# Investigation of Voltage Stability Indices to Identify Weakest Bus (TBC)

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**Abstract.** This paper proposes a new index to determine the static voltage stability of the load buses in a power network for certain operating conditions and hence identifies load buses which are close to voltage collapse. The proposed index is formulated from the quadratic equation derived from a twobus network and is computed using the apparent power and the line impedance. The proposed index shows how far the load buses from their voltage stability limit and hence the most sensitive bus can be identified according to maximum loadability. 14 bus IEEE reliability test system is used to study the performance of the proposed index for its validity. A comparison is also made between proposed index and some other indices found in the literature. The results are discussed and key conclusion drawn.

**Keywords:** Static stability indices, quadratic equation, apparent power, transmission line impedance, loadability.

## 1 Introduction

The phenomenon of voltage collapse has been observed in many countries and has been analyzed extensively in recent years [1-3]. During the last decade, voltage collapse phenomena have attracted more and more attention throughout the world, and several studies have been presented which relate to this problem [4-18]. More researchers have come to believe that voltage stability is dynamic stability and dynamic stability should be applied. However, in this paper only static analysis is presented.

The literature shows extensive work done on static voltage stability, and several voltage stability indices were derived from static power flow analysis. The values of these indices were calculated for each transmission line based on load flow calculation results. The transmission line with largest value was taken as the weakest line, and the bus with minimum loadability was taken as the most sensitive bus, and a suitable reactive power source (FACTS technology) must be connected at the appropriate location (weakest bus). This paper proposes a new index to determine the static voltage stability of the load buses in a power network for certain operating conditions, and hence identifies load buses which are close to voltage collapse. The proposed index is formulated from the quadratic equation derived from a two-bus network and is computed using the apparent power and the line impedance. The

proposed index shows how far the load buses from their voltage stability limit, and hence the most sensitive bus can be identified according to maximum loadability. The 14 bus IEEE reliability test system is used to study the performance of the proposed index for its validity. A comparison is also made between proposed index, and LQP proposed by Mohamed [19], and the FVSI index proposed by Musirin [20].

## 2 Review of Line Indices

#### Fast Voltage Stability Index (FVSI):

FVSI index was proposed by Musirin [20] and is given by,

$$\text{FVSI} = \frac{4\text{Z}Q_j}{\text{V}_i^2\text{X}} \le 1$$

where Z is the transmission line impedance, X is the transmission line reactance,  $Q_j$  is the receiving reactive power, and V<sub>i</sub> is the sending end voltage.

#### Line Stability Factor (LQP):

LQP was proposed by Mohamed [19] and is given by,

$$LQP = 4\left[\left(\frac{X}{V_i^2}\right)\left(\frac{X}{V_i^2}P_i^2 + Q_j\right)\right] \le 1$$

where  $P_i$  is the sending end active power.

#### **3 Proposed Index Formulation**

Consider the two-bus power system sketched in Fig. 1. The symbols may be identified as follows:

- V<sub>i</sub>, the sending end voltage
- V<sub>i</sub>, the receive end voltage
- Z<sub>ij</sub>, the transmission line impedance
- S<sub>i</sub>, the sending end apparent power
- S<sub>j</sub>, the receiving end apparent power

$$Bus i V_i$$

$$Bus j V_j$$

$$Bus j V_j$$

$$S_i = P_i + jQ_i$$

$$Z_{ij} = R_{ij} + jX_{ij}$$

$$S_j = P_j + jQ_j$$

Fig. 1. Single line diagram of two bus system

The quadratic equation for the voltage at receiving end is given by:

 $|V_j|^4 + |V_j|^2 \left[ 2 \left( P_j R_{ij} + Q_j X_{ij} \right) - \left| V_i \right|^2 \right] + R_{ij}^2 P_j^2 + X_{ij}^2 Q_{ij}^2 + X_{ij}^2 P_j^2 + R_{ij}^2 Q_j^2 = 0$ 

The solution for  $V_j$  is,

$$|V_j| = \pm \sqrt{\frac{|V_i|^2 - 2(P_j R_{ij} + Q_j X_{ij}) \pm \sqrt{|V_i|^2 - 2(P_j R_{ij} + Q_j X_{ij}) - 4(R_{ij}^2 + X_{ij}^2)(P_j^2 + Q_j^2)}}{2}}$$

which may be simplified by setting,

$$a = |V_i|^2 - 2(P_j R_{ij} + Q_j X_{ij})$$
 and  $b = (R_{ij}^2 + X_{ij}^2)(P_j^2 + Q_j^2)$ 

and so,

$$|V_j| = \pm \sqrt{\frac{a^2 \pm \sqrt{a^2 - 4b}}{2}}$$

Thus there are four solutions for V<sub>j</sub>: two positive, and two negative. Since V<sub>j</sub> must be non-negative, the two negative solution are not true. To derive a positive feasible solution for a real system, the expression under the square root must be positive, i.e.  $a \pm \sqrt{a^2 - 4b} \ge 0$  (or  $a^2 - 4b \ge 0$ ). Since *b* is greater than zero, *a* must be greater than for the condition to satisfied, so

$$0 < \frac{2\sqrt{b}}{a} \le 1$$

By substituting a and b we define new voltage stability index as,

$$LSZ = 2 \frac{\sqrt{(R_{ij}^2 + X_{ij}^2)(P_j^2 + Q_j^2)}}{|V_i|^2 - 2(P_j R_{ij} + Q_j X_{ij})}$$

Simplifying this expression gives,

$$LSZ = 2 \frac{|Z||S|}{|V_i|^2 - 2|Z|(P_j \cos\theta + Q_j \sin\theta)} \le 1$$

The transmission line with largest value of *LSZ* is taken as the weakest line and must receive special care to maintain voltage stability within a certain limit.

## 4 Methodology

The voltage stability analysis was conducted on IEEE 14 bus reliability test system [19] to permit an objective comparison with other indices. This system has 5 generator busbars, 9 load busbars, and 20 interconnected systems. Fig.2 shows the single line diagram for the test system. The reactive power loading is increased gradually at the chosen bus each time, until near to voltage collapse, keeping the loads on the other buses constant. From the results, the proposed index *LSZ* was calculated at each line for every load change.



Fig. 2. Single line diagram for IEEE 14 bus test system [21]

#### 5 Results and Discussions

Fig.3 (a, b, c) shows the *LSZ* is plotted versus reactive power load change at buses (10, 12, and 14). It is clear from the graphs that the line with highest LSZ is ranked as the most sensitive line. Therefore, the line connecting bus 13 to bus 14 is the most critical line, and bus 14 is ranked as the most sensitive bus (weakest bus) because it has the minimum loadability compared with the other lines. Fig.4 (a, b, c) shows FVSI, LQP, and *LSZ* are plotted versus reactive power load change together with the



Fig. 3. Lsz vs reactive power loading at buses (10, 12, 10) of 14 bus IEEE test system



Fig. 4. (a,b,c) V, L<sub>sz</sub>, FVSI, and LQP vs reactive power loading at buses (10, 12, and 14)

load voltage for selected load buses (10, 12, 14). From the graph it appears that bus 14 is the most sensitive as it reaches the stability limit for minimum loadability.

## 6 Conclusions

A study of the differences between voltage stability indices used in voltage stability analysis has been presented. These differences are recommended as important issues in the design of reactive power sources such as FACTS controllers to critical buses. A new index related to the voltage drop in the critical bus was proposed and investigated. Computed results were compared to the other previous published data [22, 23] and shown a satisfactory agreement.

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