Revenue Maximization in Customer-to-Customer Markets

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Abstract. Customer-to-customer (C2C) markets, such as eBay, provide a platform allowing customers to engage in business with each other. The success of a C2C market requires an appropriate pricing (i.e., transaction fee charged by the market owner) scheme that can maximize the market owner's revenue while encouraging customers to participate in the market. However, the choice of an optimal revenue-maximizing transaction fee is challenged by the large population of self-interested customers (i.e., sellers and buyers). In this paper, we address the problem of maximizing the market owner's revenue based on a hierarchical decision framework that captures the rationality of both sellers and buyers. First, we use a model with a representative buyer to determine the sales of products in the market. Then, by modeling sellers as self-interested agents making independent selling decisions, we show that for any transaction fee charged by the market owner, there always exists a unique equilibrium in the selling decision stage. Finally, we derive the optimal transaction fee that maximizes the market owner's revenue. We find that to maximize its revenue under certain circumstances, the market owner may even share its advertising revenues with sellers as rewards to encourage them to sell products in the market and bring more website traffic. Our results indicate that the market owner's revenue can be significantly increased by optimally choosing the transaction fee, even though sellers and buyers make self-interested and rational decisions.

Keywords: Revenue maximization, customer-to-customer market, pricing, product substitutability.

1 Introduction

Electronic commerce markets have witnessed an explosive growth over the past decade and have now become an integral part of our everyday lives. In the realm of electronic commerce, customer-to-customer, also known as consumer-to-consumer (C2C), markets are becoming more and more popular, as they provide a convenient platform allowing customers to easily engage in business with each other. A well-known C2C market is eBay, on which a wide variety of products, including second-hands goods, are sold.

As a major source of revenue, a C2C market owner charges various fees, which we refer to as *transaction fees*, for products sold in the market. For instance, eBay

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charges a final value fee and a listing fee for each sold item [1]. Hence, to enhance a C2C market's profitability, it is vital for the market owner to appropriately set the transaction fee. In this paper, we focus on a C2C market and address the problem of maximizing the market owner's revenue. The scenario that we consider is summarized as follows.

1. The market owner monetizes the market by charging transaction fees for each product sold and (possibly) through advertising in the market. For the completeness of analysis, we also allow the market owner to reward the sellers to encourage them to sell products in the market, which increases the market's website traffic and hence advertising revenues if applicable. Although rewarding sellers may seem to deviate from our initial goal of collecting transaction fees from sellers, we shall show that rewarding sellers may also maximize the market owner's revenue under certain circumstances.

2. Products are sold by sellers and purchased by buyers at fixed prices. Promotional activities (e.g., monetary rewards, rebate) and/or auctions are not considered in our study.

3. Buyers do not need to pay the market owner (e.g., membership fees) in order to purchase products in the market, and they can directly interact with sellers that participate in the market (e.g., eBay).

In the following analysis, we adopt a leader-follower model (i.e., the market owner is the leader, followed by the sellers and then by the buyers), which is described in Fig. 1. Note that, without causing ambiguity, we refer to the market owner as *intermediary* for brevity if applicable. Fig. 1 also shows interdependencies of different decisions stages. The intermediary's transaction fee decision will directly affect the sellers' participation in the market, while the sellers' selling decisions influence the buyers' purchasing decisions. Based on backward induction, we first use a model with a representative buyer, which is a collection of all the individual buyers, to determine the sales of products sold in the market. As a distinguishing feature, our model captures the (implicit) competition among the sellers, which is typically neglected in existing two-sided market research [8], and also the buyers' preference towards a bundle of diversified products. Then, we study the selling decisions made by self-interested sellers. It is shown that there always exists a unique equilibrium point at which no seller can gain by changing its selling decision, which makes it possible for the intermediary to maximize its revenue without uncertainties. Next, we formulate the intermediary's revenue maximization problem and develop an efficient algorithm to derive the optimal transaction fee that maximizes the intermediary's revenue. Finally, we conduct simulations to complement our analysis and show that the intermediary's revenue can be significantly increased by optimally choosing the transaction fee, even though sellers and buyers make self-interested and rational decisions.

The rest of this paper is organized as follows. Related work is reviewed in Section 2. Section 3 describes the model. In Section 4, we study the decisions made by the buyers and sellers, and derive the optimal transaction fee maximizing the intermediary's revenue. Finally, concluding remarks are offered in Section 5.

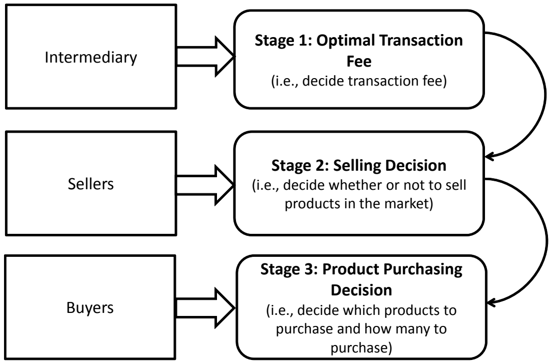

Fig. 1. Order of decision making

2 Related Works

We briefly summarize the existing related works in this section.

If the intermediary chooses to reward the sellers, then the transaction fee is essentially an incentive for sellers to sell products in the market. Various incentive mechanisms have been proposed recently. For instance, the authors in [3] proposed eliminating or hiding low-quality content to provide content producers with incentives to generate high-quality content. In [4], two scoring rules, the approval-voting scoring rule and the proportional-share scoring rule, were proposed to enable the high-quality answers for online question and answer forums (e.g., Yahoo! Answers). The authors in [5] proposed a (virtual) reward-based incentive mechanism to improve the overall task completion probability in collaborative social media networks. If the intermediary charges the sellers, then our work can be classified as market pricing. By considering a general two-sided market, the authors in [8] studied the tradeoffs between the merchant mode and the platform mode, and showed the conditions under which the merchant or platform mode is preferred. Focusing on the Internet markets, [10] revealed that a neutral network is inferior to a non-neutral one in terms of social welfare when the ratio between advertising rates and end user price sensitivity is either too high or too low.

In economics literature, C2C markets are naturally modeled as two-sided markets, where two user groups (i.e., sellers and buyers in this paper) interact and provide each other with network benefits. Nevertheless, most two-sided market research neglected the intra-group externalities (e.g., see [11][12] for a survey), which in the contexts of C2C markets indicate the sellers' competition. A few recent studies on two-sided markets explicitly considered intra-group externalities. For instance, [13] studied the optimal pricing problem to maximize the platform's profit for the payment card industry with competition among the merchants. [14] considered the sellers' competition in a two-sided market with differentiated products. More recently, considering intra-group competition, [15] studied the problem of whether an entrant platform can divert agents from the existing platform and make a profit. Nevertheless, the focus in all these works was market pricing, whereas in our work the intermediary can either charge or reward the sellers. Moreover, the existing studies on two-sided markets typically neglected product substitutability as well as buyers' "love for variety".

To summarize, this paper derives the optimal transaction fee, and determines analytically when the intermediary should subsidize sellers to maximize its revenue. Unlike general two-sided market research (e.g., [11][12]), this paper considers both the sellers' competition and the product substitutability, which are key features of C2C markets and, as shown in this paper, significantly impact the optimal transaction fee of C2C platforms.

3 Model

We first specify the basic modeling details of the intermediary, sellers and buyers, and then discuss the model extension.

3.1 Intermediary

An important and prevailing charging model in C2C markets is that, for each sold product, the intermediary charges a transaction fee that is proportional to the product price (e.g., final value fee in eBay). From the perspective of sellers, sellers pay to the intermediary when their products are sold, i.e., "pay-per-sale". In this paper, we concentrate on the "pay-per-sale" model. Nevertheless, it should be noted that other fees may also be levied on product sales, e.g., eBay charges a lump-sum listing fee for listing a product regardless of the quantities sold [1]. Investigating more sophisticated charging models (e.g., "pay-per-sale" plus lump-sum fee) is part of our ongoing research. As in many real C2C markets such as eBay, buyers do not need to pay the intermediary (e.g., membership fees) in order to purchase products in the market.

To formally state our model, we denote $\bar{x} \geq 0$ as the sales volume (i.e., quantities of sold products) in the market, and $\theta > 0$ is the transaction fee[1] that the intermediary charges the sellers for each of their sold products. For the ease of presentation, we assume in our basic model that all the products belong to the same category and have the same price and hence, θ is the same for all the products. This assumption is valid if all the sellers sell similar and/or standardized products (e.g., books, CDs) and, due to perfect competition, set the same price for their products [8][19]. Recent research support the assumption of a uniform product price by showing that price dispersion in online shopping sites is fairly small, i.e., prices offered by different sellers for the same or similar products are very close to each other [6]. Moreover, if the considered C2C market is an online labor market in which sellers "sell" their services (e.g., skills, knowledge, etc.),

[1] Note that θ is actually the percentage of the product price charged by the intermediary. However, since we later normalize the product price to 1, θ can also represent the *absolute* transaction fee charged by the intermediary.

the assumption of different services having the same price is reasonable when the offered services are of the same or similar types (see, e.g., Fiverr, an emerging C2C market where the "sellers" offer, possibly different, services and products for a fixed price of US$ 5.00 [2]). We should also make it clear that our analysis can be generalized and applied if different products are sold at different prices (see Section 3.4 for a detailed discussion). Besides the transaction fees charged for product sales, the intermediary may also receive advertising revenues by displaying contextual advertisement on its website. In general, the advertising revenue is approximately proportional to page views (i.e., the number of times that the webpages are viewed), which are also approximately proportional to sales volume in the market. Thus, overall, the advertising revenue is approximately proportional to the sales volume. Let $b \geq 0$ be the (average) advertising revenue that the intermediary can derive from each sold product. For the convenience of analysis, we assume that b is constant regardless of $\bar{x}$, i.e., the average advertising revenue is independent of the sales volume. Next, we can express the intermediary's revenue as[2]

$$\Pi_{\mathcal{I}} = (b + \theta) \cdot \bar{x}. \tag{1}$$

Remark 1: For the completeness of analysis, we allow θ to take negative values, in which case the intermediary rewards the sellers for selling their products. This may occur if the intermediary can derive a sufficiently high advertising revenue per page view and hence would like to encourage more sellers to participate in its market, which attracts more buyers and increases the website traffic (and hence, higher advertising revenues, too). In the following analysis, we use the term *transaction fee* (per sold product) to refer to θ wherever applicable, regardless of its positive or negative sign.

Remark 2: While b can be increased by using sophisticated advertising algorithms showing more relevant advertisement, we assume throughout the paper that b is exogenously determined and fixed, and shall focus on deriving the optimal θ that maximizes the intermediary's revenue.

Remark 3: As in [8], we focus on only one C2C market in this paper. Although the competition among various C2C markets is not explicitly modeled, we do consider that online buyers can purchase products from other markets (see Section 3.3 for details).

3.2 Sellers

As evidenced by the exploding number of sellers on eBay, a popular C2C market can attract a huge number of sellers. To capture this fact, we use a continuum model and assume that the mass of sellers is normalized to one. Each seller can sell products of a certain quality while incurring a lump-sum cost, which we refer to as selling cost, regardless of the sales volume. Note that the product

[2] The expression in (1) can also be considered as the intermediary's profit, if we treat b as the average advertising profit for each sold product and neglect the intermediary's recurring fixed operational cost.

quality can be different across sellers, although we assume in our basic model that the selling cost is the same for all sellers. We should emphasize that the product quality is represented by a scalar and, as a generalized concept, is jointly determined by a variety of factors including, not not limited to, product popularity, seller ratings, customer service and product reviews [7]. For instance, even though two sellers with different customer ratings sell the same product, we say that the product sold by the seller with a higher rating has a higher quality. The scalar representation of product quality, i.e., abstracting and aggregating various factors to one value, is indeed an emerging approach to representing product quality [7]. Mathematically, we denote $q_i \geq 0$ and $c > 0$ as the product quality sold by seller i and the selling cost, respectively. Without causing ambiguity, we occasionally use product q_i to refer to the product with a quality q_i. To characterize heterogeneity in the product quality, we assume that the product quality q follows a distribution in a normalized interval $[0, 1]$ across the unit mass of sellers and the cumulative distribution function (CDF) is denoted by $F(q)$ for $q \in [0, 1]$. In other words, $F(q)$ denotes the number or fraction of sellers whose products have a quality less than or equal to $q \geq 0$. In what follows, we shall explicitly focus on the uniform distribution, i.e., $F(q) = q$ for $q \in [0, 1]$, when we derive specific results, although other CDFs can also be considered and our approach of analysis still applies.[3] Note that scaling the interval $[0, 1]$ to $[0, \bar{q}]$ does not affect the analysis, but will only complicate the notations.

As stated in the previous subsection, we assume in our basic model that all the products are sold at the same price in the market. Hence, without loss of generality, we normalize the product price to 1. Denote the profit that each seller can obtain by selling a product by $s \in (0, 1)$, which is assumed to be same for all the sellers, and let $x(q_i) \geq 0$ be the sales volume for product q_i. Heterogeneous product profits (i.e., different s for different sellers) can be treated in the same way as treating heterogeneous product prices (see Section 3.4 for details). In our model, sellers are rational and each seller makes a self-interested binary decision: sell or not sell products in the considered C2C market. If seller i chooses to sell products in the market, it can derive a profit expressed as

$$\pi_i = (s - \theta) \cdot x(q_i) - c, \tag{2}$$

where θ is the transaction fee charged by the intermediary per product sale, and c is the (lump-sum) selling cost. Seller i obtains zero profit if it chooses not to sell products in the market. By the assumption of rationality, seller i chooses to sell products if and only if its profit is non-negative. It is intuitively expected that, with the same price, a product with a higher quality will have a higher sales volume (and yield a higher profit for its seller, too) than the one with a lower quality.[4] Thus, the sellers' selling decisions have a threshold structure. In particular, there exist *marginal* sellers whose products have a quality denoted

[3] The uniform distribution has been widely applied to model the diversity of various factors, such as opportunity cost [8] and valuation of quality-of-service [9].

[4] This statement can also be mathematically proved, while the proof is omitted here for brevity.

by $q_m \in [0,1]$, and those sellers whose product quality is greater (less) than q_m will (not) choose to sell products in the market. We refer to q_m as the marginal product quality. Next, it is worthwhile to provide the following remarks concerning the model of sellers.

Remark 4: In our model, a seller who sells $m \geq 1$ different products is viewed as m sellers, each of whom sells a single product, and the total selling cost is $m \cdot c$ (i.e., constant returns to scale [8]).

Remark 5: The lump-sum selling cost c accounts for a variety of fixed costs for selling products. For instance, sellers need to spend time in purchasing products from manufactures and in listing products in the market. Moreover, as charged by eBay, a small amount of lump-sum fee, i.e., listing fee, may also be charged for listing a product (although we do not explicitly consider this revenue for maximizing intermediary's revenue) [1]. As in [8], we assume that the sellers will incur a predetermined selling cost if they choose to sell products in the market. For the ease of presentation, we consider a homogeneous selling cost among the sellers, while we shall discuss the extension to heterogeneous selling costs in Section 3.4.

Remark 6: In our model, sellers always have products available if buyers would like to purchase. That is, "out of stock" does not occur.

3.3 Buyers

We adopt the widely-used representative agent model to determine how the total budget (i.e., buyers' expenditure in online shopping) is allocated across a variety of products [18]. Specifically, the representative buyer optimally allocates its total budget, denoted by T, across the available products to maximize its utility. Note that T can be interpreted as the size of the representative buyer or the online shopping market size. In addition to purchasing products sold in the considered C2C market, buyers may also have access to products sold in other online markets (e.g., business-to-customer shopping sites and/or other C2C markets), and we refer to these products as *outside* products. Similarly, we refer to those online markets where outside products are sold as *outside* markets. Focusing on the intermediary's optimal transaction fee decision, we do not consider the details of how or by whom outside products are sold. Instead, we assume that the mass of outside products is $n_a \geq 0$ and the outside product quality follows a certain CDF $\tilde{F}(q)$ with support $q \in [q_l, q_h]$, where $0 \leq q_l < q_h$ are the lowest and highest product quality of outside products, respectively. For the convenience of notation, throughout the paper, we alternatively represent the outside products using a unit mass of products with an *aggregate* quality of q_a, without affecting the analysis. Note that q_a is a function of $n_a \geq 0$, $\tilde{F}(q)$ and the utility function of the representative buyer. In particular, given a uniform distribution of outside product quality and the quality-adjusted Dixit-Stiglitz utility for the representative buyer (which we shall define later), we can readily obtain

$$q_a = \left[\frac{n_a \left(q_h^{\sigma+1} - q_l^{\sigma+1} \right)}{1+\sigma} \right]^{\frac{1}{\sigma}}, \tag{3}$$

where $\sigma > 1$ measures the product substitutability [17]. Recalling that $q_m \in [0,1]$ is the marginal product quality above which the sellers choose to sell products in the market, we write the representative buyer's utility function as $U(x(q), x_a \,|\, q_m, q_a)$, where $x(q)$ denotes the sales volume for product $q \in [q_m, 1]$ and x_a is the sales volume for outside products with an aggregate quality of q_a. Note that although there are outside products available in outside markets, we focus on only one C2C market and implicitly assume that the sellers under consideration, if they choose to sell products, can only participate in the considered C2C market [10]. In our future work, we shall explicitly consider that the sellers may sell products in multiple markets. Thus, x_a is essentially interpreted as "outside activity" of the buyers, i.e., how many products buyers purchase in outside markets. Note that $x(q)$ can be rewritten as $x(q \,|\, q_m, q_a)$, although we use the succinct notation $x(q)$ throughout the paper whenever applicable. If q_m increases (decreases), there will be fewer types of products available in the considered C2C market. Because of the continuum model, we allow $x(q)$ and x_a to take non-integer values, and $x(q)$ actually represents the sales volume *density* for a continuum of products with quality $q \in [q_m, 1]$, i.e., $x(q)$ is the sales volume that an *individual* seller with a product quality of q obtains. Next, by using a *quality-adjusted* version of the well-known Dixit-Stiglitz function [17][18] as the utility function which captures product heterogeneity as well as the buyers' "love for variety", we formulate the utility maximization problem for the representative buyer as follows

$$\begin{aligned} U(x(q), x_a \,|\, q_m, q_a) &= \left[\int_{q_m}^{1} q \cdot x(q)^{\frac{\sigma-1}{\sigma}} dF(q) + q_a \cdot x_a^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \\ s.t., \quad & \int_{q_m}^{1} x(q) dF(q) + x_a \leq T, \end{aligned} \tag{4}$$

where $\sigma > 1$ measures the elasticity of substitution between different products. In the extreme case, the products are perfectly substitutable when $\sigma = \infty$, i.e., purchasing product A and product B makes no difference except for the quality difference [17]. The inequality in (4) specifies the budget constraint, i.e., the total expenditure in purchasing products cannot exceed T. As we stated in Section 3.2, the product price is normalized to 1 and hence, the price does not appear in the inequality constraint in (4). Note that to limit the number of parameters, we assume that the price of outside products is also normalized to 1. We can also choose other values of outside product price, and it does not affect our analysis except for that the aggregate outside product quality may be changed. It is also worth mentioning that an implicit assumption underlying the problem (4) is that the aggregate quality of outside products is independent of the intermediary's transaction fee decision and other variables in the model such as q_m and $x(q)$. This can be justified by noting that there are many outside markets besides the considered C2C market and changes in one market have a negligible impact on the others. Before performing further analysis, we list the following properties satisfied by the utility function $U(x(q), x_a \,|\, q_m, q_a)$ in (4).

Property 1 (Diminishing marginal utility): $U(x(q), x_a \,|\, q_m, q_a)$ is increasing and strictly (jointly) concave in $x(q)$ and x_a, for $q \in [0, 1]$.

Property 2 (Preference towards diversified products): $\max_{x(q)\geq 0, x_a \geq 0} U(x(q), x_a \,|\, q_m, q_a)$ is decreasing in $q_m \in [0, 1]$.

Property 3 (Negative externalities): Denote by $x^*(q \,|\, q_m, q_a)$, for $q \in [0, 1]$, the optimal solution to (4). $x^*(q \,|\, q_m, q_a)$ is continuous and strictly increasing in $q_m \in [0, 1]$, increasing in $q \in [0, 1]$, and decreasing in q_a for $q_a \in [0, \infty)$. In particular, $x^*(0 \,|\, q_m, q_a) = 0$ for all $q_m \in [0, 1]$ and $q_a \geq 0$.

We briefly discuss the above properties. Property 1 captures the effects of diminishing marginal utility when the representative buyer purchases more products [17]. Property 2 models the phenomenon that buyers will benefit from the participation of sellers in the market. This is particularly true for online markets, where the buyers prefer to be given available options for a diversified bundle of products. Thus, when $q_m \in [0, 1]$ increases, i.e., fewer sellers sell products in the market, the representative buyer's (maximum) utility decreases. Property 3 reflects the "crowding effects", i.e., lower q_m or more (types of) products available increases competition among the sellers. Specifically, an individual seller will obtain a lower sales volume if more sellers choose to sell products in the market or the aggregate outside product quality is higher [19].

Remark 7: Although we focus on the utility function defined in (4) for the ease of presentation, our analysis of product purchasing and product selling decisions applies to any other utility functions that satisfy Properties 1–3.

3.4 Model Extension

To keep the model succinct and highlight our hierarchical framework that captures the customer rationality, we only present the basic model in this paper. In this subsection, we briefly discuss how our basic model is extended to better capture a real market. In particular, we emphasize heterogeneous selling costs and heterogeneous product prices.

Heterogeneous Selling Costs. The assumption that all the sellers have the same (homogeneous) selling cost can be relaxed to consider that different sellers have heterogeneous selling costs. Specifically, as in [20], we assume that there are $K \geq 1$ possible values of selling costs, denoted by $c_1, c_2, \ldots, c_K$, where $0 < c_1 \leq c_2 \cdots \leq c_K$, and refer to sellers with the selling cost of c_k as type-k sellers. Under the continuum model, the (normalized) mass of type-k sellers is $n_k > 0$ such that $\sum_{k=1}^{K} n_k = 1$. To model the product quality heterogeneity, we consider that the product quality of type-k sellers follows a continuous and positive CDF denoted by $F_k(q) > 0$ for $q \in [0, 1]$. Thus, the fraction of type-k sellers whose product quality is less than or equal to $q \in [0, 1]$ is given by $n_k F_k(q)$. Following a framework of analysis similar to the one illustrated in Fig. 1, we can show that there exists a unique equilibrium outcome in the selling decision stage, and develop a recursive algorithm to derive the optimal transactions fee to maximize the intermediary's revenue.

Heterogeneous Product Prices. To explain how the assumption of a uniform price for all the products can be relaxed, we consider a scenario that the product price is expressed as a function $p(q)$ in terms of the quality.[5] To limit the number of free parameters, we still assume that the price for outside products is normalized to 1. Hence, the budget constraint in (4) becomes $\int_{q_m}^{1} x(q) \cdot p(q) dF(q) + x_a \leq T$, while the objective function in (4) remains unchanged. Then, buyers will purchase more products that have higher values of "quality/price" (i.e., $q/p(q)$) instead of higher values of q. Moreover, according to the distribution of product quality, we can easily derive the distribution of $q/p(q)$. As a result, we can view $q/p(q)$ as if it were the product quality "q" in our basic model. Next, because of the price heterogeneity, a seller' profit may not always increase with the sales volume. To tackle this problem, we can normalize the sellers' profits with respect to their own net profits per product without affecting the binary selling decisions. For instance, if the profits of seller A and seller B are $(s_A - p_A \cdot \theta) \cdot x_A - c$ and $(s_B - p_B \cdot \theta) \cdot x_B - c$, then the corresponding normalized profits are $x_A - c/(s_A - p_A)$ and $x_B - c/(s_B - p_B)$, respectively, where p_A, s_A and x_A are seller A's product price, product profit, and sales volume, respectively, and similar definitions for seller B. Note that θ is the percentage that the intermediary charges as the transaction fee based on the product price, while in our basic model the normalized product price is 1 and hence the product price term does not appear in (1) or (2). It can be seen that the normalized profits of sellers are obtained by dividing (2) by $s - \theta$, except for the heterogeneous selling costs. Thus, the analysis of selling decisions can be performed following the "heterogeneous selling costs" model that we discussed above. To sum up, if we view $q/p(q)$ as if it were the product quality "q" in our basic model, then the analysis in this paper still applies, although there may not exist a closed-form expression for the optimal transaction fee θ^* to maximize the intermediary's revenue (since the intermediary's profit expression changes) and we may need to resort to numerical methods to find it.

4 Revenue Maximization in C2C Markets

In this section, based on the model described above, we study the problem of optimizing the transaction fee in the presence of self-interested sellers and buyers. We proceed with our analysis using backward induction.

[5] We can also consider that products of the same quality may have different prices, but this significantly complicates the notations and explanation.

4.1 Optimal Product Purchasing

By considering the quality-adjusted Dixit-Stiglitz utility defined in (4) and uniform distribution of the product quality, we can obtain explicitly the closed-form solution as follows

$$x^*(q) = \frac{T(\sigma+1)q^\sigma}{(\sigma+1)\cdot q_a^\sigma + \left(1 - q_m^{\sigma+1}\right)}, \tag{5}$$

for $q \in [q_m, 1]$, $x^*(q) = 0$ for $q \in [0, q_m)$, and $x_a^* = \frac{T(\sigma+1)q_a^\sigma}{(\sigma+1)\cdot q_a^\sigma + \left(1-q_m^{\sigma+1}\right)}$. The details of deriving (5) are omitted for brevity. After plugging $x^*(q)$ and x_a^* into (4), the maximum utility derived by the representative buyer is given by

$$U^*(x^*(q), x_a^*) = T\left[q_a^\sigma + \frac{1 - q_m^{\sigma+1}}{\sigma+1}\right]^{\frac{1}{\sigma-1}}, \tag{6}$$

which is decreasing in $q_m \in [0, 1]$. Note that the other concave utility functions can also be considered, although an explicit closed-form solution may not exist.

4.2 Equilibrium Selling Decision

Based on the representative buyer's product purchasing decision, we now analyze the self-interested selling decisions made by sellers (i.e., Stage 2 in Fig. 1). Due to rationality, sellers will not choose to sell products if they cannot obtain non-negative profits. Essentially, interaction among the sellers can be formalized as a non-cooperative game with an infinite number of players, the solution to which is (Nash) equilibrium. The intermediary's revenue will become stabilized if the product selling stage reaches an equilibrium. Thus, the existence of an equilibrium point is important and relevant for the intermediary to maximize its long-term revenue. At an equilibrium, if any, no sellers can gain more profits by deviating from their decisions. In other words, the fraction of sellers choosing to sell products on the intermediary's C2C market does not change at the equilibrium, or equivalently, the marginal product quality $q_m \in [0, 1]$ becomes invariant. Next, we study the equilibrium selling decision by specifying the equilibrium marginal product quality denoted by q_m^*.

If $q_m^* = 1$, then no (or a zero mass of) sellers can receive a non-negative profit by selling products in the market. This implies that, with $q_m^* = 1$, we have $x^*(1|1, q_a)\cdot(\theta+s) - c \leq 0$. If there are some sellers choosing to sell products at the equilibrium (i.e., $q_m^* \in [0, 1)$), then according to the definition of marginal product quality, we have $x^*(q_m^*|q_m^*, q_a)\cdot(\theta+s) - c = 0$. Hence, we can show that

$$q_m^* \triangleq Q(q_m^*) = \left[\left\{\frac{c\cdot\left[(\sigma+1)\cdot(q_a)^\sigma + 1 - (q_m^*)^{\sigma+1}\right]}{T(\sigma+1)(s-\theta)}\right\}^{\frac{1}{\sigma}}\right]_0^1, \tag{7}$$

where $[\nu]_0^1 = \max\{1, \min\{0, \nu\}\}$. Thus, an equilibrium selling decision exists if and only if the mapping $Q(q_m^*)$, defined in (7), has a fixed point. Next, we formally define the equilibrium marginal product quality in terms of q_m^* as below.

Definition 1: q_m^* is an *equilibrium* marginal product quality if it satisfies $q_m^* = Q(q_m^*)$.

We establish the existence and uniqueness of an equilibrium marginal product quality in Theorem 1, whose proof is omitted for brevity. For the proof technique, interested readers may refer to [20] where we consider a user-generated content platform.

Theorem 1. *For any $\theta \in [-s, b]$, there exists a unique equilibrium $q_m^* \in (0, 1]$ in the selling decision stage. Moreover, q_m^* satisfies*

$$\begin{cases} q_m^* = 1, & \text{if } x^*(1 \mid 1, q_a) \cdot (s - \theta) \leq c, \\ q_m^* \in (0, 1), & \text{otherwise}, \end{cases} \tag{8}$$

where $x^*(1 \mid 1, q_a)$ is obtained by solving (4) with $q_m \to 1$.[6] □

Theorem 1 guarantees the existence of a unique equilibrium point and shows that if the seller with the highest product quality cannot obtain a profit (due to high selling cost, high transaction fee, etc.), then no sellers choose to sell products in the market at equilibrium. For notational convenience, we denote the value of θ that satisfies $x^*(1 \mid 1, q_a) \cdot (s - \theta) = c$ by

$$\bar{\theta} \triangleq s - \frac{c}{x^*(1 \mid 1, q_a)} = s - \frac{c \cdot (q_a)^\sigma}{T}. \tag{9}$$

Then, it follows from Theorem 1 that the intermediary can gain a positive revenue if and only if $\theta \in (-b, \bar{\theta})$. Nevertheless, if $\bar{\theta} \leq -b$, then the intermediary's revenue is always zero. Hence, we assume $\bar{\theta} > -b$ throughout the paper. Based on the uniqueness of q_m^* for any $\theta \in [-b, s]$, we can express $q_m^* = q_m^*(\theta)$ as a function of $\theta \in [-b, s]$. While there exists no simple closed-form expression of $q_m^*(\theta)$, it can be easily shown that $q_m^*(\theta) \in (0, 1)$ is strictly increasing in $\theta \in [-b, \bar{\theta})$ (i.e., fewer sellers choose to sell products in the market when the transaction fee θ increases) and $q_m^*(\theta) = 1$ for $\theta \in [\bar{\theta}, s]$.

4.3 Optimal Transaction Fee

Based on decisions made by the buyers and sellers, we study the optimal transaction fee θ that maximizes the intermediary's steady-state revenue (i.e., revenue obtained when the product selling decision stage reaches equilibrium). Mathematically, we formalize the problem as

$$\theta^* = \arg \max_{\theta \in [-b, \bar{\theta}]} (b + \theta) \cdot \bar{x}, \tag{10}$$

[6] When $q_m \to 1$, only a negligible fraction of sellers choose to sell products in the market.

where $\bar{x} = \int_{q_m^*}^{1} x^*(q \mid q_m^*, q_a) dF(q)$. The decision interval is shrunk to $[-b, \bar{\theta}]$, since $\theta \in (\bar{\theta}, s]$ always results in a zero revenue for the intermediary, where $\bar{\theta}$ is defined in (9). In the following analysis, a closed-form optimal transaction fee $\theta^* \in [-b, s - \frac{c \cdot (q_a)^\sigma}{T}]$ is obtained and shown in Theorem 2.

Theorem 2. *The unique optimal transaction fee $\theta^* \in [-b, \bar{\theta}]$ that maximizes the intermediary's revenue is given by*

$$\theta^* = s - \frac{c \cdot \left[(\sigma + 1) \cdot (q_a)^\sigma + 1 - z^{\sigma+1}\right]}{T(\sigma + 1) \cdot z^\sigma}, \tag{11}$$

where $z \in [q_m^*(-b), 1]$ is the unique root of the equation[7]

$$-\frac{T \cdot (q_a)^\sigma \cdot (b + s)}{\left[(\sigma + 1) \cdot (q_a)^\sigma + 1 - z^{\sigma+1}\right]^2} + \frac{c}{(\sigma + 1)^3} \cdot \frac{\sigma + z^{\sigma+1}}{z^{2\sigma+1}} = 0. \tag{12}$$

Proof. Due to space limitations, we only provide the proof outline. Instead of directly solving (10)*, we first find the optimal (equilibrium) marginal product quality, which is the root of* (12)*. Then, based on the marginal user principle, we can obtain the optimal transaction fee θ^* maximizing the intermediary's revenue. The detailed proof technique is similar to that in [20].* ■

Next, we note that, to maximize its revenue, the intermediary may even reward the sellers for selling products in the market, i.e., $\theta^* < 0$. In particular, "rewarding" should be applied if one of the following cases is satisfied:

1. Total budget T (i.e., market size) is sufficiently small;
2. Selling cost c is sufficiently large;
3. Profit of each sold product s is sufficiently small;
4. Aggregate outside product quality q_a is sufficiently large;
5. Advertising revenue for each sold product b is sufficiently large.

In the first four cases, few sellers can receive a non-negative profit by selling products without being economically rewarded by the intermediary (e.g., if the selling cost c is very high, then sellers need to receive subsidy from the intermediary to cover part of their selling costs). The last case indicates that if the intermediary can derive a sufficiently high advertising revenue for each sold product, then it can share the advertising revenue with the sellers to encourage them to sell products in the market such that the intermediary can increase its total advertising revenue. In Fig. 2, we illustrate the impacts of transaction fees on the intermediary's revenue. Note that the numeric settings for Fig. 2 are only for the purpose of illustration and our analysis applies to any other settings. For instance, with all the other parameters being the same, a larger value of T indicates that the buyers spend more money in online shopping (i.e., the online shopping market size is bigger). In practice, the intermediary needs to obtain

[7] $q_m^*(-b)$ is the equilibrium point in the product selling stage when $\theta = -b$.

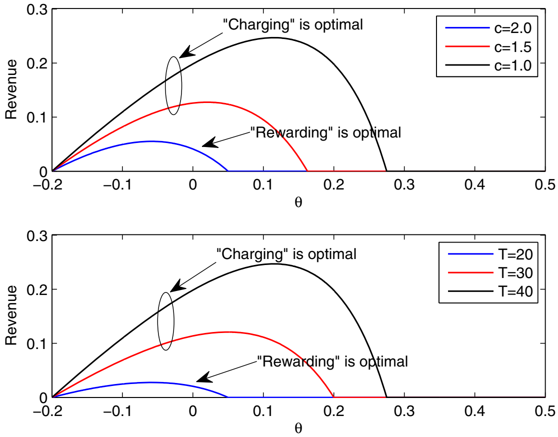

Fig. 2. Revenue versus transaction fee. $\sigma = 2.0$, $b = 0.2$, $s = 0.5$, $q_a = 3.0$. Upper: $T = 40$; Lower: $c = 1.0$.

real market settings by conducting market surveys, data analysis, etc. [8]. The upper plot Fig. 2 verifies that the intermediary should reward the sellers if the selling cost is high, while the lower plot indicates the intermediary should share its advertising revenue with sellers in an emerging online shopping market (i.e., the market size is small). We also observe from Fig. 2 that by optimally choosing the transaction fee θ^*, the intermediary can significantly increase its revenue compared to setting a non-optimal transaction fee (e.g., $\theta = 0$). For instance, the upper plot in Fig. 2 shows that with an optimal transaction fee and $c = 1.0$, the intermediary's revenue increases by nearly 30% compared to $\theta = 0$ (i.e., the intermediary only relies on advertising revenues). Due to the space limitation, we omit more numerical results and the analytical condition specifying when the intermediary should reward sellers (i.e., $\theta^* < 0$) to maximize its revenue.

5 Conclusion

In this paper, we studied a C2C market and proposed an algorithm to identify the optimal transaction fee to maximize the intermediary's revenue while taking into account the customer rationality. We first used the *representative* buyer model to determine how the buyers' total budget is allocated across a variety of products. Then, we showed that there always exists a unique equilibrium point at which no seller can gain by changing its selling decision. Next, we formalized the intermediary's revenue maximization problem and, by using the quality-adjusted Dixit-Stiglitz utility function function and the uniform distribution of product qualities, derived the closed-form optimal solution explicitly. We discussed qualitatively the impacts of the aggregate outside product quality and product substitutability on the intermediary's revenue. Extension to heterogeneous selling costs and product prices were also addressed. Our results showed that a significant increase in the intermediary's revenue can be achieved using our proposed algorithm. Future research directions include, but are not limited to: (1) competition among different markets; (2) intermediary's investment decisions; and (3) optimal transaction fee maximizing social welfare.

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