3: The non-cooperative multi-channel ALOHA game (10) is given by $\Gamma = (\mathcal{N}, \mathcal{P}, R)$, where $\mathcal{N} = \{1, 2, ..., N\}$ denotes the set of players (or users), $\mathcal{P}$ denotes the set of multi-strategies and $R : \mathcal{P} \to \mathbb{R}^N$ denotes the payoff (i.e., rate) function.

Next, we examine the unconstrained and constrained NEP solutions of this game (10).

3 Unconstrained Rate Maximization

In this section, we characterize unconstrained NEP solutions of this game (10). Here, we set $P_{max} = 1$ in (10). When considering unconstrained solutions, we are mainly interested in the case where $K \geq N$ to avoid collisions. Practically, each user monitors the channel utilization $v_n(k)$ for all $k = 1, ..., K$ (i.e., the complete $\mathbf{P}_{-n}$ is not required), and tries to access only a single available channel, which is the best response to all users' strategy $\mathbf{P}_{-n}$ (5).

Theorem 1. *Assume that $P_{max} = 1$ in (10). Then:*

a) For any NEP, each user's probability vector is a standard unit vector with probability 1 (i.e., each user tries to access a single channel with probability one and does not try to access other channels).
b) The network converges to a NEP in N iterations.

Proof. The proof is given in [14].

We infer from Theorem 1 that the unconstrained distributed rate maximization is equivalent to a channel assignment problem, where each user chooses a single channel. Once a channel is taken by some user, no other user can access the same channel, since it has a zero utility. A good distributed solution to (10) is obtained via distributed opportunistic access [15] combined with the Gale-Shapley algorithm [16] to achieve a stable channel assignment, as was done in [4, 5]. For details the reader is referred to [14].

In the general case where $N = K$ any permutation that avoids a collision is a NEP. For instance, in the case of 3 users and 3 channels, the following multi-strategy is a NEP:

$$\mathbf{P} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} , \tag{12}$$

since any user that unilaterally modifies its strategy gets a zero utility (due to collision or no-transmission). In this case we have $N!$ NEPs.

In the case where $K > N$ any permutation that avoids a collision and maximizes every users' rate (given other users' strategies) is a NEP. For instance, consider the case of 2 users and 3 channels and assume that $u_1(3) \leq u_1(2)$ and $u_2(3) \leq u_2(1)$. The following multi-strategy is a NEP:

$$\mathbf{P} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} , \tag{13}$$

since non of the users can increase its utility by unilaterally modifying its strategy $\mathbf{p}_n$. As a result, there exist $(K \cdot (K-1) \cdots (K-N+1))$ NEPs at most.

In the case where $N > K$ any permutation is a NEP if at least K users access different K channels. For instance, in the case of 3 users and 2 channels, the following multi-strategy is a NEP:

$$\mathbf{P} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} , \tag{14}$$

since any user that unilaterally modifies its strategy gets a zero utility (due to a collision or accessing the virtual channel). Note that a better NEP can be obtained if users 2 or 3 access the virtual channel (i.e., do not transmit).

4 Constrained Rate Maximization

We now discuss the more interesting case, where $N > K$. In this case, unconstrained solutions lead to collisions or to zero utilities for some users. Therefore, constrained solutions should be used. According to Theorem 1, setting $P_{max} < 1$ is necessary to avoid collisions (otherwise, all users access a single channel with probability one). First, we show the following result:

Theorem 2. *Assume that $P_{max} < 1$ in (10). Let $r_n(k^*) = \max_k \{r_n(k)\}$, where $r_n(k)$ is defined in (8). Then, each user n plays the strategy:*

$$p_n(k) = \begin{cases} 1 - P_{max} \, , & \text{if } k = 0 \\ P_{max} \, , & \text{if } k = k^* \\ 0 \, , & \text{otherwise} \end{cases} , \tag{15}$$

with probability 1.

Proof. The proof is given in [14].

We infer from Theorem 2 that in each iteration each user will access a single channel with probability P_{max} and will not try to access other channels. However, in contrast to the unconstrained solutions, other users can still access occupied channels, since the utility is strictly positive in all channels. We discuss the convergence later.

As a result of Theorem 2, we obtain a best response algorithm, given in Table 1. The proposed algorithm solves the constrained rate maximization problem (10). In the initialization step, each user selects the channel with the maximal collision-free rate $u_n(k)$. This can be done by all users simultaneously in a single iteration. Then, each user occasionally monitors the channels utilization and updates its strategy by selecting the channel with the maximal achievable rate $r_n(k)$ given the channels utilization.

Table 1. Proposed best response algorithm

%——initializing——————————

- for all $n = 1, ..., N$ users do:
- estimate $u_n(k)$ for all $k = 1, ..., K$
- $k^* \leftarrow \arg\max_k \{u_n(k)\}$
- $p_n(k^*) \leftarrow P_{max}$
- end for

%——end initializing——————————

- repeat:
- estimate $v_n(k)$ for all $k = 1, ..., K$
- compute $r_n(k) = u_n(k)v_n(k)$ for all $k = 1, ..., K$
- $k^* \leftarrow \arg\max_k \{r_n(k)\}$
- $p_n(k^*) \leftarrow P_{max}$
- until convergence

Next, we examine the convergence of the proposed algorithm. In contrast to the unconstrained solutions, convergence of the algorithm is not guaranteed in N iterations. However, in the following we use the theory of potential games to show that the constrained rate maximization (10) indeed converges in finite time. In potential games, the incentive of all players to change their strategy can be expressed as a single global function, the potential function. In exact potential games, the improvement that each player can get by unilaterally changing its strategy equals to the improvement in the potential function. Hence, any local maximum of the potential function is a NEP. The existence of an exact bounded potential function corresponding to the constrained rate maximization problem (10) implies that the convergence of the proposed algorithm is guaranteed. Furthermore, the convergence is in finite time, starting from any point and using any updating dynamics across users.

Definition 4 [13]: A game $\Gamma = \left(\mathcal{N}, \mathcal{P}, \tilde{R}\right)$, is an exact potential game if there is an exact potential function $\phi : \mathcal{P} \to \mathbb{R}$ such that for every user $n \in \mathcal{N}$ and for every $\mathbf{P}_{-n} \in \mathcal{P}_{-n}$ the following holds:

$$\begin{aligned} &\tilde{R}_n(\mathbf{p}_n^{(2)}, \mathbf{P}_{-n}) - \tilde{R}_n(\mathbf{p}_n^{(1)}, \mathbf{P}_{-n}) \\ &\quad = \phi(\mathbf{p}_n^{(2)}, \mathbf{P}_{-n}) - \phi(\mathbf{p}_n^{(1)}, \mathbf{P}_{-n}) \;, \\ &\qquad\qquad \forall \mathbf{p}_n^{(1)}, \mathbf{p}_n^{(2)} \in \mathcal{P}_n \;. \end{aligned} \tag{16}$$

Theorem 3. *The constrained rate maximization (10) can be formulated as an exact potential game. Specifically, a global bounded exact potential function exists to this game.*

Proof. The proof is given in [14].

Corollary 1: Any sequential update dynamics of the multi-channel ALOHA game (10) converges to a NEP in finite time, starting from any point. Specifically, the proposed best response algorithm, given in Table 1, converges to a NEP in finite time.

5 Simulation Results

In this section we provide numerical examples to illustrate the algorithm performance. Here, we focus on the constrained rate maximization. We simulated a network with $N = 30$ users, $K = 10$ channels, and the following parameters: the channels are distributed according to Rayleigh fading distribution, i.i.d across users and channels. The bandwidth W of each channel was set to 10MHz, and the SNR was set to 20dB. The entries of the collision-free rate matrix U are $u_n(k) = W \log(1 + \text{SNR})$Mbps. We set $P_{max} = K/N = 1/3$. We compare between two algorithms: 1) The totally greedy algorithm, in the sense that each user transmits over the channel that maximizes its collision-free rate $u_n(k)$ without considering the channel utilization; 2) The proposed best response algorithm, given in Table 1. We initialize the proposed algorithm by the totally greedy algorithm solution, as described in Table 1.

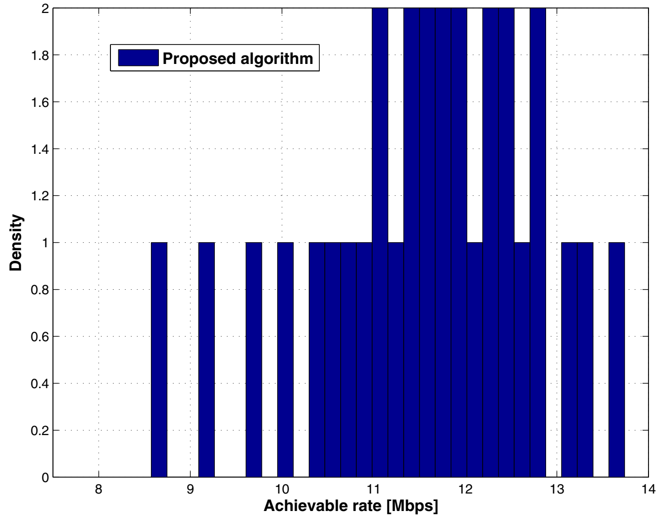
In Fig. 1(a) and 1(b) we present the average density of the rates achieved by the proposed algorithm and by the totally greedy algorithm, respectively. It can be seen that the rates variance achieved by the proposed algorithm is much lower than the rates variance achieved by the the totally greedy algorithm. In Table 2 we compare between the algorithms performance. It can be seen that the average rate achieved by the proposed best response algorithm outperforms the average rate achieved by the totally greedy algorithm by roughly 15%. The average number of iterations until convergence of the proposed best response algorithm is less than 9.

Table 2. Performance comparison

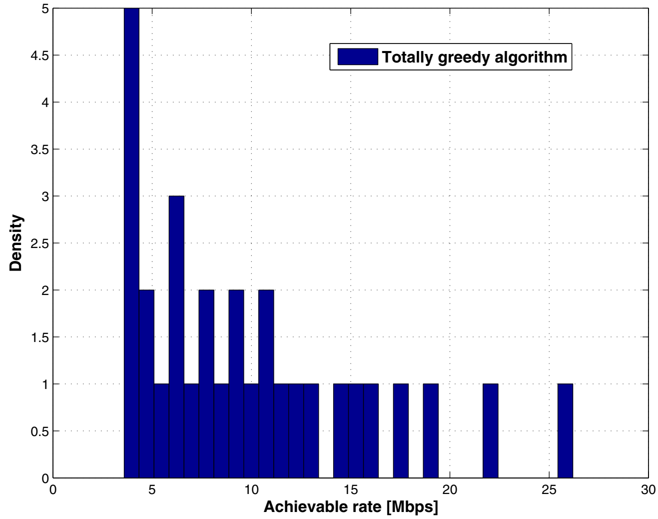
	Proposed algorithm	Totally greedy
Average rate [Mbps]	11.56	10.08
Variance	1.45	34.16
Average number of iterations	8.75	1

6 Conclusion

In this paper we investigated the problem of distributed rate maximization of networks applying the multi-channel ALOHA random access protocol. We characterized the NEPs of the network when users solve the unconstrained rate maximization. In this case, for any NEP, we obtained that each user tries to access a single channel with probability one and does not try to access other channels. Next, we limited each user's total access probability and solved the problem under a total probability constraint, to overcome the problem of collisions when the number of users is much larger than the number of channels. We characterized the NEPs when user rates are subject to a total transmission probability constraint. We proposed a simple best-response algorithm that solves the constrained rate maximization, where each user updates its strategy using its local CSI and by monitoring the channel utilization. We used the theory of potential games to prove convergence of the proposed algorithm. Finally, we provided numerical examples to demonstrate the algorithms performance.



(a) Performance of the proposed best response algorithm, given in Table 1.



(b) Performance of the totally greedy algorithm.

Fig. 1. Average density of the rates achieved by the proposed algorithm and by the totally greedy algorithm

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