# **Token-Based Incentive Protocol Design for Online Exchange Systems**

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**Abstract.** In many online exchange systems, agents provide services to satisfy others agents' demands. Typically, the provider incurs a (immediate) cost and hence, it may withhold service. As a result, the success of the exchange system requires proper incentive mechanisms to encourage service provision. This paper studies the design of such systems that are operated based on the exchange of tokens, a simple internal currency which provides indirect reciprocity among agents. The emphasis is on how the protocol designer should choose a protocol - a supply of tokens and suggested strategies - to maximize service provision, taking into account that impatient agents will comply with the protocol if and only if it is in their interests to do so. Agents' interactions are modeled as a repeated game. We prove that the these protocols have a simple threshold structure and the existences of equilibria. Then we use this structural property to design exchange strategies that maximize the system efficiency. Among all protocols with the same threshold, we find that there is a unique optimal supply of tokens that balances the token distribution in the population and achieves the optimal efficiency. Such token protocols are proven to be able to achieve full efficiency asymptotically as agents become sufficient patient or the cost becomes sufficient small.

**Keywords:** token protocols, repeated games, agents, efficiency.

#### **1 Introduction**

Resource sharing services are currently proliferating in many online systems. For example, In BitTorrent, Gnutella and Kazaa, individual share files; in Seti@home individuals provide computational assistance; in Slashdot and Yahoo!Answers, individuals provide content, evaluations and answers to questions. The expansion of such sharing and exchange services will depend on their participating members (herein referred to as agents) to contribute and share resources with each other. However, the participating agents are s[elf-in](#page-10-0)terested and hence, they will try to "free-ride", i.e. they will derive services from other agents without contributing their own services in return. Empirical studies show that this free-riding problem can be quite severe: in Gnutella system for instance, almost 70% of users share no files at all [1].

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To compel the self-interested agents to cooperate, incentive schemes can be designed which rely on the information that individual agents have. Typically, this information is about the past reciprocation behavior of other agents in the system which can be complete or partial. Such incentives schemes can be classified into two categories: personal reciprocation (direc[t r](#page-10-1)eciprocation) and social reciprocati[on](#page-10-2) (indirect reciprocation) . In the first category [3][7], agents are able to "recognize" (identify) each other and exchange resources depending on their own past mutual interactions. While simple to implement, such incentive schemes cannot be efficiently deployed in systems where anonymous agents interact infrequently with the same partner or in systems with large number of agents. There has been considerable literature on reputation-based schemes for various applications, which appertains to the second category of incentive schemes. Reputation is used as a way to achieve cooperation among self-interested users in [8]. This framework is generalized in [13], where also protocols are designed using social norms based on reputation. However, an important limitation of such solutions is their centralized nature: the provision of service depends on the reputation of both the client and server, some central authority is required to keep track of and verify reputations. Moreover, reputation schemes are also vulnerable to collusion attacks: a set of colluding cheaters can mutually increase their reputation by giving each other positive feedback while giving others negative feedback.

In this paper, we focus on pure strategies and design a new framework for providing incentives in social communities, using tokens. Agents exchange tokens for services: the client who receives service from a server pays for that service with a token which the provider will later use to obtain service when it becomes a client. In this setting, there is potentially a great deal of scope for a designer to improve the social welfare of the system by carefully designing of the token exchanges. The extent to which this potential can be realized depends of course on the degree of control the designer can exert. Here we ask what the designer can achieve by imposing a system that relies solely on the exchange of intrinsically worthless *tokens* or *fiat money*. Our emphasis in this paper is on the *design* of such a system; in particular, how the designe[r sh](#page-10-3)[oul](#page-10-4)[d c](#page-10-5)[hoo](#page-10-6)se a protocol - a supply of tokens and suggested strategies - to maximize the system efficiency. Among all such choices/recommendations, the designer should select one that maximizes the social welfare/system efficiency - or at least approaches this maximum. We characterize the equilibria (in terms of the system parameters), show that they have a particularly simple form, and determine the achievable system efficiency. When agents are patient, it is possible to design equilibria to nearly optimal efficiency.

This work connects to a number of economic literatures [11][10][6][14]. We go further than these papers in that we emphasize the design of equilibria and the designer's goal of efficiency. In particular, we identify equilibria that are asymptotically efficient, which these papers do not do. In the computer science and engineering literature, token approaches are also adopted in various systems [12][4][2]. However, they either assume that agents are compliant, rather than self-interested, and do not treat incentives and equilibrium or mainly focus on

#### 250 J. Xu, W. Zame, and M. van der Schaar

simulations rather than rigorous theoretical justifications. The work closest to ours is probably [5][9] which treats a rather different model in a "scrip" system. More importantly, it assumes that agents adopt threshold strategies but we rigorously prove that threshold strategies are the only equilibrium strategy.

The rest of this paper is organized as follows. Section 2 introduces the proposed token exchange model, defines equilibrium strategies and formulates the optimal protocol design problem. Section 3 describes the nature of equilibrium. Section 4 discusses efficiency of equilibrium protocols and designs the optimal protocol - optimal token supply and optimal threshold. Section 5 illustrates the simulation results. Finally concluding remarks are made in Section 6.

## **2 System Model**

In the environment we consider, a continuum (mass 1) of agents each possess a unique resource that can be duplicated and provided to others. (In the real systems we have in mind, the population is frequently in the tens of thousands, so a continuum model seems a reasonable approximation.) The benefit of receiving this resource is b and the cost of producing it is c ; we assume  $b > c > 0$ , so that social welfare is increased when the service is provided, but the cost is strictly positive, so that the server has a disincentive to provide it. Agents care about current and future benefits/costs and discount future benefits/costs at the constant rate  $\beta \in (0,1)$ . Agents are risk neutral so seek to maximize the discounted present value of a stream of benefits and costs.

Time is discrete. In each time period, a fraction  $\rho \leq 1/2$  of the population is randomly chosen to be a client and matched with randomly chosen server; the fraction  $1-2\rho$  is unmatched. (No agent is both a client and a server in the same period.) When a client and server are matched, the client chooses whether or not to request service, the server chooses whether or not provide service (i.e., transfer the file) if requested. This client-server model describes the world where an agent has demand at times and also is matched by the system to provide service at other times.

The parameters  $b, c, \beta, \rho$  completely describe the environment. Because the units of benefit  $b$  and cost  $c$  are arbitrary (and tokens have no intrinsic value), only the benefit/cost ratio  $r = b/c$  is actually relevant. We consider variations in the benefit/cost ratio r and the discount factor  $\beta$ , but view the matching rate  $\rho$  as immutable.

#### **2.1 Tokens and Strategies**

In a single server-client interaction, the server has no incentive to provide services to the client. The mechanism we study for creating incentives to provide involves the exchange of tokens. Tokens are indivisible, have no intrinsic value, and can be stored without loss. Each agent can hold an arbitrary non-negative finite number of tokens, but cannot hold a negative number of tokens and cannot borrow.

The protocol designer creates incentives for the agents to provide or share resources by providing a supply of tokens and recommending strategies for agents when they are clients and servers. The recommended strategy is a pair  $(\sigma, \tau)$ :  $\mathbb{N} \to \{0,1\}$ ;  $\tau$  is the client strategy and  $\sigma$  is the server strategy. It is obvious that the strategy should only depend on agents' current token holding because the future matching process is independent of the history.

## **2.2 Equilibrium**

Because we consider a continuum population and assume that agents can observe only their own token holdings, the relevant state of the system from the point of view of a single agent can be completely summarized by the fraction  $\mu$  of agents who do not request service when they are clients and the fraction  $\nu$  of agents who do not provide service when they are servers. If the population is in a steady state then  $\mu$ ,  $\nu$  do not change over time.

Given  $\mu, \nu$  the strategy  $(\sigma, \tau)$  is optimal or a best response for the current token holding of  $k$  if the long-run utility satisfies

$$
V(k|\mu, \nu, \sigma, \tau) \ge V(k|\mu, \nu, \sigma', \tau')
$$

for alternative strategies  $\sigma', \tau'$ . Because agent discount the future at the constant rate  $\beta$ , the strategy  $(\sigma, \tau)$  is optimal if and only if it has the one-shot deviation property: there does not exist a continuation history h and a profitable deviation  $(\sigma', \tau')$  that differs from  $(\sigma, \tau)$  followed by the history h and nowhere else; i.e. for the server strategy

$$
\sigma(k) = 0 \Rightarrow \beta V(k|\sigma, \tau, \mu, \nu) \ge -c + \beta V(k+1|\sigma, \tau, \mu, \nu) \n\sigma(k) \in (0, 1) \Rightarrow \beta V(k|\sigma, \tau, \mu, \nu) = -c + \beta V(k+1|\sigma, \tau, \mu, \nu) \n\sigma(k) = 1 \Rightarrow \beta V(k|\sigma, \tau, \mu, \nu) \le -c + \beta V(k+1|\sigma, \tau, \mu, \nu)
$$

for the client strategy

$$
\tau(k) = 0 \Rightarrow \beta V(k|\sigma, \tau, \mu, \nu) \ge b + \beta V(k-1|\sigma, \tau, \mu, \nu) \n\tau(k) \in (0, 1) \Rightarrow \beta V(k|\sigma, \tau, \mu, \nu) = b + \beta V(k-1|\sigma, \tau, \mu, \nu) \n\tau(k) = 1 \Rightarrow \beta V(k|\sigma, \tau, \mu, \nu) \le b + \beta V(k-1|\sigma, \tau, \mu, \nu)
$$

Write  $EQ(r, \beta)$  for the set of protocols  $\Pi$  that constitute an equilibrium when the benefit/cost ratio is r and the discount factor is  $\beta$ . Conversely, given  $\Pi$  write  $\Phi(\Pi)$  for the set  $\{(r,\beta)\}\$  of pairs of benefit/cost ratios r and discount factors β such that  $\Pi$  is an equilibrium protocol. Note that  $EQ, Φ$  are correspondences and are inverse to each other.

## **2.3 Invariant Distribution**

If the designer chooses the protocol  $\Pi = (\alpha, \sigma, \tau)$  and agents follow the recommendation, we can easily describe the evolution of the token distribution (the distribution of token holdings). Note that the token distribution must satisfy two feasibility conditions:

$$
\sum_{k=1}^{\infty} \eta(k) = 1, \qquad \sum_{k=0}^{\infty} k\eta(k) = \alpha
$$

 $\mu, \nu$  are computed as

$$
\mu = \sum_{k=0}^{\infty} (1 - \tau(k)) \eta(k), \nu = \sum_{k=0}^{\infty} (1 - \sigma(k)) \eta(k)
$$

Evidently,  $\mu$  is the fraction of agents who do not request service, and that  $\nu$  is the fraction of agents who do not server (assuming they follow the protocol).

To determine the token distribution next period, it is convenient to work backwards and ask how an agent could come to have k tokens in the next period. Given the protocol  $\Pi$  the (feasible) token distribution  $\eta$  is invariant if  $\eta_+ = \eta$ ; that is,  $\eta$  is stationary when agents comply with the recommendation  $(\sigma, \tau)$ .

#### **2.4 Problem Formulation**

The goal of the protocol designer is to provide agents with incentives to provide service. Define the system efficiency as the probability that the service provision is successfully carried out when two agents are paired given the system parameters  $b, c, \beta$ . Using the definition of  $\mu, \nu$ , by the Law of Large Numbers, the efficiency is computed in the straightforward manner,

$$
\texttt{Eff}\left(\varPi|b,c,\beta\right)=\left(1-\mu\right)\left(1-\nu\right)
$$

Taking into account that impatient agents will comply with the protocol if and only if it is in their interests to do so, the protocol needs to be an equilibrium given the system parameters. Formally, the design problem are thus to choose the protocol  $\Pi =$  $\Pi$ : $(\beta,r) \in \Phi(\Pi)$  $\arg \max$  Eff  $(\Pi | \beta, r)$ .

## **3 Equilibrium Strategies**

The candidate protocols are enormous, directly focusing on the efficiency hence is impossible. Therefore, we explore whether there exist some special structures of the optimal strategies which may simplify the system design.

#### **Proposition 1.** *Given*  $b, c, \beta, \mu, \nu,$

- *1. The optimal client strategy*  $\tau$  *is*  $\tau(k) = 1$  *for every*  $k \geq 1$ *; that is, "always request service when possible".*
- *2. The optimal server strategy* σ *has a threshold property; that is, there exists* K such that  $\sigma(k)=1, \forall k < K$  and  $\sigma(k)=0, \forall k \geq K$ .

*Proof.* 1. Suppose there is some  $b, c, \beta, \mu, \nu$  such that  $\tau(k) < 1$ . If this client strategy is optimal, it implies that the marginal value of holding  $k - 1$  tokens is at least  $b/\beta$ , i.e.  $V(k) - V(k-1) \ge b/\beta > b$ . Consider any realized continuation history following the decision period. We estimate the loss in the expected utility having one less token. Because there is only one deviation in the initial time period, the following behaviors are exactly the same. The only difference occurs at the first time when the token holding drops to 0 when it is supposed to buy. At this moment, the agent cannot buy and losses benefit b. Therefore the loss in the utility is  $\beta^t b$  for some t depending on the specific realized history. Because this analysis is valid for all possible histories, the expected utility is strictly less than b. This violates the optimality condition. Hence, it is always optimal for the agent to spend the token if possible.

2. (sketch) Based on the result of part 1, we study an arbitrary server strategy  $\sigma$ . The utilities of holding different numbers of tokens are inter-dependent with each other

$$
V(0) = \sigma(0) \rho (1 - \mu) (-c + \beta V(1))
$$
  
+ (\rho (\sigma(0) (\mu - 1) + 2) + 1 - 2\rho) \beta V(0)  

$$
V(k) = \sigma(k) \rho (1 - \mu) (-c + \beta V (k + 1))
$$
  
+ (\rho (1 - \nu) (b + \beta V (k - 1))  
+ (\rho (\sigma (k) \mu + \nu + 1 - \sigma (k)) + 1 - 2\rho) \beta V(k),  

$$
\forall k = 1, 2, ..., K - 1
$$
  

$$
V(k) = \rho (1 - \nu) (b + \beta V (k - 1))
$$
  
+ (\rho (\nu + 1) + 1 - 2\rho) \beta V(k),  $\forall k = K, K + 1, ...$ 

Using these equations, it can be shown that if a strategy is an equilibrium, the marginal utilities  $M(k) = V(k+1) - V(k)$  are decreasing sequences. Therefore, there exists a threshold K such that  $M(k) \ge c/\beta, \forall k < K$  and  $M(k) > c/\beta, \forall k \ge$ K.

In view of Proposition 1, we suppress client strategy  $\tau$  entirely, assuming that clients always request service whenever possible. Therefore we frequently write  $\Pi = (\alpha, \sigma)$  instead of  $\Pi = (\alpha, \sigma, \tau)$ . Moreover, we only need to focus on threshold server strategies in the following analysis.

Existence of equilibrium is not trivial. To see why, fix a benefit/cost ratio and consider a threshold protocol  $\Pi = (\alpha, \sigma_K)$ . If the discount factor is small, agents will not be willing to continue providing service until they acquire  $K$  tokens; if  $\beta$  is large, agents will not be willing to stop providing service after they have acquired K tokens - and it is not obvious that there will be any discount factor  $\beta$  that makes agents be willing to do so. The following theorem claims that such  $\beta$  can always be found.

**Proposition 2.** For each threshold strategy protocol  $\Pi = (\alpha, \sigma_K)$  and bene*fit/cost ratio*  $r > 1$ *, the set*  $\beta : \Pi_K \in EQ(r, \beta)$  *is a non-degenerate interval*  $[\beta^L, \beta^H]$ .

*Proof.* (sketch) We first see that  $M(K-1) > c/\beta$ ,  $M(K) < c/\beta$  is a necessary and sufficient condition for a strategy to be an equilibrium. This is established on the properties of marginal utilities. Define  $F(\beta) = M(K-1|\beta) - c/\beta$ ,  $G(\beta) =$  $M(K|\beta) - c/\beta$ . Hence, the necessary and sufficient condition becomes  $F(\beta)$  $0, G(\beta) < 0.$ 

It can be shown that there exists a unique  $\beta^L \in (0,1)$ , such that  $F(\beta) \geq$  $0, \forall \beta \in (\beta^L, 1)$  and equality holds only for  $\beta^L$ . Next we show that there exists a unique  $\beta^H \in (\beta^L, 1)$  such that  $G(\beta) \leq 0, \forall \beta \in (\beta^L, \beta^H)$  and equality holds only for  $\beta^H$ . To see that such  $\beta^H$  exists, we prove the  $G(\beta)$  is strictly increasing in  $\beta$ ,  $G(\beta^L)$  < 0 and  $G(1) > 0$ . Therefore, there must exist an non-degenerate interval  $[\beta^{\vec{L}}, \beta^H]$  that makes a pure threshold strategy an equilibrium.

If the discount factor is given, the existence of equilibrium can be similarly characterized by the benefit/cost ratio.

**Proposition 3.** For each threshold strategy protocol  $\Pi = (\alpha, \sigma_K)$  and discount *factor*  $\beta \in (0,1)$ *, the set*  $r : \Pi_K \in EQ(r, \beta)$  *is a non-degenerate interval*  $[r^L, r^H)$ *.* 

*Proof.* (sketch) The proof is similar to the proof of Theorem 2 but this time we write  $F(r) = M(K-1|r) - c/\beta$  and  $G(r) = M(K|r) - c/\beta$  as functions of r. Using similar arguments, we can show that  $F(r) \geq 0, \forall r \in (r^L, \infty)$  and  $G(r) < 0, \forall r \in (r^L, r^H)$  and  $r^L < r^H$ .

From the design perspective, it is important to understand the set of strategies that can be equilibria for given system parameters. This will be more clear when we show that the system efficiency not only depends on the strategy (threshold) but also the token supply. If the token supply is not designed properly with regard to the threshold, there will be strict efficiency loss. Due to this reason, understanding the equilibrium thresholds for the system parameters is of paramount importance.

# **4 Protocol Design**

The protocol designer is interested in maximizing the probability of service provision Eff =  $(1 - \mu)(1 - \nu)$ . We also define it as the system efficiency. It is directly dependent on the fractions of request  $(1 - \mu)$  and service  $(1 - \nu)$ , which are determined by the recommended strategy and token distribution in the population.

The token holding distribution is a joint impact of the recommended strategy and token supply. Using the definition of the token distribution and its transition equations, we are able to characterize it for the threshold strategy which is completely determined by the feasibility conditions and the relationship

$$
\eta(k) = \left(\frac{1-\eta(0)}{1-\eta(K)}\right)^{k} \eta(0), \forall k = 0, 1, ..., K - 1
$$

We will use it in determining the optimal token supply in the next subsection.

## **4.1 Optimal Token Supply**

In general it seems hard to determine the efficiency of a given protocol or to compare the efficiency of different protocols. However, for a given threshold strategy, we can find the most efficient protocol and compute its efficiency. Write  $\Pi_K = (K/2, \sigma_K).$ 

**Proposition 4.** For a given threshold strategy  $\sigma_K$ ,  $\Pi_K$  is the most efficient *protocol; i.e.,*  $Eff(\alpha, \sigma_K) \leq Eff(\Pi_K)$  *for every per capita supply of tokens*  $\alpha$ *. Moreover,*

$$
\mathit{Eff}(\varPi_K)=1-\frac{1}{\left(K+1\right)^2}
$$

*Proof.* It is convenient to first solve the following maximization problem

maximize 
$$
(1 - x_1)(1 - x_2) = 1 - x_1 - x_2 + x_1x_2
$$
  
subject to  $x_1(1 - x_1)^K = x_2(1 - x_2)^K$   
 $0 \le x_1, x_2 \le 1$ 

To solve this problem, set  $f(x) = x(1-x)^{K}$ , a straightforward calculus exercise shows that if  $0 \le x_1 \le 1/(K+1) \le x_2 \le 1$  and  $f(x_1) = f(x_2)$  then,

(a)  $x_1 + x_2 \ge 1/(K+1)$  with equality achieved only at  $x_1 = x_2 = 1/(K+1)$ . (b)  $x_1x_2 \leq 1/(K+1)$  with equality achieved only at  $x_1 = x_2 = 1/(K+1)$ .

Putting (a) and (b) together shows that the optimal solution to the maximization problem is to have  $x_1 = x_2 = 1/(K + 1)$  and the maximized objective function value is

$$
\max(1 - x_1)(1 - x_2) = \left(1 - \frac{1}{K + 1}\right)^2
$$

Now consider the threshold K strategy and let  $\eta$  be the corresponding invariant distribution. If we take  $x_1 = \eta_o$ ,  $x_2 = \eta_d$  then our characterization of the invariant distribution shows that  $f(x_1) = f(x_2)$ . By definition,  $\text{Eff} = (1 - x_1)(1 - x_2)$ so

$$
\texttt{Eff} = \left(1 - \frac{1}{K+1}\right)^2
$$

Taken together, these are the assertions which were to be proved.

Proposition 4 identifies a sense in which there is an optimal quantity of tokens. This optimal token supply balances the token distribution in the population in the sense that there are not too many agents who do not serve or too many agent who cannot request service. However, these most efficient protocols (for a given threshold) need not be equilibrium protocols; i.e. such combinations of token supply and threshold need not be feasible for all system parameters. For example, given the benefit/cost ratio  $r$ , it does not exclude the possibility that for some discount factor  $\beta$ , we cannot find any threshold protocol with the corresponding optimal token supply that is an equilibrium. However, we disclaim this conjecture by showing that the sustainable discount factor intervals overlap between consecutive threshold protocols with optimal token supply. Based on this overlap property, the following proposition describes the equilibrium threshold in the limiting case.

**Proposition 5.** *1. for each fixed discount factor*  $\beta < 1$  lim  $Eff = 1$ ; 2. for each fixed benefit-cost ratio  $r > 1$   $\lim_{\beta \to 1} Eff = 1$ .

*Proof.* (sketch) We prove the first part. The second part is similarly proved. Consider two protocols  $\Pi_1 = (K/2, \sigma_K)$  and  $\Pi_2 = ((K+1)/2, \sigma_{K+1})$  which are have consecutive thresholds. The corresponding intervals of discount factors that sustain equilibrium are  $[\beta_1^L, \beta_1^H]$  and  $[\beta_2^L, \beta_2^H]$ . We assert that

$$
\beta_1^L<\beta_2^L<\beta_1^H,\beta_2^L<\beta_1^H<\beta_2^H
$$

In words, the sustainable ranges of the discount factors overlap between two consecutive threshold protocols. To see this, arithmetical exercises show that for  $M_{\Pi_1}(K|\beta_2^L) > c/\beta_2^L$  which leads to  $\beta_2^L > \beta_1^L$ ;  $M_{\Pi_2}(K|\beta_1^H) > c/\beta_1^H$  which leads to  $\beta_2^L < \beta_1^H$ . The assertion follows immediately by combining this overlapping result and Proposition 4.

As agents become arbitrarily patient or the benefit/cost ratio become arbitrarily large, it is possible to choose equilibrium protocols that achieve efficiency arbitrarily close to full efficiency (i.e.,  $Eff \rightarrow 1$ ).

# **5 Simulations**

In Fig. 1 we illustrate the sustainable region of the pair  $(\beta, r)$  of the discount factor and the benefit/cost ratio for various threshold protocols. For a larger threshold to be an equilibrium, larger discount factors or larger benefit/cost ratios are required. Moreover, fix one of  $\beta$  and r, for given threshold, there is always an continuous interval for the other parameter to make the threshold protocol an equilibrium.



**Fig. 1.** Threshold equilibrium region



**Fig. 2.** Efficiency loss of a fixed threshold protocol

Fig. 2 shows the efficiency of a optimal equilibrium protocol and a fixed threshold protocol. First, the optimal system efficiency goes to 1 as the agents becomes sufficient patient ( $\beta \rightarrow 1$ ). Second, it compares the achievable efficiency with the efficiency of a protocol for which the strategic threshold is constrained to be  $K = 3$ . The enormous efficiency loss induced by choosing the wrong protocol supports our emphasis on the system design in accordance to system parameters.

# **6 Conclusions**

In this paper, we designed token-based protocols - a supply of tokens and recommended strategies -to encourage cooperation in the online exchange systems where a large population of anonymous agents interact with each other. We focused on pure strategy equilibrium and proved that only threshold strategies can emerge in equilibrium. With this threshold structural results in mind, we showed that there also exists an unique optimal quantity of tokens that maximizes the efficiency given the threshold. It balances the population in such a way that there are not too many agents who do not serve or too many agents who cannot pay with tokens. Moreover, the proposed protocols asymptotically achieve full efficiency when the agents become perfectly patient or the benefit/cost ratio goes to infinity. This paper characterizes the performance of the online exchange systems operated on tokens and emphasizes the importance of a proper token protocol. Importantly, the token supply serves as a critical design parameter that needs to be well understood based on the intrinsic environment parameters.

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