

# A Dynamic-Blind Estimation of the $\lambda$ -MRC Combiner's Weight for a Decode-and-Forward Based Receiver

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**Abstract.** This paper develops a new dynamic-blind estimation of the combiner's weight  $\lambda$  suggested for decode-and-forward cooperative based receivers and provides an empirical form for it. The estimation is performed by applying a monotonically decreasing formula on the incoming bit stream. The performance evaluation is investigated using the analytical probability of error ( $P_e$ ) closed-form of the decode-and-forward cooperative transmissions. An adjacency is illustrated between the performances using the values of  $\lambda$  obtained by the proposed estimation against the use of optimum values of  $\lambda$  found by iterative search. A practical implementation of the decode-and-forward cooperative algorithm is also considered and the performance analysis is studied for different channel assignments. It is demonstrated that the proposed dynamic-blind estimation of  $\lambda$  provides in an overall performance on the verge of that of using optimum values of the combiner's weight  $\lambda$ .

**Keywords:** Blind combiner, Dynamic-Blind estimation, Decode-and-Forward algorithm,  $\lambda$ -MRC.

## 1 Introduction

The early papers introducing the concept of decode-and-forward cooperative diversity for multiple access communication systems [1-6] were in concurrence with one basic combiner model; the sub-optimum  $\lambda$ -MRC. The combiner's weight  $\lambda$ ,  $\lambda \in [0,1]$ , is defined as a measure of the base-station's (BS) confidence in the cooperative message delivered by the partner. The weight has the potential to control the amount of cooperation required to provide higher quality of service. The value of  $\lambda$  is a function of the current channel conditions and the inter-user channel probability of error  $P_{e12}$ , a quantity which the BS may or may not have access to. Considering a practical implementation of this cooperative framework, it is difficult to find an optimum expression for  $\lambda$  ( $\lambda_{\text{optimum}}$ ) as a function of  $P_{e12}$  due to the non-convex property of the overall probability of error  $P_e$  expression. Consequently three solutions arise; finding a sub-optimum expression ( $\lambda_{\text{sub-optimum}}$ ), finding an optimized solution using a signal processing tool, and finding an efficient estimation of  $\lambda$ . The two former solutions are

expected to result in more residual errors, either due to imperfections in the feedback from the users concerning the value of  $P_{e12}$  or estimation errors in the values of the Rayleigh fading parameters, compared to an adaptive estimation of  $\lambda$ .

This paper reaches for a dynamic-blind estimation of  $\lambda$  that provides a performance at the verge of the minimum probability of error ( $P_e$ ) for a specific transmission conditions. The proposed estimation procedure begins by a bit-by-bit calculation of the difference ( $\delta$ ) between the Bs's combined decision statistics of the user's received signal during the odd intervals and during even intervals respectively. It is rational that this difference is a measure of the deviation between the user's and partner's information either due to channel degradation or estimation imperfections of the partner's cooperative bit. Next, a monotonically decreasing expression is applied to the difference ( $\delta$ ) to obtain the value of  $\lambda$  that is supposed to provide a minimized  $P_e$  for the currently received bit. The analytical closed-form of the probability of error ( $P_e$ ) of a decode-and-forward cooperative transmission introduced in [1-6] is used to evaluate the performance using the proposed  $\lambda$  values against the minimum  $P_e$  curves. These reference curves are found by an iterative search, for a specific transmission conditions, and refer to the optimum value of  $\lambda$  in each case.

Then, a practical implementation of a decode-and-forward DS-CDMA cooperative algorithm based on the use of the complete complementary (CC) codes set is considered [11]. The highlighted cooperative transmission framework and the sub-optimum  $\lambda$ -MRC are used as a reference and its performance is evaluated against the performance using the proposed dynamic-blind  $\lambda$  values under several channel models and similar performances are observed. The paper is organized as follows; section 2 introduces a full description the  $\lambda$ -MRC applied for the decode-and-forward cooperative algorithms, section 3 discuss the proposed dynamic-blind estimation of the combiner's weight  $\lambda$ , section 4 investigates an actual implementation of a DS-CDMA decode-and-forward cooperative transmission, section 5 display the simulation results, and section 6 is the paper conclusions.

## 2 The $\lambda$ -MRC Combiner; An Overview

In a decode-and-forward multiple-access cooperative transmission procedure, the BS receives the users' data over 2 intervals [1-6]. The first interval's data  $Y^{odd}$  is sent by the user itself while the second interval's data  $Y^{even}$  is an additive combination of the user's data sent by the partner and himself. Where  $Y^{odd}$  is given by (1) and similarly for  $Y^{even}$  in (2).

$$Y^{odd} = K_{10} X_1^{odd} + K_{20} X_2^{odd} + Z^{odd} \tag{1}$$

$$Y^{even} = K_{10} X_1^{even} + K_{20} X_2^{even} + Z^{even} \tag{2}$$

Where  $X_k$  is the  $k^{th}$  user transmitted signal,  $X_k^{odd} = a_{k,u} b_k C_k$ ,  $X_k^{even} = a_{k,u} b_k C_k + a_{k,u+1} \hat{b}_{k+1} C_{k+1}$ ,  $a_{k,u}$  denotes the power factor of the  $k^{th}$  source during the transmitting interval  $u$ ,  $C_k$  is an index of the spreading code,  $b$  is the transmitted bit,  $\hat{b}$  is an estimated bit,  $K_{ij}$  are the Rayleigh fading coefficients from the source  $i$  to

the destination  $j$  having mean  $\xi_{ij}^2$  and finally  $Z$  the white zero-mean Gaussian noise with spectral height  $N_i/2$ .

It was shown that the optimal detector of user 1 based on these received signals is given by (3).

$$(1 - P_{e12})A^{-1}e^{v_1^T y} + P_{e12}Ae^{v_2^T y} \underset{-1}{\overset{1}{>}} (1 - P_{e12})A^{-1}e^{-v_1^T y} + P_{e12}Ae^{-v_2^T y} \quad (3)$$

Where  $y = [y_{odd} \quad y_{even}]^T \frac{\sqrt{N_c}}{\sigma_0}$ ,  $v_1 = [K_{10}a_{11} \quad (K_{10}a_{12}) + (K_{20}a_{22})]^T \frac{\sqrt{N_c}}{\sigma_0}$ ,  $v_2 = [K_{10}a_{11} \quad (K_{10}a_{12}) - (K_{20}a_{22})]^T \frac{\sqrt{N_c}}{\sigma_0}$ , and  $A = \exp(K_{10}K_{20}a_{11}a_{22}N_c/\sigma_0^2)$ .

Obviously, this detector form was found complex and does not have a closed form expression and a  $\lambda$ -MRC sub-optimum detector was suggested instead as described by (4).

$$\hat{b}_1 = \text{sign}([\gamma_1 \quad \lambda(\gamma_2 + \gamma_3)]y) \quad (4)$$

Where  $\gamma_1 = (K_{10}a_{11})\frac{\sqrt{N_c}}{\sigma_0}$ ,  $\gamma_2 = (K_{10}a_{12})\frac{\sqrt{N_c}}{\sigma_0}$ ,  $\gamma_3 = (K_{20}a_{22})\frac{\sqrt{N_c}}{\sigma_0}$ , and  $\lambda \in [0,1]$  is a measure of the BS's confidence in the bits estimated by the partner. The probability of  $P_e$  for the  $\lambda$ -MRC was shown in [1] as given by (5).

$$P_e = (1 - P_{e12})Q\left(\frac{v_1^T v_\lambda}{\sqrt{v_\lambda^T v_\lambda}}\right) + P_{e12}Q\left(\frac{v_2^T v_\lambda}{\sqrt{v_\lambda^T v_\lambda}}\right) \quad (5)$$

Where  $v_\lambda = [\gamma_1 \quad \lambda(\gamma_2 + \gamma_3)]^T$ ,  $v_1 = [K_{10}a_{11} \quad (K_{10}a_{12}) + (K_{20}a_{22})]^T \frac{\sqrt{N_c}}{\sigma_0}$ , and  $v_2 = [K_{10}a_{11} \quad (K_{10}a_{12}) - (K_{20}a_{22})]^T \frac{\sqrt{N_c}}{\sigma_0}$ .

The value of the  $\lambda$  weight is directly related to the inter-user channel error ( $P_{e12}$ ) between the user and the partner. For a perfect inter-user channel  $P_{e12}=0$ , the optimal detector in (3) collapses to the detector in (5) with  $\lambda=1$ . As  $P_{e12}$  increases, the inter user channel becomes unreliable, the value of the best  $\lambda$  decreases towards zero. Accordingly the receiver model suggested in (5) is considered a modified MRC combiner, the branch with the partner's uncertain bit is weighed less than the branch with the bits coming directly from the desired user.

### 3 The Proposed Blind $\lambda$ -Combiner

This paper reaches for an important development on the decode-and-forward receiver's combiner, the  $\lambda$ -MRC, that is; finding a dynamic-blind estimation to calculate the value of the combiner's weight  $\lambda$ . The combiner's weight  $\lambda$  is defined to be a measure of the BS's confidence in the cooperative bit delivered by the partner compared to that delivered by the user itself. It is mandatory to find an expression that

calculates this value to reduce the implementation complexity for any practical implementation of the decode-and-forward cooperative transmission.

The proposed estimate of  $\lambda$  relies on calculating the bit-by-bit difference ( $\delta$ ) between the Bs's combined statistics of the odd duration information,  $y_{odd}$ , and that of the even duration,  $y_{even}$ . Where  $y_{odd}$  is expressed by (6),  $y_{even}$  by (7) and the difference ( $\delta$ ) by (8).

$$y_{odd} = K_{10}a_{11}b + n_{odd} \tag{6}$$

$$y_{even} = K_{10}a_{12}b + K_{20}a_{22}\hat{b} + n_{even} \tag{7}$$

$$\begin{aligned} \delta &= y_{odd} - y_{even} \\ &= K_{10}a_{11}b - (K_{10}a_{12}b + K_{20}a_{22}\hat{b}) + n \end{aligned} \tag{8}$$

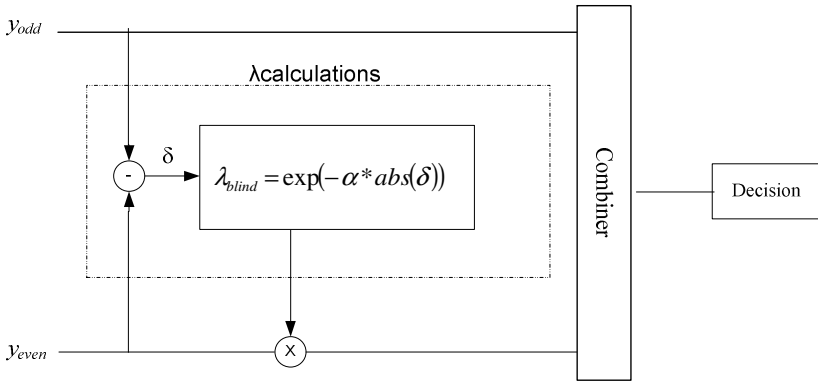
The difference ( $\delta$ ) is a normally distributed random variable with zero mean and variance equals to  $\sigma_{\delta}^2 = \sigma_{y_{odd}}^2 + \sigma_{y_{even}}^2$ . It is a measure of the deviation between the user's and partner's information delivered to the base station for a specific bit. This deviation results from the channel degradation or the partner's bit estimate error that arises from the quality of the inter-user channel error ( $P_{e12}$ ). Obviously, the value of  $\delta$  reduces to a definition very close to that of  $\lambda$ . Considering the definition of  $\lambda$  and its range, we suggest an empirical form that relates the weight  $\lambda$  and the difference  $\delta$  in a monotonically decreasing expression with parameter  $\alpha$  as described in (9) and by the block diagram in Fig.1.a.

$$\lambda_{blind} = \exp(-\alpha * abs(\delta)). \tag{9}$$

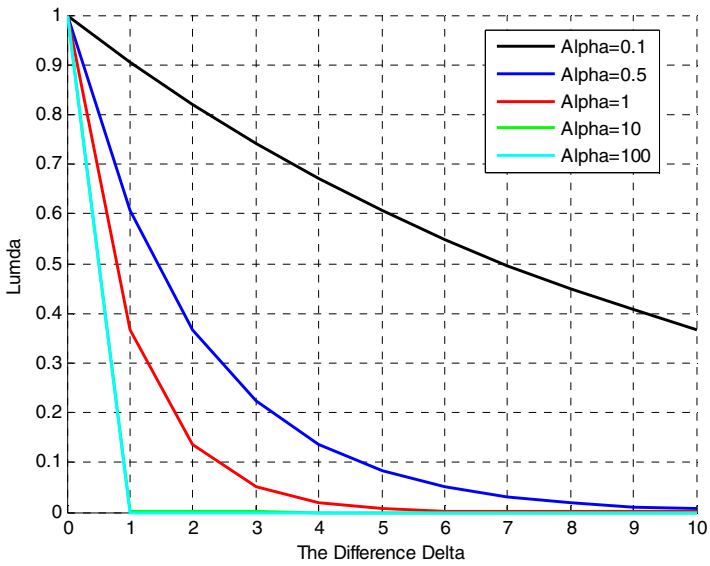
The relation between the two variables for different values of  $\alpha$  is shown in Fig.1.b. An iterative search is done to find the optimum value of  $\alpha$  that describes the relation between  $\lambda$  and  $\delta$  and results in a value of  $\lambda$  that minimizes the  $P_e$  expression in (5). Each time the resulting performance is evaluated against the minimum  $P_e$  for specific simulation conditions found by an iterative search for a range of  $\lambda$ . Using a wide range of  $\alpha$  value from  $10^{-4}$  to  $10^4$ , we found that  $\alpha=0.1$  is the target value as shown in Fig.2.a. Therefore, the empirical expression describing the relation between the combiner's weight  $\lambda$  and  $\delta$  reduces to (10).

$$\lambda_{blind} = \exp(-0.1 * abs(\delta)). \tag{10}$$

We refer to the proposed calculation of  $\lambda$  as the dynamic-blind as this value is calculated for every received bit and utilizes the incoming bit stream without any prior knowledge of the transmission environment. Using the empirical expression of  $\lambda$ , the probability of error  $P_e$  performance against the reference curves is exploited for different values of  $P_{e12}$  in Fig.2.b. It is observed that the curves using the proposed estimation of  $\lambda$  are on the verge of the reference curves especially for low values of  $P_{e12}$ . The simulations considered a range of  $\gamma_1^2$  from -3 to 10 db,  $\gamma_2=1.2$ , and  $\gamma_3=0.8$ .

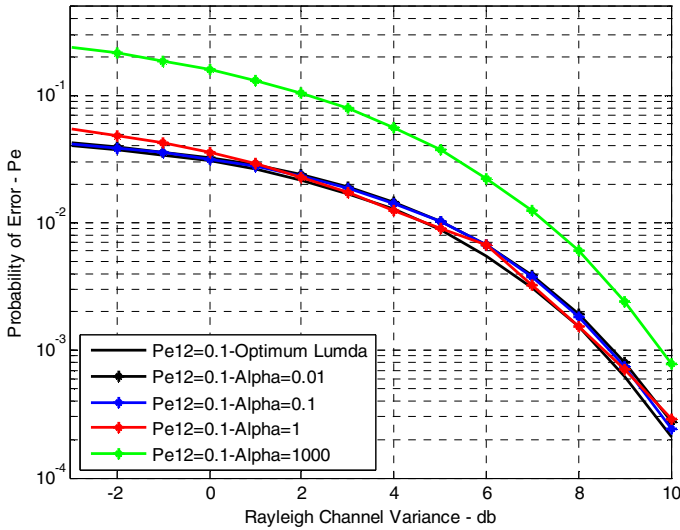


(a)

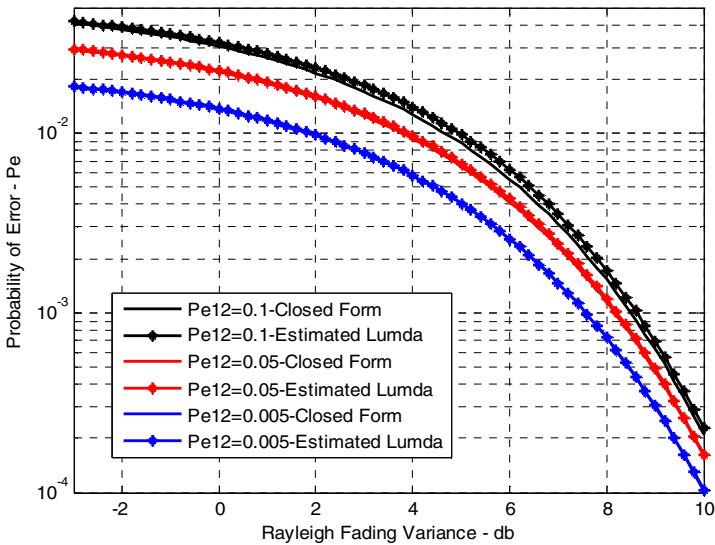


(b)

**Fig. 1.** The block diagram representing the empirical expression of  $\lambda$  in (a) and the relation between the weight  $\lambda$  and the difference  $\delta$  for different values of  $\alpha$  in (b)



(a)



(b)

**Fig. 2.** The analytical closed-form of the probability of error  $P_e$  performance for different values of  $\alpha$  under the effect of  $P_{e12}=0.1$  versus  $\gamma_1^2$  range from -3 to 10 db in (a) and the analytical closed-form of the probability of error  $P_e$  performance using the  $\lambda$ -MRC against the use of the dynamic-blind  $\lambda$  estimation for different values of  $P_{e12}= 0.5, 0.1, 0.05, 0.005,$  and  $0.0001$  versus  $\gamma_1^2$  range from -3 to 10 db in (b)

## 4 An Actual Implementation of a Decode-and-Forward Cooperative Algorithm

In this section, an actual implementation of a decode-and-forward cooperative algorithm is considered in order to evaluate the efficiency of the proposed  $\lambda$  calculation under various channel conditions. The highlighted cooperative algorithm is presented in [11]; it considers a decode-and-forward multiple-access cooperation transmission framework using the complete complementary (CC) code sets. These codes have particular correlation properties [7-9]. Along each set, the autocorrelation sum of its codes is impulsive. Besides, the codes in different sets are orthogonal and particularly the cross correlation sum of these codes along the set size vanishes for all shifts.

The algorithm supports a number of users equals to the number of code sets and performs the transmission over several parallel channels following the number of codes per set. The transmission procedure goes as follows;  $K$  users are assigned a CC code set of  $M$  codes, each user spread his information using each of the  $M$  codes separately resulting in  $M$  different signals sent through  $M$  parallel channels following the multi-band DS-CDMA proposed in [10]. Then the users'  $M$  signals are transmitted through different frequency bands and each band carries the summation of all users' respective CDMA signals.

The cooperative transmission is achieved through two symbol duration; the odd and even durations. The odd symbol durations are used to send the user own bit, using  $M$  carriers, to the base station and partners. Meanwhile, partners are assigned to detect and estimate the received information. Then during the even symbol durations the users cooperate and transmits the sum of its own bit (previously sent in the odd duration) and the partner's estimated bit, each spread by the appropriate spreading code. Equation (11) observes user  $k$  signal in the proposed framework during two intervals. While the transmitting signals during the odd and even intervals are observed in (12) and (13) respectively.

$$\begin{aligned} X_1^k &= b_j^k C_1^k & b_j^k C_1^k + \hat{b}_j^p C_1^p \\ X_2^k &= b_j^k C_2^k & b_j^k C_2^k + \hat{b}_j^p C_2^p \end{aligned} \tag{11}$$

$$T_{odd}^k(t) = \sum_{n=-\infty}^{\infty} b_j^k C_{m,n}^k h(t - nT_c - \tau^k) \tag{12}$$

$$T_{even}^k(t) = \sum_{n=-\infty}^{\infty} (b_j^k C_{m,n}^k h(t - nT_c - \tau^k) + \hat{b}_j^p C_{m,n}^p h(t - nT_c - \tau^p)) \tag{13}$$

Where  $h(t)$  is the impulse response of the chip wave-shaping filter,  $T_c$  the chip duration,  $j$  is the data index,  $p$  is the partner index,  $b_j^k$  is the user  $k$ 's bit, and  $\hat{b}_j^p$  is the partner estimated bit,  $m=1,2,\dots,M$  is the band and code index and  $C_{m,n}$  denotes the code sequence  $m$  of length  $n$  from the set  $C$ .

The proposed receiver follows the generalized model of a decode-and-forward cooperative receiver as in [1]; the base station receiver is composed of a matched filter and a  $\lambda$ -MRC combiner. The considered implementation uses a matched filter is formed of  $M$  parallel branches to match the user's  $M$  spreading codes. Equation (14) and (15) describe the output of the correlators of user  $k$  signal in the odd and even symbol respectively. Finally, the  $\lambda$ -MRC combines the user information extracted during both odd and even intervals. Note that, parameter  $n'$  is the same as  $n$  after passing through the band pass filter.

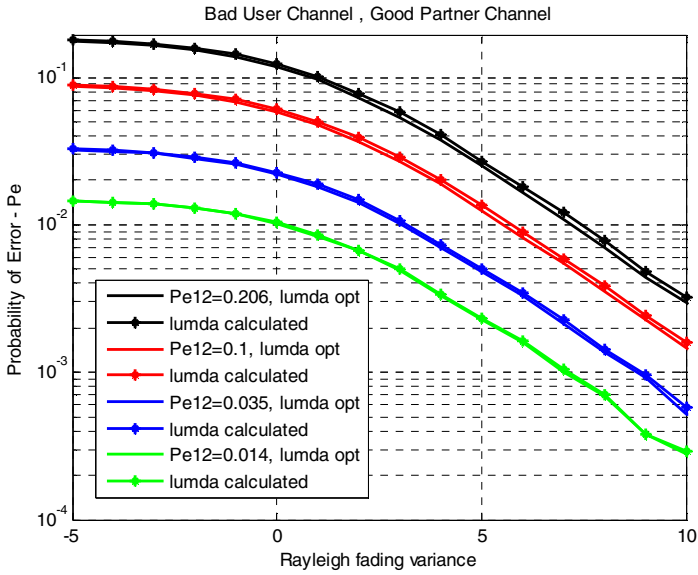
$$R_{odd}^k(t) = \sqrt{E_c} \sum_{n=-\infty}^{\infty} b_j^k \left( \sum_{n'=0}^{N-1} \left( \sum_{m=1}^M C_{m,n'}^k \cdot C_{m,n'}^k \right) \right) x(t - nMT_c) \quad (14)$$

$$R_{even}^k(t) = \sqrt{E_c} \sum_{n=-\infty}^{\infty} \left( b_j^k \left( \sum_{n'=0}^{N-1} \left( \sum_{m=1}^M C_{m,n'}^k \cdot C_{m,n'}^k \right) \right) + \hat{b}_j^p \left( \sum_{n'=0}^{N-1} \left( \sum_{m=1}^M C_{m,n'}^k \cdot C_{m,n'}^p \right) \right) \right) x(t - nMT_c) \quad (15)$$

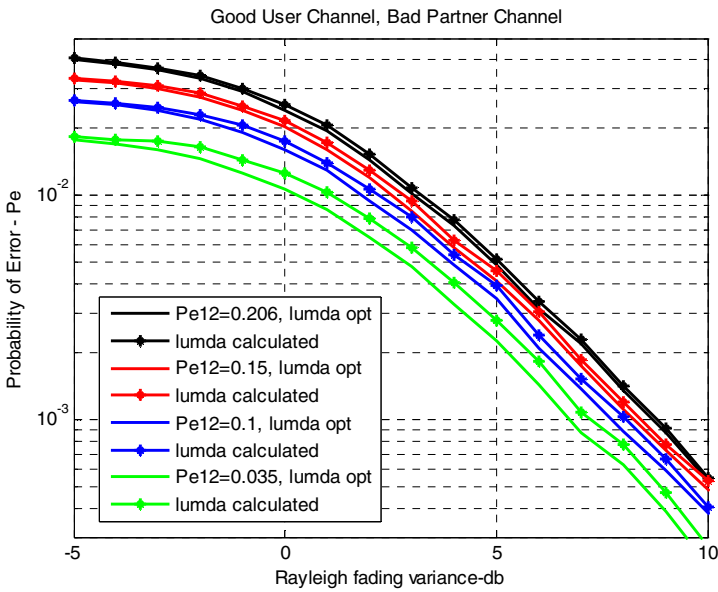
## 5 The Simulation Results

The  $P_e$  performance is analyzed for different values of inter-user channel, under the effect of a Rayleigh flat fading channel and a unity power AWGN. The Rayleigh channel variance is ranging from -5 to 10 db during the odd symbol duration while during the even duration two cases are presented the bad user-good partner and the good user-bad partner cases. For the bad user-good partner case, the Rayleigh fading coefficient is 0.4 for the user and 1 for the partner. The performance using the proposed  $\lambda$  calculations for different values of  $P_{e12}$  considering these simulations conditions is shown to be on the verge of the minimum  $P_e$  performance curves as illustrated in Fig.3.a. While for the good user-bad partner case, the Rayleigh fading coefficient is 0.8 for the user and 0.3 for the partner. The results show to be close to the minimum curves for high values of  $P_{e12}$  and a noticeable deviation arises for low values of  $P_{e12}$  as observed in Fig.3.b. These slight differences occur due to the in-significant differences between  $y_{odd}$  and  $y_{even}$  at low  $P_{e12}$  compared to the fading effect. Generally, the proposed dynamic-blind estimation of  $\lambda$  results in efficient performances under different channel assignments compared to the reference performance curves and illustrates to be simple and reliable.





(a)



(b)

**Fig. 3.** The performance analysis of the complete complementary codes based CDMA cooperative transmission using the  $\lambda$ -MRC combiner using  $\lambda$  selected by an iterative search against the proposed dynamic-blind  $\lambda$  calculation. Under different values of inter-user channel  $P_{e12}$  equals 0.206, 0.15, 0.1, 0.085, 0.035, and 0.014 versus the Rayleigh flat fading channel variance for bad user-good partner in (a) and good user- bad partner in(b)

## 6 Conclusions

In this paper, a dynamic-blind calculation of the combiner's weight  $\lambda$  and its empirical expression were introduced. Using the closed-form expression, the minimum  $P_e$  under specific channel conditions was compared to the performance using the dynamic-blind values of  $\lambda$  as the combiner's weight. The proposed values of  $\lambda$  showed an efficient performance against the reference values. Then, a practical implementation of a decode-and-forward cooperative multiple access transmission was discussed and the performance evaluation was provided under different channel assignments. The proposed dynamic-blind estimation of  $\lambda$  showed a performance on the verge of the minimum  $P_e$  for all the studied transmission conditions, with slight differences in some cases.

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