Evolving the Asymmetry of the Prisoner's Dilemma Game in Adaptive Social Structures

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Abstract. Empirical studies show that most real social networks exhibit both a significant average connectivity and marked heterogeneity. While the first precludes the emergence of cooperation in static networks, it has been recently shown that the latter induces a symmetry breaking of the game, as cooperative acts become dependent on the social context of the individual. Here we show how adaptive networks can give rise to such diversity in social contexts, creating sufficient conditions for cooperation to prevail as a result of a positive assortment of strategies and the emergence of a symmetry breaking of the game. We further show that realistic heterogeneous networks of high average connectivity where cooperation prevails can result from a simple topological dynamics.

Keywords: Evolution of Cooperation, Evolutionary Game Theory, Distributed Prisoner's Dilemma, Dynamic Networks, Evolutionary Dynamics.

1 Introduction

Conventional Evolutionary Game Theory (EGT) predicts that natural selection favors the selfish and strong [1, 2], in contrast with empirical evidence which shows that cooperation is widespread in nature. The issue of cooperation has been traditionally dealt with in EGT making use of the Prisoner's Dilemma (PD), and several mechanisms have been proposed which make cooperation evolutionary viable [3, 4]. Among those, the structure of the network along which individuals interact drastically affects the chances of cooperation. While homogeneous networks (degree-homogeneous) open a small window of opportunity for cooperation to thrive [5-9], heterogeneous networks (degree-heterogeneous) induce a remarkable boost of cooperative behavior [6, 10-13]. This enhancement, however, is limited to social networks exhibiting low average connectivity, whereas data on realistic networks [14-19] shows that values of the average connectivity (z) up to 170 are possible. This requires yet another mechanism to allow the survival of cooperation.

In this work we shall explore this new mechanism, making use of a PD game in which the benefits collected by the participants may be proportional to the costs expended. Besides the conventional scenario in which every cooperator contributes the same amount to each game they participate in, we shall also explore the limit in

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which every cooperator contributes the same overall amount, irrespective of the total number of games they participate in. This is particularly relevant whenever heterogeneous networks are at stake. In such setting the evolution of the interaction network (see below) may break the original symmetric game into an asymmetric game, as the actual game played by each player becomes dependent on their social context [11, 20].

Here, we use a simple adaptive network model [21] that combines strategy evolution with topological evolution [3, 22, 23]. We consider individuals with limited cognitive capacities and investigate the necessary conditions for cooperation to thrive. We will show that network heterogeneity, which emerges as a result of a co-evolution of strategy and topology, is crucial for the appearance and stability of cooperative action. This break of symmetry is naturally induced by a simple dynamics in which individuals revise their contacts uniquely based on their myopic self-interest.

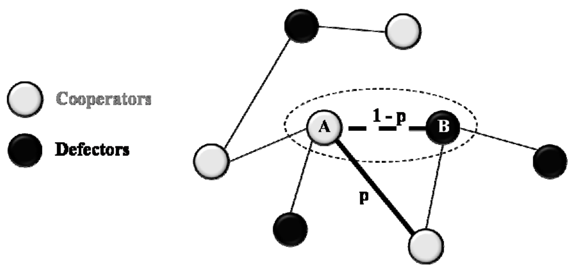

Fig. 1. Readjusting social ties. Cooperators and defectors interact along the links of a network. A (B) is dissatisfied (satisfied) since B (A) is defector (cooperator). Consequently A wants to change the link whereas B does not. The success of the rewiring will depend on the fitness values *Π*(A) and *Π*(B) of A and B, respectively. With probability *p* (see section 3.3) A redirects the link to a random neighbor of B. Otherwise, with a probability 1 - *p*, A will stay linked to B. Other possibilities may occur depending on the strategies of the chosen individuals (see section 3.3).

2 Co-evolution of Strategy and Topology

We consider a population of individuals that can be either cooperators (*C*) or defectors (*D*). They only keep information on their first neighbors, and engage in 2-person PD, where *C*s contribute a cost *c* whereas *D*s do not contribute any cost. The total amount is multiplied by an enhancement factor *F* and then shared equally between the two players. Hence, a player *i* (*i* =1, 2) using strategy s_i ($s_i = 1$ if *C*, 0 if *D*) will get the payoff $P_i = Fc(s_1 + s_2)/2 - cs_i$ [20]. For 1 < F < 2 we get the payoff ranking characteristic of a two-person PD.

In the equation above we have considered that *C*s contribute a fixed cost *c* per game. We can consider a somewhat different scenario in which *C*s now distribute the same cost *c* among all games they play. In this case, if player *i* is a *C*, she/he will pay a cost $c_i = c/k_i$, where k_i is the player's connectivity (number of neighbors). Consequently, the payoff that a player gets from this game is $P_i = F(c_1s_1 + c_2s_2)/2 - c_is_i$.

It is reasonable to make this distinction in the costs paid by *C*s when we allow different individuals to have different number of neighbors (k), a situation that naturally occurs on a heterogeneous graph. For instance, the interaction between two *C*s directly connected but with different ks will result in a higher payoff for the player with the smaller number of links. This will translate in a symmetry breaking of the game.

Following the convention of Ref. [20] we will refer to the game where *C*s contribute a fixed cost per game as the *conventional prisoner's dilemma* (CPD); and to the game where *C*s contribute a fixed cost per individual we call the *distributed prisoner's dilemma* (DPD).

In addition, both *C*s and *D*s are able to decide, on an equal footing, which ties they want to maintain and which they want to change. Given an edge with individuals A and B at the extremes, we say that A (B) is satisfied with that edge if the B (A) is a *C*, being dissatisfied otherwise. If, for instance, A is satisfied, then they will keep the link. If not, then they will compete with B to rewire the link (see Fig. 1 and section 3.3), rewiring being attempted to a random neighbor of B.

This is justifiable on the fact that individuals, who have a limited knowledge of their social environment, will look for new social ties by proxy [24]. In this sense, A is more likely to encounter one of B's friends and become neighbors with them. In addition, selecting a neighbor of an inconvenient partner may turn out to be a good choice, since this partner also tries to establish links with *C*s, making it more likely that the rewiring results in a tie to a *C*.

The fact that in our model *C*s and *D*s interact via social ties they both decide upon establishes a coupling between individual strategy and population structure: the game payoff induces an entangled co-evolution of strategy and structure. The adaptive nature of the social structure explained above introduces a new time scale, τ_a, not necessarily equal to the time scale associated with strategy evolution, τ_e. We define a ratio of time scale $W = \tau_e/\tau_a$, which determines the cooperative state of the population at the end of the evolution.

Indeed, whenever $\tau_e << \tau_a$, that is, $W \approx 0$, we recover the results of [6, 25]. On the other hand, with increasing W, individuals become ever more proficient at adapting their ties. In general, however, one expects the two time scales to be of comparable magnitude in realistic situations (cf. Figs. 2 and 3).

More intuitively, W provides a measure of individuals' responsiveness to adverse ties: large values of W reflect populations in which individuals react promptly to adverse ties, whereas smaller values of W reveal the opposite behavior.

3 Materials and Methods

We place individuals on the nodes of a graph, to a total of N. A total of N_E links represent the social ties between individuals. Graphs will evolve in time as individuals change their ties. The average connectivity $z = 2N_E/N$ is conserved since we do not create nor destroy links. We require that graphs remain connected at all times. To enforce this condition we impose that nodes connected by a single link cannot lose this link. Each simulation starts from a homogeneous random graph in which all nodes have the same number of links randomly connected to other nodes [26].

We also computed the cumulative degree distribution $D(k) = N^{-1}\sum_{i=k}^{N-1} N_i$.

N_i indicates the number of nodes with i links so $D(k)$ gives the probability of finding nodes in the graph with degree greater or equal to k. The maximum value of the connectivity of a graph is k_{max} which provides a simple measure of the heterogeneity of a graph since $D(k) = 0$ for $k > k_{max}$.

Whenever $W > 0$, evolution of strategy and structure proceed together under asynchronous updating. Choice of type of update event depends on W. If we assume, without loss of generality, $\tau_e = 1$, then a strategy update event is chosen with probability $1/(1+W)$, a structural update event being selected otherwise.

A strategy update event is defined in the following way, corresponding to the so-called pairwise comparison rule [27]: One node A is chosen at random and another node B is chosen randomly among A's first neighbors. The individuals A and B interact with all their first neighbors, according to CPD or DPD. As a result, they accumulate the total payoffs $\Pi(\text{A})$ and $\Pi(\text{B})$, respectively. The individual A will imitate the strategy of B with a probability that increases with the payoff difference, which is given by the Fermi distribution function $p = 1/[1+e^{-\beta[\Pi(\text{B})-\Pi(\text{A})]}]$.

The value of $\beta \geq 0$ (which plays the role of an inverse temperature in statistical physics), controls here the intensity of selection [27]: $\beta \to 0$ leads to neutral drift whereas $\beta \to \infty$ leads to the so-called imitation dynamics, often used to model cultural evolution.

*C*s and *D*s interact via the links of a network. Two individuals, A and B, connected by one link, may be satisfied or dissatisfied. In Fig. 1, B is satisfied, whereas A is not, since A (B) is a C (D). Therefore, A wants to change the link whereas B does not. The action taken depends on the fitness $\Pi(\text{A})$ and $\Pi(\text{B})$ of A and B, respectively. With a probability p, defined above in terms of the Fermi distribution, A redirects the link to a random neighbor of B. With probability $1 - p$, A stays linked to B. Whenever both A and B are satisfied, nothing happens. When both A and B are dissatisfied, rewiring takes place such that the new link keeps attached to A with probability p and attached to B with probability $1 - p$.

We start our simulations from a homogeneous random graph [26], in which all nodes have the same number of links (z), randomly linked to arbitrary nodes. The population size is $N = 10^3$ with average connectivities $z = 20$, 30, and 40 (the value $z = 30$ used in Fig. 2 and Fig. 3, right panel, reflects the mean value of the average connectivities reported in [13] for socials networks). We always start with 50% of *C*s randomly distributed in the population. In all cases we used $c = 1$ for the cost of cooperation ($c_i = 1/k_i$ for an individual i playing the DPD).

Each value in the figures corresponds to an average over 10^4 different randomly generated configurations and graphs. In each of those we average and evolution over $9\text{x}10^5$ generations after discarding a transient period of 10^5 generations.

At the end of each evolution we also computed the maximal connectivity k_{max} associated with the final graph and the cumulative degree distribution, which are the basis of the results plotted in Figs. 3.

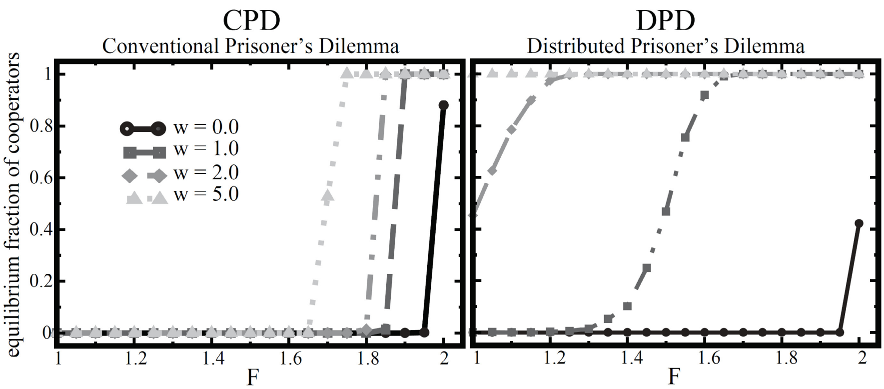

Fig. 2. Co-evolution for different time scales. Equilibrium fraction of cooperators as a function of the enhancement factor F using a homogeneous random graph of $z = 30$ and $\beta = 1.0$. **Left panel:** Under CPD it is difficult for cooperation to emerge unless we allow a fast adaptation of the network structure. As *W* increases, the rate of link rewiring also increases, and so does the viability of cooperation. **Right panel:** Under DPD, in addition to the adaptive assortment of *C*s, cooperation benefits from the break of symmetry associated with the nature of the dilemma and emerging heterogeneity of the network.

4 Results and Discussion

The results of Fig. 2 show the fraction of *C*s that survive at the end of evolution (see section 3.4) for different values of *W*. We plot the graphs for the interval $1 < F < 2$, for both the CPD and the DPD. For $W = 0$ cooperation can be hardly sustained, since the network remains static and equal to the initial homogeneous random network (see above). Moreover, in homogeneous networks the CPD and DPD games are equivalent, as the differences between both amounts to a rescaling of the intensity of selection. It is only when we give individuals the chance to change their social ties, than we begin to see differences. As *C*s (*D*s) seek for *C*s to cooperate (exploit), *C*s tend to acquire a higher number of links when compared with *D*s. This self-organized heterogeneity benefits the emergence of cooperation [10], in particular when highly connected nodes are occupied by *C*s [28].

Yet, in the DPD paradigm, as the network changes, the actual game played by each individual may also change with her degree. The DPD can represent a situation where individuals have limited resources and therefore, as the network becomes more heterogeneous with increasing *W*, so do the amounts contributed by different *C*s. As shown in Ref. [20], in the DPD paradigm the condition for a highly connected *C* to become advantageous becomes less stringent the larger their connectivity. On the contrary, under the CPD paradigm, the cost of cooperation plays a major role in the overall fitness of the cooperative hub, which means that the larger their connectivity, the harder it will be for the cooperative hub to become advantageous with respect to any D in their neighborhood. Consequently, when compared with the CPD paradigm, the DPD will promote cooperation as it benefits from the additional break of symmetry of the game induced by evolution of the social structure.

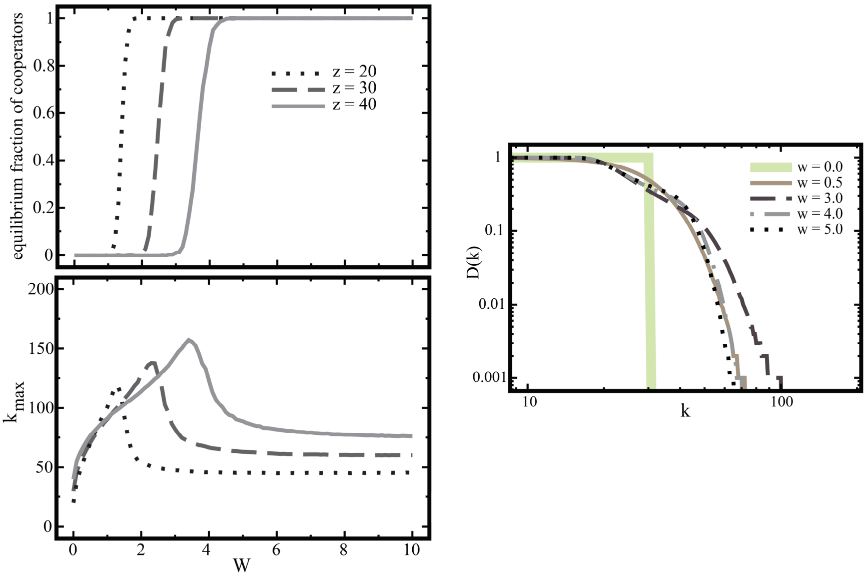

Fig. 3. Co-evolution for different networks. CPD game for F = 1.8 and β = 1.0 using homogeneous random graphs. **Upper left panel**: Equilibrium fraction of cooperators as a function of W for different values of z. For each value of z, there is a critical value of the time scale W_{crit}, above which cooperators wipe out defectors. **Lower left panel**: Maximum value of the connectivity in the population as a function of W. W_{crit} increases monotonically with z. **Right panel**: Cumulative distributions for different values of W. Starting from a homogeneous random network with $k_{max} = z = 30$, just as we increase W, the distribution widens, resulting in both single scale networks ($W = 0.5$, solid brown line) and broad-scale networks ($W > 3$, dotted black and grey lines). W_{crit} is also the value for which the heterogeneity of the associated network reaches a maximum. The results obtained for DPD are qualitatively the same.

Nevertheless, in both games, for a sufficient large W, we will get a full cooperative scenario: The quicker the individuals are able to alter their social ties, the easier it is for Ds to become extinct. This behavior is better understood from the upper left panel of Fig. 3: For small W, Cs never survive long, but, as W approaches a critical value W_{crit}, they become increasingly better at wiping out Ds. The W_{crit} increases monotonically with z, which makes sense because there are more links to be rewired. In practice, W_{crit} is determined as the value of W at which cooperation reaches 50%. Thus, the survival of cooperation relies on the capacity of individuals to adjust to adverse ties, even when the average connectivity is high.

Fig. 3 also provides evidence of the detailed interplay between strategy and structure. On one hand, strategy updating promotes a local assortment of strategies, since Cs breed Cs and Ds breed Ds. On the other hand, structural updating promotes local assortative interactions between *C*s and disassortative interactions between *D*s and *C*s. When both are active, strategy update will promote assortativity among *C*s, but will restrain disassortativity between *D*s and *C*s, which overall will benefit the emergence of cooperation. Additionally, since graphs will become heterogeneous for any W > 0, as a result of structural update (we are starting from homogeneous graphs), it will become easier for strategy update to promote cooperation, and even more when we play the DPD (see Fig. 2).

From the left panels of Fig. 3, the overall onset of increase of heterogeneity qualitatively follows the wave of cooperation for the corresponding z [21]. Indeed, the overall heterogeneity of the graph increases with W until it reaches a maximum at W_{crit}, above which heterogeneity again decreases down to a stationary value [21]. This is clearly shown in the right panel of Fig. 3 for a CPD with an enhancement factor F = 1.8. The results shown suggest that the adaptive dynamics of social ties accounts for the heterogeneities observed in realistic social networks [16]. The DPD produces results similar to the ones represented in Fig. 3. Also, similar analytic results were already obtained in a simpler model of link rewiring [29].

Our results show that to understand the emergence of cooperative behavior in a realistic scenario, one should consider simultaneously the evolution of the social network of interactions and the evolution of individual strategies. We show how an adaptive social network can easily transform a defection dominance scenario into a different one where cooperation may thrive. Moreover, the co-evolutionary process of strategy and structure can produce realistic heterogeneous networks. Hence, besides providing a bottom-up answer to the problem of cooperation, the proposed mechanism also shows how complex social topologies can result from simple social dynamical processes, exclusively based on local assumptions. In addition, the emergence of such heterogeneous structures with diverse social contexts becomes particularly relevant whenever individuals contributions are correlated with the social context they are embedded in. In this regime, network dynamics is able to remove the game symmetry of the PD in homogeneous networks, opening a route for cooperation to thrive.

Finally, the DPD used here relies on the fact that all cooperators are effectively assessed as cooperators, irrespectively of the amount contributed. In fact, such assessment should rely on a social norm [30], which may evaluate an action as "Good" or "Bad". From this perspective, our setting considers a social norm where the act of giving is seen as more important than the amount given, under which, as we show, cooperation prevails.

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