Noncooperative Games for Autonomous Consumer Load Balancing over Smart Grid

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Abstract. Traditionally, most consumers of electricity pay for their consumptions according to a fixed rate. With the advancement of Smart Grid technologies, large-scale implementation of variable-rate metering becomes more practical. As a result, consumers will be able to control their electricity consumption in an automated fashion, where one possible scheme is to have each individual maximize its own utility as a noncooperative game. In this paper, noncooperative games are formulated among the electricity consumers in Smart Grid with two real-time pricing schemes, where the Nash equilibrium operation points are investigated for their uniqueness and load balancing properties. The first pricing scheme charges a price according to the average cost of electricity borne by the retailer and the second one charges according to a time-variant increasing-block price, where for each scheme, a zero-revenue model and a constant-rate revenue model are considered. The Nash equilibrium is shown to exist for four different combined cases corresponding to the two pricing schemes and the two revenue models, and is unique for three of the cases under certain conditions. It is further shown that both pricing schemes lead to similar electricity loading patterns when consumers are only interested in minimizing the electricity costs without any other profit considerations. Finally, the conditions under which the increasing-block pricing scheme is preferred over the average-cost based pricing scheme are discussed.

Keywords: Game Theory, Noncooperative Game, Nash Equilibrium, Smart Grid, Real Time Pricing, Increasing-Block Pricing.

1 Introduction

In the traditional power market, electricity consumers usually pay a fixed retail price for their electricity usage. This price only changes on a seasonal or yearly basis. However, it has been long recognized in the economics community that charging consumers a flat rate for electricity creates allocative inefficiencies, i.e., consumers do not pay equilibrium prices according to their consumption levels [1]. This was shown through an example in [2], which illustrates how flat pricing causes deadweight loss at off-peak times and excessive demand at the peak times. The latter may lead to small-scale blackouts in a short run and excessive capacity

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buildup over a long run. As a solution, variable-rate metering that reflects the real-time cost of power generation can be used to influence consumers to defer their power consumption away from the peak times. The reduced peak-load can significantly reduce the need for expensive backup generation during peak times and excessive generation capacity.

The main technical hurdle in implementing real-time pricing has been the lack of cost-effective two-way smart metering, which can communicate real-time prices to consumers and their consumption levels back to the energy provider. In addition, the claim of social benefits from real-time pricing also assumes that the consumer demand is elastic and responds to price changes while traditional consumers do not possess the equipments that enable them to quickly alter their demands according to the changing power prices. Significant research efforts on real-time pricing have involved estimating the consumer demand elasticity and the level of benefits that real time pricing can achieve [1,3,4]. Fortunately, the above requirements on smart metering and consumer adaptability are being fulfilled [5] as technology advances in cyber-enabled metering, power generation, power storage, and manufaturing automation, which is driven by the need for a Smart Grid.

Such real-time pricing dynamics have been studied in the literature mainly with game theory [6–8]. In particular, the authors in [6] provided a design mechanism with *revelation principle* to determine the optimal amount of incentive that is needed for the customers to be willing to enter a contract with the utility and accept power curtailment during peak periods. However, they only considered a fixed pricing scheme. In [7], the authors studied games among consumers under a certain class of demand profiles at a price that is a function of day long aggregate cost of global electricity load of all consumers. However, the case with real-time prices was not investigated in [7]. In [8], a noncooperative game was studied to tackle the real-time pricing problem, where the solution was obtained by exploring the relationship with the congestion games and potential games. However, the pricing schemes that we study are not amenable to transformations described in [8].

In this paper we formulate noncooperative games [9,10] among the consumers with two real-time pricing schemes under more general load profiles and revenue models. The first pricing scheme charges a price according to the instantaneous average cost of electricity production and the second one charges according to a time-varying version of increasing-block price [11]. We investigate consumer demands at the Nash equilibrium operation points for their uniqueness and load balancing properties. Furthermore, two revenue models are considered for each of the schemes, and we show that both pricing schemes lead to similar electricity loading patterns when consumers are interested only in the minimization of electricity costs. Finally we discuss the conditions under which the increasing-block pricing scheme is preferred over the average-cost based pricing scheme.

The rest of the paper is organized as follows. The system model and formulation of the noncooperative game are presented in Section 2. The game is analyzed with different real-time pricing schemes under different revenue models in Sections 3 and 4, where the Nash equilibrium properties are investigated. We conclude the paper in Section 5.

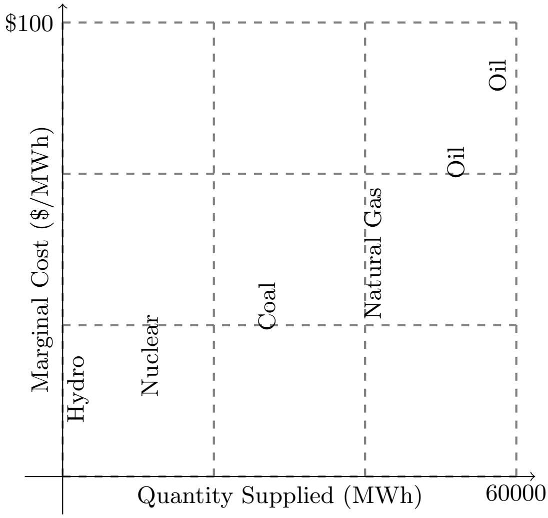
2 System Model and Game Formulation

2.1 System Model

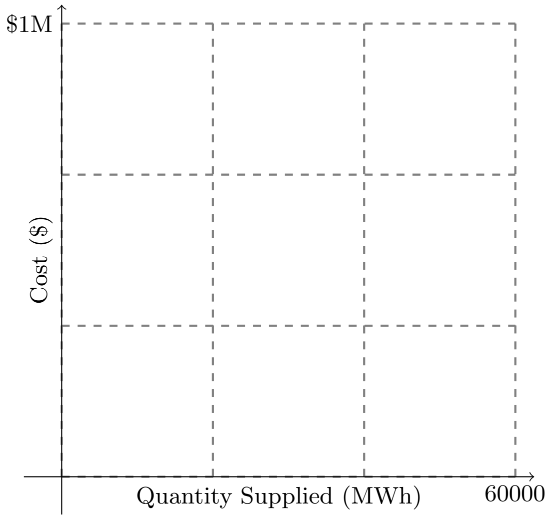
We study the transaction of energy between a single electricity retailer and multiple consumers. In each given time slot, each consumer has a demand for electric energy (measured in Watt-hour, Wh). The job of the retailer is to satisfy demands from all the consumers. The electricity supply of the retailer is purchased from a variety of sources over a wholesale electricity market and the retailer may possess some generation capacity as well. These sources may use different technologies and fuels to generate electricity, which leads to different marginal costs of electricity at the retailer, where the marginal cost is the incremental cost incurred to produce an additional unit of output [12]. Mathematically, the marginal cost function is expressed as the first derivative of the total cost function. Examples of the marginal cost function and the corresponding total cost are presented in Fig. 1(a) and Fig. 1(b), respectively, which are based on real world data from the wholesale electricity market [3]. Naturally, the retailer attempts to satisfy demands by procuring the cheapest source first. This results in a non-decreasing marginal cost of the supply curve, as illustrated through the example in Fig. 1(a). The retailer charges each consumer a certain price for its consumption in order to cover the cost, where the sum payments by all the consumers should be enough to cover the total cost and certain profit margin set by the retailer or regulatory body. In our model we assume that all these are incorporated within the marginal cost of electricity.

While the retailer aims to procure sufficient supply to meet the sum demand of its consumers in each time slot, in reality, the supply is limited by the generation capacity available in the wholesale electricity market. Thus, the maximum sum load that the retailer can service bears an upper limit and we model this capacity limit by setting the marginal cost of electricity to infinity when the sum load exceeds a predetermined threshold. Each consumer has an energy demand in each time slot and it pays the retailer at a price that is set by the retailer such that, in each time slot, the sum of payments made by all consumers meets the total cost in that slot. As such, a particular consumer's share of this bill depends on the retailer's pricing scheme, which is a function of the demands from all the consumers. Accordingly, as the total load varies over time, each consumer operates over a time-variant price with time-slotted granularity. We assume that each consumer has a total demand for electricity over each day[1], which can be distributed throughout the day in a time-slotted manner, to maximize certain utility function. Next, we model such individual load balancing behaviors as a noncooperative game.

[1] Here we adopt one day as an operation period that contains a certain number of time slots. Obviously, such a choice has no impact on the analytical results in this paper.



(a) Marginal cost as a function of supply



(b) Total cost as a function of supply

Fig. 1. A hypothetical marginal cost of supply and the corresponding total cost curve as seen by the retailer in the wholesale market within a single time slot. Supply is from five different sources: hydroelectric, nuclear, coal, natural gas, and oil. Two different generators may use different technologies for power generation thus incurring different marginal costs with the same fuel (e.g., the two different cost levels for oil in Fig. 1(a)).

2.2 Noncooperative Load Balancing Game

The noncooperative game between these consumers is formulated as follows. Consider a group of N consumers, who submit their daily demands to a retailer in a time-slotted pattern at the beginning of the day (which contains T time slots). These consumers are selfish and aim to maximize their individual utility/payoff functions; hence they do not cooperate with each other to manage their demands. Each consumer i has a minimum total daily requirement of energy, $\beta_i \geq 0$, which is split over the T time slots. Let x_t^i denote the ith consumer's demand in the tth time slot. A consumer can demand any value $x_t^i \geq 0$ (negativity constraint) with $\sum_t x_t^i \geq \beta_i$ (demand constraint). Let $\mathbf{x}^i = \{x_1^i, x_2^i, \ldots, x_t^i, \ldots, x_T^i\}$, represent the ith consumer's demand vector, which is called the strategy for the ith consumer. Let $\mathbf{x}_t = \{x_t^1, \ldots, x_t^N\}$, represent the demand vector from all consumers in time slot t with $x_t = \sum_i x_t^i$. Let $\mathbf{x}$ represent the set $\{\mathbf{x}^1, \ldots, \mathbf{x}^N\}$.

The payoff or utility for consumer i is denoted by π^i which is the difference between the total revenue it generates from the purchased electricity and its cost. In particular, let E_t^i, a function of x_t^i, represent the revenue generated by the ith consumer in the tth time slot and M_t^i, a function of $\mathbf{x}_t$, represent its payment to the retailer for purchasing x_t^i. Then the payoff π^i, to be maximized by consumer i, is given by

$$\pi^i = \sum_{t \in \{1,\ldots,T\}} \left(E_t^i - M_t^i \right).$$

Since M_t^i is a function of $\mathbf{x}_t$, we see that the consumer payoff is influenced by its load balancing strategy and those of other consumers.

We consider the problem of maximizing the payoff at each consumer by designing the distributed load balancing strategy $\mathbf{x}^i$'s, under two real-time pricing schemes set by the retailer. The first one is the average-cost based pricing scheme and the second one is the increasing-block pricing scheme. Specifically, for the first scheme the retailer charges the consumers the average cost of electricity procurement that is only dependent on the sum demands, x_t, from all the consumers. For the second scheme, the retailer charges according to a marginal cost function that depends on the vector of demands from all consumers, $\mathbf{x}_t$.

Let $C(x)$ represent the cost of x units of electricity, to the retailer, from the wholesale market (an example function is plotted in Fig. 1(b)). Then under the average-cost based pricing, the price per unit charged to the consumers is given by

$$A(x_t) = C(x_t)/x_t, \tag{1}$$

and at time t consumer i pays

$$M_t^i = x_t^i A(x_t) \tag{2}$$

for consuming x_t^i units of electricity. It is easy to see that $\sum_i M_t^i = C(x_t)$, i.e., with average-cost based pricing the total payment made by the consumers covers the total cost to the retailer. Note that $C'(x_t)$ gives the marginal cost function in the wholesale market, henceforth denoted by $\mathcal{C}(x_t) = C'(x_t)$ in the context of increasing-block pricing (an example marginal cost curve is plotted in Fig. 1(a)). For reasons we discussed earlier, in the context of electricity market, the marginal cost $\mathcal{C}(x_t)$ is always non-negative and non-decreasing such that $C(x_t)$ is always positive, non-decreasing, and convex. Briefly, we note that as the retailer capacity is constrained by a predetermined upper limit U, we model this constraint as $C(x_t) = \infty$, $\forall x_t > U$; obviously $x_t^i \leq U$ is an implicit constraint on the demand x_t^i for any rational consumer.

The second scheme is a time-variant version of the increasing-block pricing scheme [11]. With a typical increasing-block pricing scheme, consumer i is charged a certain rate b_1 for its first z_1 units consumed, then charged rate b_2 ($> b_1$) for additional z_2 units, and charged rate b_3 ($> b_2$) for additional z_3 units, and so on. The b's and z's describe the marginal cost price for the commodity. In our scheme we design a marginal cost function, which retains the increasing nature of increasing-block pricing, such that it depends on $\mathbf{x}_t$ and the function $C(\cdot)$. Consumer i pays an amount determined by the marginal cost function $\mathcal{M}(x, \mathbf{x}_t)$, applicable to all consumers at time slot t. In particular consumer i pays

$$M_t^i = \int_0^{x_t^i} \mathcal{M}(x, \mathbf{x}_t) dx \tag{3}$$

for consuming x_t^i units of electricity where $\mathcal{M}(\cdot)$ is chosen as

$$\mathcal{M}(x, \mathbf{x}_t) = \mathcal{C}\left(\sum_j \min\left(x, x_t^j\right)\right),$$

such that $\sum_i M_t^i = C(x_t)$ is satisfied. An intuition behind this pricing scheme is to penalize consumers with relatively larger demands. Note that in this case, $x_t^i \leq U$ is implicitly assumed by letting $C(\cdot) = \infty$ $\forall x_t^i > U$ and hence $M_t^i = \infty$ $\forall x_t^i > U$.

For each of the two pricing schemes, we study two different revenue models. For the first one we set E_t^i as zero for all consumers over all time slots, which leads to payoff maximization being the same as cost minimization from the point of view of the consumers. For the second one we assign consumer i a constant revenue rate ϕ_t^i at each time slot t, which gives $E_t^i = \phi_t^i x_t^i$ and leads to payoff maximization being the same as profit maximization.

3 Nash Equilibrium with Average-Cost Pricing

For the average-cost pricing, the payment to the retailer in slot t by consumer i is given by (2).

3.1 Zero-Revenue Model

In this case the revenue is set to zero as $E_t^i = 0$, which results in payoff maximization being the same as cost minimization for each consumer. Specifically, the payoff for consumer i is given by $\pi^i = -\sum_t M_t^i$. The consumer load balancing problem for consumer i, for $i = 1, \ldots, N$, is given by the following optimization problem:

$$\begin{aligned} \text{maximize} \quad & \pi^i(\mathbf{x}^i) = -\sum_t M_t^i \\ \text{subject to} \quad & M_t^i = x_t^i A(x_t), \quad \forall t, \\ & \sum_t x_t^i \geq \beta_i, \\ & x_t = \sum_j x_t^j, \quad \forall t, \\ & 0 \leq x_t^i, \quad \forall t. \end{aligned}$$

As cost to the retailer becomes infinity whenever the total demand goes beyond the capacity threshold for the wholesale market, i.e., when $C(x_t) = \infty \quad \forall x_t > U$, the price to consumers will become infinite and their payoff will go to negative infinity. Thus any consumer facing an infinite cost at a particular time slot can manipulate the demand vector such that the cost becomes finite, which is always feasible under the assumption that sum load demand over all times slots is less than sum supply availability. This implies that, at Nash equilibrium, sum demand x_t will be less than the capacity threshold U, $\forall t$, which allows for a redundant constraint $x_t^i \leq U$, $\forall i, t$, as $x_t^i \leq \sum_i x_t^i = x_t \leq U$. Such a redundant but explicit constraint in turn makes the feasible region for $\mathbf{x}$, denoted by $\mathcal{X}$, finite and hence compact. The compactness property is utilized to prove the Kakutani's theorem [13] which in turn is required to show the existence of NEP solution.

By the results in [14] we can show that there exists an NEP strategy for all agents with the cost function used here and the NEP solution exists for the proposed noncooperative consumer load balancing game.

On the other hand, the cost function M_t^i does not satisfy the conditions for being a type-A function, defined in [14]. Therefore, the corresponding uniqueness result in [14] cannot be extended to our formulation. In [15] we show that our problem is equivalent to an atomic flow game [16] with splittable flows and different *player types* (i.e., each player controls a different amount of total flow) over a generalized nearly-parallel graph, which has strictly semi-convex, non-negative, and non-decreasing functions for cost per unit flow. By the results of [16], we can prove that the NEP solution for the load balancing game is unique [15].

In the following, we discuss the properties for the unique NEP solution for the proposed load balancing game.

Lemma 1. *With the average-cost based pricing and zero revenue, at the Nash equilibrium the price of electricity faced by all consumers is the same over all time slots.*

The proof is provided in [15].

Lemma 2. *If $C(\cdot)$ is strictly convex, at the Nash equilibrium, the sum of demands on the system, x_t, keeps the same across different time slots.*

The proof is provided in [15].

Lemma 3. *If $C(\cdot)$ is strictly convex, at Nash equilibrium, each consumer will distribute its demands equally over the T time slots.*

The proof is provided in [15].

Remark: Under the average-cost based pricing scheme with zero revenue, if one particular consumer increases its total demand of electricity, the price $A(\cdot)$ increases, which in turn increases the payments for all other consumers as well. Theoretically one consumer may cause indefinite increases in the payments of all others; and in this sense this scheme does not protect the group from reckless action of some consumer(s). This issue will be addressed by our second pricing scheme as we will show in Section 4.

3.2 Constant-Rate Revenue Model

In this case, the rate of revenue generation for each consumer at each time slot is taken as a non-negative constant ϕ_t^i. Thus, $E_t^i = \phi_t^i \times x_t^i$. The consumer load balancing problem for each consumer i is given by the following optimization problem:

$$\begin{aligned}
\text{maximize} \quad & \pi^i(\mathbf{x}^i) = \sum_t \left(E_t^i - M_t^i\right) \\
\text{subject to} \quad & E_t^i = \phi_t^i x_t^i, \quad \forall t, \\
& M_t^i = x_t^i A(x_t), \quad \forall t, \\
& \sum_t x_t^i \geq \beta_i, \\
& x_t = \sum_j x_t^j, \quad \forall t, \\
& 0 \leq x_t^i, \quad \forall t.
\end{aligned}$$

We assume that $\beta_i = 0,\ \forall i$, and the rate of revenue is larger than the price of electricity such that we do not end up with any negative payoff or the trivial solution $x_t^i = 0,\ \forall i, t$.

Here again, if the sum demand in a given time slot t exceeds the retailer's capacity threshold U, the consumers will face an infinite price for their consumption. This implies that, at Nash equilibrium the sum demand x_t will never exceed the capacity threshold U, as we assume that sum load demand over all time slots is greater that sum load available. This again allows for the redundant constraint $x_t^i \leq U,\ \forall i, t$, as $x_t^i \leq \sum_i x_t^i = x_t \leq U$, which in turn makes the feasible region for $\mathbf{x}$, $\mathcal{X}$, finite and hence compact.

The proof for the existence of NEP for this game under the given assumptions is provided in [15].

Lemma 4. *At the Nash equilibrium, the consumer(s) with the highest revenue rate (ϕ_t^i) within the time slot, may be the only one(s) buying the power in that time slot.*

The proof is provided in [15]. Thus if consumer i has the maximum rate of revenue, either it is the only consumer buying non-zero power x_t^i such that $\phi_t^i = A(x_t^i)$ or $\phi_t^i < C'(0)$ and hence $x_t^i = 0$ in that time slot, which leads to a unique Nash equilibrium for the sub-game. If in a given time slot multiple consumers experience the same maximum rate of revenue, the sub-game will turn into a Nash Demand Game [17] between the set of consumers given by $\{\arg\max_k \phi_t^k\}$, which is well known to admit multiple Nash equilibriums. Thus the overall noncooperative game has a unique Nash equilibrium if and only if, in each time slot, at most one consumer experiences the maximum rate of revenue.

4 Nash Equilibrium with Increasing-Block Pricing

In this section we study the load balancing game with the time-variant increasing-block pricing scheme. Under this scheme consumer i pays M_t^i for x_t^i units of electricity, which is given by (3) with $\mathcal{M}(x, \mathbf{x}_t)$ the marginal cost function posed to the consumer. Thus, as defined before, we have

$$\mathcal{M}(x, \mathbf{x}_t) = \mathcal{C}\left(\sum_j \min\left(x, x_t^j\right)\right).$$

As an example, if the demands from different consumers at time slot t are identical, i.e., if $x_t^i = x_t^j, \ \forall i, j$, we have,

$$\mathcal{M}(x, \mathbf{x}_t) = \mathcal{C}(Nx).$$

4.1 Zero-Revenue Model

In this case the payment by consumer i is given by (3)

$$M_t^i = \int_0^{x_t^i} \mathcal{M}(x, \mathbf{x}_t) dx.$$

The consumer load balancing problem for each consumer i is given by the following optimization problem:

$$\begin{aligned} \text{maximize} \quad & \pi^i(\mathbf{x}^i) = -\sum_t M_t^i \\ \text{subject to} \quad & M_t^i = \int_0^{x_t^i} \mathcal{M}(x, \mathbf{x}_t) dx, \quad \forall t, \\ & \sum_t x_t^i \geq \beta_i, \\ & 0 \leq x_t^i, \quad \forall t. \end{aligned}$$

If the sum demand x_t in a time slot t exceeds U, the price of electricity for the consumer with the highest demand (indexed by $\hat{j}$) becomes infinite. As we retain the assumption that sum load demand over all time slots is greater that sum load available, consumer $\hat{j}$ can rearrange its demand vector such that either the sum demand becomes within the capacity threshold or consumer $\hat{j}$ is no longer the highest demand consumer (then the new customer with the highest demand performs the same routine until the sum demand is under the threshold). This implies that, at the Nash equilibrium point we have $x_t \leq U$. Similarly, we now have the redundant constraint $x_t^i \leq U, \ \forall\ i,\ t$, which in turn makes the feasible region $\mathcal{X}$ finite and hence compact.

The proof for the existence of NEP for this game under the given assumptions is provided in [15]. When each consumer tries to minimize its total cost while satisfying its minimum daily energy requirement β_i, we have the following result.

Lemma 5. *If $C(\cdot)$ is strictly convex, the Nash equilibrium is unique and each consumer distributes its demand uniformly over all time slots.*

The proof is provided in [15].

Remark: Notice that under the zero-revenue model, the NEP point is the same with both increasing-block pricing and average-cost based pricing. For both the cases, at NEP, we have $x_t^i = \beta_i/T, \ \forall i, t$. However, even though the loading pattern is similar, the payments M_t^i made by the consumers will differ and, with increasing-block pricing, will likely be lesser for consumers with relatively lower consumption. In addition, with increasing-block pricing, the maximum payment M_t^i made by the ith consumer given x_t^i demand will be $C(Nx_t^i)/N$, irrespective of what other consumers demand and consume. Thus this addresses the issue faced under the average-cost based pricing and zero-revenue model, in which one consumer can increase their demand indefinitely and cause indefinite increase in the payments of all other consumers.

4.2 Constant-Rate Revenue Model

The consumer load balancing problem for consumer i is given by the following optimization problem:

$$
\begin{aligned}
\text{maximize} \quad & \pi^i(\mathbf{x}^i) = \sum_t \left(E_t^i - M_t^i\right) \\
\text{subject to} \quad & E_t^i = \phi_t^i x_t^i, \quad \forall t, \\
& M_t^i = \int_0^{x_t^i} \mathcal{M}(x, \mathbf{x}_t) dx, \quad \forall t, \\
& \sum_t x_t^i \geq \beta_i, \\
& 0 \leq x_t^i, \quad \forall t.
\end{aligned}
$$

Here again, we assume $\beta_i = 0, \ \forall i$, to avoid any negative payoffs and we could agree for the redundant constraint $x_t^i \leq U, \ \forall \ i, \ t$, which in turn makes the feasible region for $\mathcal{X}$ finite and hence compact.

The proof for the existence of NEP for this game under the given assumptions is provided in [15]. With the average-cost based pricing scheme under the constant-rate revenue model, we see that in a given time slot, if a single consumer enjoys the maximum rate of revenue, it will be the only consumer who is able to purchase power. We show here that with the increasing-block pricing scheme under constant-rate revenue model, the result is different.

For a given time slot t, consumer i has an incentive to increase their demand x_t^i as long as the payoff increases, i.e., $\partial \pi^i / \partial x_t^i > 0$. Therefore at the equilibrium the following holds for all consumers:

$$\begin{aligned} &\frac{\partial \pi^i}{\partial x_t^i} \leq 0 \\ &\Rightarrow \phi_t^i \leq \frac{\partial M_t^i}{\partial x_t^i} = \mathcal{M}(x_t^i, \mathbf{x}_t). \end{aligned} \tag{4}$$

Additionally, if $\phi_t^i < \mathcal{M}(x_t^i, \mathbf{x}_t)$, J_t^i can be reduced by reducing x_t^i. This implies that if $x_t^i > 0$, at the equilibrium we have

$$\phi_t^i \geq \mathcal{M}(x_t^i, \mathbf{x}_t). \tag{5}$$

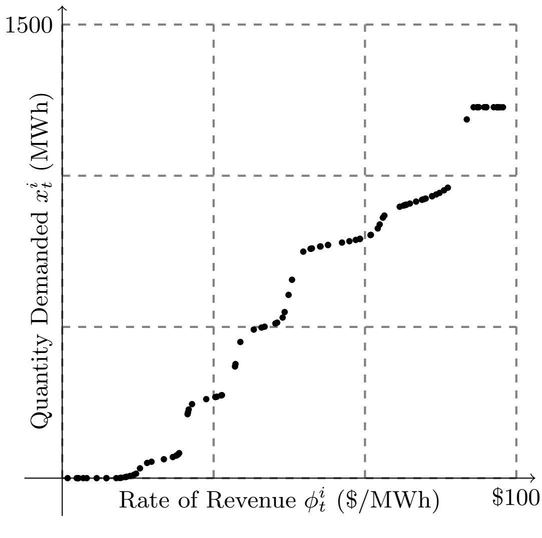


Fig. 2. Demand x_t^i versus the rate of revenue (ϕ_t^i) at equilibrium. Each dot represents a particular consumer $i = \{1, \dots, 100\}$.

Thus (4) and (5) together imply that, if $x_t^i > 0$, we have $\phi_t^i = \mathcal{M}(x_t^i, \mathbf{x}_t)$. Together we can write the following set of necessary conditions for equilibrium,

$$\begin{array}{ll} \phi_t^i = \mathcal{M}(x_t^i, \mathbf{x}_t) & \text{if } \phi_t^i \geq \mathcal{M}(0, \mathbf{x}_t), \\ x_t^i = 0 & \text{if } \phi_t^i < \mathcal{M}(0, \mathbf{x}_t). \end{array} \tag{6}$$

For illustration, we simulate a scenario consisting of 100 consumers, who have their rate of revenue ϕ_t^i generated from a uniform distribution ranging over \$0 − \$100/MWh, where the marginal cost to the retailer $\mathcal{C}(\cdot)$ is given by Fig. 1(a). In Fig. 2 we plot the demand x_t^i versus the rate of revenue (ϕ_t^i) at a given time slot t, where x_t^i is evaluated over $i = \{1, \ldots, 100\}$. The equilibrium is obtained by iterative updates of $\mathcal{M}(\cdot)$ and $\mathbf{x}_t$ until convergence within an error tolerance as in (6).

Thus, unlike with the average-cost pricing, where only the consumer with the maximum rate of revenue could purchase electricity at the equilibrium, any consumer may procure a non-zero amount of energy as long as its own rate of revenue is larger than $\mathcal{M}(0, \mathbf{x}_t)$.

5 Conclusion

In this paper we formulated noncooperative games among the consumers of Smart Grid with two real-time pricing schemes to derive autonomous load balancing solutions. The first pricing scheme charges consumers a price that is equal to the average cost of electricity borne by the retailer and the second scheme charges consumers an amount that is dependent on the incremental marginal cost which is shown to protect consumers from irrational behaviors. Two revenue models were considered for each of the pricing schemes, for which we investigated the Nash equilibrium operation points for their uniqueness and load balancing properties. For the zero-revenue model, we showed that when consumers are interested only in the minimization of electricity costs, the Nash equilibrium point is unique with both the pricing schemes and leads to similar electricity loading patterns in both cases. For the constant-rate revenue model, we showed the existence of Nash equilibrium with both the pricing schemes and the uniqueness results with the average-cost based pricing scheme.

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