Prime Slaughter: Playful Prime Numbers

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Abstract. Starting from the difficulty of creating playful representation of domain-specific abstract concepts, this study discusses the design of Prime Slaughter, a computer game aimed at facilitating individual sense-making of abstract mathematical concepts. Specifically the game proposes a transposition of primality and factorization into playful interactions, addressed to primary and early secondary school children. Taking into account individual needs expressed by children regarding play, during a participatory design processes aimed at enhancing learning in museums, Prime Slaughter allows for multiple forms of play and their integration. A simple working prototype has already been developed; it will be tested and re-designed through participatory workshops, involving a group of children in our target group.

Keywords: Non-formal learning, computer games, factorization, play.

1 Introduction

This paper explores the transposition of abstract mathematical concepts into playful visual interaction, by means of computer games. Factorization was chosen as it is representative of algorithmic procedures that are challenging for the learner, yet needed in many fields, being related to fractions (when adding or simplifying them) and to prime numbers. Testing for primality and factorization are also central elements of computer science curricula, where they are considered classical and formative examples of numerical algorithms. Before designing our game, named Prime Slaughter (PS), a survey of math-related games was conducted; the results revealed that these applications are mostly computer-augmented exercises or (virtual) manipulative environments. Genuine math games are rare, especially if one is interested in those related to theory and abstract concepts. PS is therefore an attempt at designing such a game.

The theoretical framework adopted for PS is based on studies about the role of goal-directed activities in learning. Moreover, considering findings from a 1 year participatory design study conducted by one of the authors, about facilitating playful learning in museums, and the lack of playful math-related games, PS has been conceived as an actual computer game, in which the combination of different forms of play are explored and mathematical notions are transposed into goal-directed activities within the gameplay.

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The following sections of the paper proposes a survey of math-related games in section 2, section 3 instead presents findings from the museum project and the theoretical framework. Section 4 discusses the design of the game, and section 5 presents conclusions and future work.

2 Math-Related Games

The first step toward the creation of PS was to conduct a review of existing games about math learning, specifically focusing on theory coverage and problem solving techniques. Three kinds of games were identified: computer-augmented exercises, exploratory/manipulative environments, and finally very few genuine games.

The category of computer-augmented exercises is the most represented and it includes game such as: *BBC bitesize* and the *Toon University: Prime Factoring*. *BBC bitesize* is a collection of computer-augmented exercises, requiring players to give the right answer. An interesting aspect of *BBC bitesize* is that it provides a concrete context for abstract mathematical operations: for instance in the KS1 section, in the division game, after an answer has been given an animation shows a division as partitioning diamonds among a series of wagons. When a wrong answer is given, some of the wagons are left empty or some diamonds fall on the ground after all wagons have being used. The interface is targeted at primary school children, it has nice and colorful graphics supported by sound effects and voice recordings, which are used to warn about mistakes and about the functions of the buttons. Similarly the *Toon University: Prime Factoring* game requires the players to answer to mathematical questions, by shooting the right answer with a cannon. The goal of the game is to identify the right answer among 3 choices, after having performed mental calculations. This game has a quite low graphic quality and does not offer representations of the steps needed to find the right answer, in this sense is very different from our vision, but represents a typical example of free online math games.

Another approach is to offer a virtual environment, in which the players are supposed to learn by exploring mathematical concepts. An example is provided by *ABCya! Fraction Tiles,* which allows to explore how parts make up a whole, interacting through a desktop-like user interface and a palette of fraction tiles, from 1 to 1/12. The tiles can be taken from the palette and arranged as construction blocks. The width of each tile is proportional to the size of the fraction it represents, so for example two ¼ fractions one aside of the other are as long as a single ½ fraction. The goal here is to let the children explore fractions, freely or under supervision of teachers that could provide challenges or tasks.

An different, and interesting case is provided by the series of *Pixeline* games, which are technically well designed and engaging. These games seem to combine computer-augmented exercises, taken from school curricula, with an exploratory environment. They offer a good context to make exercises more concrete and fun, which are solved by the players through the avatar Pixeline. However, the goal is still solving exercises, theory and problem solving procedures are not directly represented.

The category of genuine games, which better fits our vision, is represented mainly by *number ninjas* (by Armor Games). There the player is a number, who can move around in a side-scrolling world and has the goal to collect shurikens (ninja's throwing stars) and kill other numbers. The player's character is a natural number, initially 1; each shuriken corresponds to an operation, so by killing number 3 with a *plus* shuriken the character becomes number 4 (and 3 dies). In advanced levels there might be multiple ways to finish, by adding, subtracting or multiplying the character's initial value with the available numbers disseminated in the level. The general rule is that if the character becomes more than 9 or less than 0, it will die. This rule contributes to make the solution of the levels more challenging. Decisions involving application of operations to numbers are essential parts of the game, which makes it very interesting and inspiring for us.

In conclusion, the result of our survey suggests that math is typically not effectively transposed into gameplay. And even if some games involve to multiplication and division, we could find not any game that tries to cover more abstract operations, comparable to prime factorization or primality testing.

3 Different Forms of Play

The PS concept is based on findings and reflections from another ongoing research project (conducted by one of the authors [2]) about playful learning and the transposition of abstract notions into playful interactions, specifically regarding museum learning practice and children around 10 years old. A one year Participatory Design (PD) process has been conducted in cooperation with an after-school institution, where a group of 25 children was involved in co-designing a playful learning game about urban development in history [3]. Insights collected through this process, and the related theoretical framework, provided inspiration to try the same approach in factorization and primality. An analogy was identified between the two projects, as in both games the children are supposed to experience theoretical notions and problem solving activities, playing as characters within the provided narrative.

Fig. 1. Different play styles; the mixed group is facing the camera

The theoretical framework adopted for the museum project is based on Rogoff's theory of apprenticeship [4]. According to Rogoff children learn by engaging in goal-directed activities within informal contexts, usually supported by adults when reaching the boundary of their current knowledge [4]. In this study we intend to combine the use of goal-directed activities within play, which is supposed to mediate between the children and the learning content, also in the case they would like to explore the learning domain by themselves. Play is then intended, as in [5], a self-driven activity, which could take different forms according to individual interests.

Observations conducted during the PD sessions have concretely showed that children may express different forms of play, even while playing at the same game and within the same group of players. Some children adopted a form of *military* board games play, usually related to games like chess, Stratego by Milton Bradley Board Games and Monopoly by Hasbro. They placed their tangibles on the board and challenged each other, as they were competing to win control over the other player's land. Other children engaged in a form of *designerly* play, suitable for construction bricks or games like SimCity [1]. They acted as urban planners, arranging their settlement and introducing new buildings or other features (humans, animals, and plants). Some other children combined the two forms of play, they generally started by planning their settlements, in groups or individually, and then engaged in a military play with other children. An interesting case was provided by a group of two girls and two boys: they all started by planning their settlement, but the girls took the designerly phase more seriously and used a lot of time for it. The boys instead placed the tangibles and engaged as soon as possible in the military play form. They had to wait for the girls to be ready, so they started playing by themselves, and after the girls had completed their settlement, they played together in the military way. At the same time other subgroups or individuals played on a parallel basis, as urban planners or fighting landowners, almost as they were in another room (see Fig. 1).

These observations provided a valuable grounding to determine requirements for the museum project and also inspired us to create Prime Slaughter for a different domain. Combining these data and the theoretical framework provided by Rogoff, it was decided to explore a scenario, in which primality and factorization are transposed into playful goal-directed activities embedded in the game. The players are supposed to explore individually theory and problem solving techniques they learned in school, in a tangible and playful way. Then if needed, they may ask support from teachers or parents. In this sense both learning, play, and adults participation are intended as self-driven activities.

4 The Design of Prime Slaughter

Abstractions are commonly used in subjects such as math and computer science, as they allow to effectively represent complex meanings. However, such abstractions are difficult to grasp for novices, therefore, direct manipulation has been introduced for instance in the field of algorithm animation, to allow learners to intuitively experience the semantics of abstract concepts, interacting with their visual representations [7].

In our view PS should follow a similar approach, allowing players to learn abstract notions and procedures related to factorization, through direct manipulation, experiencing the meaning of factorization within a computer game framework.

Moreover, PS should be playable as a real game and not as computer-augmented exercises. By playable we mean also that the children should perceive PS as a game, from their individual perspective of what play is. Therefore, based on the data from the museum project, it was decided to start by supporting both a designerly, generative form of play and a military, dramatic form of play, and possibly emergent combinations of these forms. Furthermore, in our view a playable game should be fun and engaging also for players who do not have solid knowledge of the learning content, in order to motivate them in playing again and learn more.

Starting to design PS we analyzed the typical elements and mechanics of 2D action adventure games, such as *The Legend of Zelda* by *Nintendo,* and *generative games* (or simulation-like games in which the player actively manages or builds part of the simulation), such as *SimCity* by *Maxis,* to gather requirements and support respectively military and designerly forms of play. Then we analyzed the actions involved in prime factorization, such as division, multiplication, and possible visualizations of abstract concepts like the primality or divisibility of a natural number. Finally we defined mappings between math and game dynamics, to create an engaging gameplay. According to our analysis classic 2D action-adventure games have typically a main character, the hero, who is supposed to kill enemies, often represented as monsters, and to explore the world to find tricks or hidden artifacts, which allow to defeat more effectively different monsters. A narrative element, increasing difficulty of levels, and rewards enabling the player to become more effective, provide short terms goals and motivation to continue playing, which may fit well within goal-directed activities. The hero is usually characterize by energy level, experience points, magic or special abilities, and items collected in the game. Long term progress requires strategy in managing experience points or bonuses, and a deeper understanding of the rules of the game, of what is possible to do, including unexpected side-effects of certain actions or artifacts. Similar features are present also in generative games, however, the role of the player is significantly different. The player is often an external agent, who is supposed to act on the world, adding or manipulating some of its features, for instance city infrastructures in SimCity. The goal is to create certain configurations and explore the implications as the simulation unfolds. To keep playing the player simply needs to fulfill some minimal requirements, otherwise she will run out of resources and lose. Finally all these games are enriched by nice colorful graphics and compelling audio effects, providing feedback on the players' performance, and engaging narrative elements disseminated throughout the game.

Factorization requires to see a natural number as a unique collection of its prime factors, in whatever order: for instance 12 is also $2 \times 2 \times 3$. Regarding actions, a simple algorithm to find the prime factors of an integer number N, is to start from 2 and try to divide N by ever larger primes. And this, in turn, requires to be able to identify prime numbers. Therefore, integer division and multiplication are central concepts in this domain. Divisibility offers yet another way to perceive a number: a number is related to all its factors. For example, the factors of 12 are {1,2,3,4,6,12}. Therefore, a central goal of PS is to allow players to familiarize with the concepts that a natural number has multiple representations, and some properties may become more are clearly visible by manipulating the number to create a different representation.

Hence, initial design phases of PS consisted of an exploration of possible visualizations of the division and multiplication operations, through the creation of simple concept arts, inspired by the games we analyzed. We decided for a representation of division as splitting an object (a cube of jello in the Fig. 2, on the left) into 2 or 3 parts. This representation seems to fit well with the action of slaying of monsters, a typical element of action-adventure games. It was then decided that the monsters would represent numbers to be sliced by the players, also to make fun of the tension often associated to math learning.

Multiplication is instead depicted (in Fig. 2, at the center) as the process of pruning a young tree, to allow it to grow more branches. A tree initially has a value of 1, and a single branch; the top of this tree will have a single leaf of value 1. Cutting the tree in 3 would generate 3 new branches, each with a leaf of value 3. Cutting the tree again in 2, would result in a single trunk, 3 branches and 6 smaller branches. Interestingly, this form of pruning enables players to freely explore the space of the possible tree shapes. Therefore, we decided to map the duality division/multiplication onto the duality killing/growing.

Summarizing in PS the hero is supposed to collect points by fighting monsters with a sword, representing a prime number by which the monsters can be split (as visible in Fig. 2, on the right). The rule for PS is that a monster of value N (or N-monster) can only be sliced by a sword of value M, where M is a factor of N. The sliced monster divides into many smaller monsters; for instance, a sword of value 2 can slice a 12-monster into 2 monsters of value 6. Trying to slice a monster with the wrong sword (e.g. attacking a 9-monster with a 2-sword) results in damage for the hero, who loses energy points, and ultimately dies.

At the beginning of the game, the player's character has a 2-sword (sword of value 2) and monsters are reduced to prime numbers after a few hits; it would be impossible to kill them without acquiring new proper prime-swords. Even further in the slicing process, every number-monster will eventually be reduced to a 1-monster, impossible to kill, since 1 is only divisible by 1. Monsters reduced to value 1 are short lived, and spontaneously disappear. Taking all these points into consideration, it was decided that when a number-monster is prime (for example an 11-monster) it can be frozen by the hero through magic, and transformed into a sword. This double role of the monsters as enemies and material to construct new swords, is intended to enforce the idea that prime numbers are a single fundamental concept throughout the game, and monsters and swords have ultimately the same nature, i.e. they are primes. Finally, number-monsters have different sizes in the game, in relation to their number of prime factors. Hence, a 12-monster has size 3, while a 17-monster has size 1, and 1-monsters have size 0, the minimal possible. In this way the player can use size as a visual indicator of how close the monster is to being prime (and dead).

In alternative to killing monsters, the player can choose to explore the *Natural Bonsais* level, a forest populated by trees representing each a natural number, and like bonsais can be shaped in various forms. Despite many generative games do not have an avatar for the player, in Natural Bonsais the hero is important as it allows to keep the whole game coherent. Moreover, to allow the players to freely choose how to play, it was decided that the killing and the pruning play styles are inherently situated in the levels: some levels only have monsters, others only bonsais, and monsters cannot move across the levels.

In the current prototype, a bonsai starts as a seed with a natural value associated, for example 12; then it can be pruned by the hero's sword, but only if the value of the sword is a factor of the value of the bonsai, in this case 2 or 3. If a newly born 12-bonsai is pruned by a 3-sword, it grows 3 more branches, on top of the existing tree trunk; each of the new branches will have a leaf of value 12/3, i.e. 4. The process of pruning and growth continues until the leaves are primes, then the leaves are replaced by fruits, each with the same value as the leaf that generated it (apples in the current prototype).

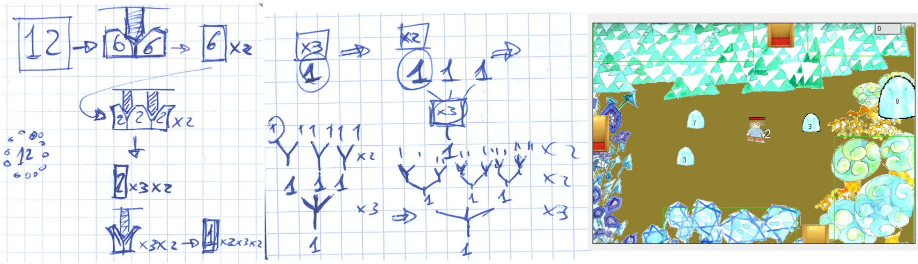

Fig. 2. Splitting numbers could represent division (on the left); pruning a tree can be like multiplication: each new cut creates many new branches (central image). On the right an actual screenshot of the same level in the current prototype.

When the hero collects a number-fruit, his sword is changed accordingly. Points are given during the pruning process, but not when fruits are collected. Old trees can be felled by freezing them, i.e. using the same magic that freezes prime-monsters. When an old tree disappears from the level, a new one is planted at the current position of the hero; this set of rules are meant to suggest players to often fell and re-plant their trees, in order to acquire more points. Moreover, since the bonsai level has no enemies and the player has plenty of time to reflect before acting, making mistakes is punished more severely than in the other levels: for example trying to prune a 12-bonsai with a 5-sword results in reduction of 25% of the hero's current energy. Since the levels of PS contain monsters with values from 1 to 20, the hero needs to collect a number of swords, with all the values of the primes from 1 to 20: 2, 3, 5, 7, 11, 13, 17, 19. The hero in PS can only hold one sword at any given time, so all other swords are kept in an armory. In case the hero dies, he will be re-born in his armory, and all swords collected before dying will still be available. To win the game, the player can collect at least 400 points (by slicing as many monsters as possible) or she can collect all 8 prime-swords in her armory.

The game has goals inspired by the action-adventure genre, i.e. collecting items, increasing the score, or killing monsters. But the actual elements of the game and the actions involved in achieving those goals are related to division, multiplication and factorization. In this sense our game is envisioned as supporting forms of tangential learning, in which learning is not the main focus of players' actions and occurs indirectly [8]. Understanding the built-it mathematical concepts should make you a better player, but you should enjoy playing even with little previous knowledge. Moreover, mathematical concepts should be easily recognizable by player that already studied them in school. Finally, conceptual (mathematical) errors are mapped onto damage points.

5 Conclusion and Future Work

The design of a new game is proposed, aimed at supporting learning of abstract mathematical concepts, specifically factorization. The game is targeted to primary and early secondary school children and is intended to allow them to experience the meaning and dynamics of factorization through direct manipulation. Based on data collected from an ongoing research project about playful learning in museums, and transposition of abstract notions into playful interactions, and from a survey about existing math-related games, it was decided that the game, called Prime Slaughter, should be a fun and playful math game. We map abstract concepts into goal-directed activities within the game framework, and support different forms of play. Taking inspiration from 2D action-adventure games and simulation games, the action of dividing has been transposed into slicing of jello number-monsters with a sword, also representing a natural number, and multiplication has been transposed into pruning and shaping trees in a bonsais forest. The current prototype will be soon tested through a series of participatory workshops, involving children within our target group, to evaluate if the game is really playable as expected and how it actually support learning. Insights collected through the tests will be used to design better version of the game.

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