

Image Super-Resolution Based on Alternative Registration, Blur Identification and Reconstruction

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Abstract. A solution to the problem of obtaining a high-resolution image from several low-resolution images is provided. In general, this problem can be broken up into three sub-problems: registration, blur identification, and reconstruction. Conventional super-resolution approaches solve these sub-problems independently. In this paper, we propose a method to simultaneously solve all the sub-problems. The proposed method minimizes a nonlinear least squares error function. The cost function is alternatively minimized with respect to registration parameters, blurring operators, and high resolution image. The objective and subjective results are shown to demonstrate the effectiveness of the proposed method.

Keywords: Super-resolution, image registration, blurs identification.

1 Introduction

Super-resolution (SR) is an approach to obtain high-resolution (HR) image(s) from a set of low-resolution (LR) images. In its most general form, HR image reconstruction can be broken up into three sub-problems: registration, blur identification, and reconstruction. Image registration is the process to align images to the reference image [1]. Image blur identification is the process to estimate the blurring operators appearing in the observation model. The reconstruction step uses the resulting registered and restored LR images to reconstruct the desired HR image. SR approaches can be categorized based on the cue, by which the SR process is performed, into motion-based approaches and non-motion-based approaches. In previous works, a few approaches have assumed no scene motion, and use other cues such as lighting or varying zoom. Most of the motion-based approaches either pre-register the inputs using standard registration techniques [2,3], or assume that a perfect registration is given a priori before carrying out the super-resolution estimate [4]. However, the steps taken in super-resolution are seldom truly independent, and this is too often ignored in current super-resolution techniques [2,3,4]. Recently, the dependency between registration and reconstruction steps has been taken into consideration [5,6,7]. However, these approaches assumed known or pre-estimated blurring operators which are seldom known in the practical situations. The main idea behind the proposed method is that; due to the dependency of the SR steps, an improvement in the HR image leads to more accurate registration parameters and

blurring coefficients. Hence, we introduce an alternative minimization of a nonlinear least squares cost function with respect to the HR image, registration parameters, and blurring coefficients which can greatly improve the performance of SR. Three cases of the proposed approach are examined. First, perform simultaneous registration and reconstruction with known blurring operators. Second, estimate blurring operators assuming the HR image is known. In this case, we use the initially interpolated image as HR image and we simultaneously estimate motion operator. Finally, we perform simultaneous registration, blur identification, and reconstruction.

2 Problem Description

Assume that K LR frames of the same scene in lexicographical order denoted by Y^k ($1 \leq k \leq K$), are observed, and they are generated from the HR frame denoted by X . The observation of K LR frames are modeled by the following degradation process:

$$\begin{aligned} Y^k &= DB^k F^k X + DB^k \Delta F^k X + V^k \\ &= DB^k F^k X + \mathcal{V}^k \end{aligned} \quad (1)$$

Where F^k , B^k and D^k are the motion operator, the blurring operator (due to camera), and the down-sampling operator respectively, X is the unknown HR frame, Y^k is the k -th observed LR frame, V^k is an additive random noise for the k -th frame, and \mathcal{V}^k is the combination of additive noise and motion error.

Throughout this work, we assume that D is known, the additive noise is Gaussian with zero mean, and the blurring operators are shift invariant hence they can be described as moving averaging as $B^k X = \sum_{l=-\rho}^{\rho} \sum_{m=-\rho}^{\rho} b_{l,m,k} S_x^l S_y^m X$ where $b_{l,m,k}$ ($-\rho \leq l, m \leq \rho$) are the coefficients of the blurring operator and S_x^l and S_y^m are shifting operators by l and m pixels in x and y direction respectively. We will use B^k and $\sum_{l=-\rho}^{\rho} \sum_{m=-\rho}^{\rho} b_{l,m,k} S_x^l S_y^m X$ interchangeably. Therefore the problem is to reconstruct HR image, X , while estimating blurring operators, B^k , and motion operators F^k .

3 Blind Super-Resolution

3.1 The Proposed Cost Function

The SR problem can be viewed as the minimization of a nonlinear least squares error function [1]. The cost function is described by the difference between pixel intensities of the warped, blurred, and down-sampled version of the estimated HR image and observed LR images. The proposed cost function is described as follows:

$$J(X, B, p) = \sum_{k=1}^K \left\| DB^k X \left(W_1^k(x; p) \right) - Y^k \right\|_2^2 + \lambda(X, B, p) \|CX\|_2^2 \quad (2)$$

Where x is a $MN \times 2$ matrix containing the pixels' coordinates and W is the warping operator. The second term is used to regularize the cost function with the smoothness constraint of the HR image. C is a general high pass operator, and λ is the regularization factor. Throughout this paper we will use $F^k X$ and $X(W^k(x, p))$ interchangeably. The warping operator is defined as

$$W(x; p) = \begin{pmatrix} 1 + p_1 & p_2 & p_3 \\ p_4 & 1 + p_5 & p_6 \end{pmatrix} \begin{pmatrix} X_1 & \dots & X_{MN} \\ Y_1 & \dots & Y_{MN} \\ 1 & \dots & 1 \end{pmatrix} \quad (3)$$

Where p_1, \dots, p_6 are the registration parameters. Instead of minimizing Eq. (2) with respect to p , we proposed to minimize

$$\begin{aligned} J(X, B, p) &= \sum_{k=1}^K \left\| DB^k X(W^k(x; p)) - Y^k(W^k(x; \Delta p)) \right\|_2^2 + \lambda(X, B, p) \|CX\|_2^2 \\ &= \sum_{k=1}^K \left\| DB^k X(W^k(x; p)) - Y^k(W^k(x; 0)) + \nabla Y^k \frac{\partial W}{\partial p} \Delta p \right\|_2^2 \\ &\quad + \lambda(X, B, p) \|CX\|_2^2 \end{aligned}$$

With respect to Δp by getting the derivative with respect to Δp equals zero, we get

$$\Delta p = H^{-1} \sum_{k=1}^K \left[\nabla Y^k \frac{\partial W}{\partial p} \right]^T \left[Y^k(W(x; \Delta p)) - DB^k X(W^k(x, p)) \right] \quad (4)$$

$$\text{Where } H = \sum_{k=1}^K \left[\nabla Y^k \frac{\partial W}{\partial p} \right]^T \left[\nabla Y^k \frac{\partial W}{\partial p} \right] \quad (5)$$

The HR image (X) and blurring operator parameters ($b_{i,j}, -\rho \leq i, j \leq \rho$) can be updated by the increment in the inverse direction of the gradient of the cost function with respect to X and $b_{i,j}$ respectively. The gradient of $J(X, B, p)$ with respect to X may be approximated by

$$\nabla_X J(X, B, p) \cong 2 \sum_{k=1}^K F^k T B^k T D^T (DB^k F^k X - Y^k) + \lambda(X, B, p) C^T C X, \quad (6)$$

Where we assume that the gradient of the weight $\lambda(X, B, p)$ with respect to X is negligible with respect to that of the L2-norm. The gradient of $J(X, B, p)$ with respect to $b_{l,m,k}$ is given by

$$\nabla_{b_{l,j,k}} J(X, B, p) = 2X^T F^k T S_y^{-j} S_x^{-i} D^T \left(\sum_{l=-\rho}^{\rho} \sum_{m=-\rho}^{\rho} b_{l,m} D S_x^l S_y^m F^k X - Y^k \right) \quad (7)$$

The optimization of $J(X, B, p)$ is performed with respect to p , X and $b_{l,m,k}$ alternatively. Where, the blurring coefficients are constrained to be bi-symmetric, non-negative, and unit-sum.

3.2 Adaptive Regularization and Step Size

The regularization parameter, $\lambda(X, B, p)$, controls the trade-off between fidelity to the data and smoothness of the solution. The following choice of the regularization parameter [8] can be useful:

$$\lambda(X, B, p) = \tau \left[\sum_{k=1}^K \|DB^k F^k X - Y^k\|_2^2 + \lambda(X, B, p) \|CX\|_2^2 \right] \quad (8)$$

Implying

$$\lambda(X, B, p) = \frac{\sum_{k=1}^K \|DB^k F^k X - Y^k\|_2^2}{\frac{1}{\tau} - \|CX\|_2^2} \quad (9)$$

Where τ is chosen so that λ is non-negative; therefore it can be chosen as [8]

$$\frac{1}{\tau} \geq \|CX\|_2^2 = \|C\|_2^2 \|X\|_2^2 \geq \|X\|_2^2. \quad (10)$$

The step size $\beta_X^{(n)}$ and $\beta_B^{(n)}$ are calculated by minimizing the cost function

$$J(X^{(n+1)}, B^{(n)}, p^{(n)}) = J(X^{(n)} - \beta_X^{(n)} \nabla_X^{(n)} J, B^{(n)}, p^{(n)}) \quad (11)$$

and

$$J(X^{(n)}, B^{(n+1)}, p^{(n)}) = J(X^{(n)}, B^{(n)} - \beta_B^{(n)} \nabla_B^{(n)} J, p^{(n)}) \quad (12)$$

with respect to $\beta_X^{(n)}$ and $\beta_B^{(n)}$ respectively. Then $\beta_X^{(n)}$ and $\beta_B^{(n)}$ are obtained as $\beta_X^{(n)} = \frac{\zeta^{(n)}}{\eta^{(n)}}$, where

$$\begin{aligned} \zeta^{(n)} = & \sum_{k=1}^K X^{(n)} (\nabla_X^{(n)} J)^T F^k T B^k T D^T (DB^k F^k X^{(n)} - Y^k) \\ & + \lambda(X^{(n)}, B^{(n)}, p^{(n)}) (\nabla_X^{(n)} J)^T C^T C X^{(n)} \end{aligned}$$

$$\eta^{(n)} = \sum_{k=1}^K X^{(n)} (\nabla_X^{(n)} J)^T F^k T B^k T D^T DB^k F^k X^{(n)} + \lambda(X^{(n)}, B^{(n)}, p^{(n)}) (\nabla_X^{(n)} J)^T C^T C \nabla_X^{(n)} J$$

and $\beta_B^{(n)} = \vartheta^{(n)} / \gamma^{(n)}$, where

$$\vartheta^{(n)} = \sum_{k=1}^K X^{(n)T} F^k T \sum_{i=-\rho}^{\rho} \sum_{j=-\rho}^{\rho} \nabla_{b_{i,j}}^{(n)} J S_y^{-j} S_x^{-i} D^T (DB^k F^k X^{(n)} - Y^{(n)}),$$

$$\gamma^{(n)} = \sum_{k=1}^K X^{(n)T} F^k T \sum_{i=-\rho}^{\rho} \sum_{j=-\rho}^{\rho} \nabla_{b_{i,j}}^{(n)} J S_y^{-j} S_x^{-i} D^T (DB^k F^k \nabla_X^{(n)} J).$$

4 Simulation Results

In this section, the simulation results are shown for an example including human face image. For that example we used four low-resolution images to reconstruct one HR image. The simultaneous registration and reconstruction is compared with independent registration and reconstruction algorithm in the three cases.

The regularization term is defined as follows:

$$\|CX\|_2^2 = \sum_{l=-\varepsilon}^{\varepsilon} \sum_{m=-\varepsilon}^{\varepsilon} \alpha^{|l|+|m|} \|X - S_x^{-1} S_y^{-m} X\|_2^2. \quad (17)$$

The parameters are used as follows: $\alpha = 0.1$ and $\varepsilon = 1$. Since only LR versions of X are available, we used $1/\tau$ as the summation of squared L2-norm of these LR images as $\frac{1}{\tau} = 2 \sum_{k=1}^K \|Y^k\|_2^2$.

In this experiment we estimate the HR image in case of unknown blurring operator. In this case, the blurring operator, the registration parameters and the HR image are estimated simultaneously. Figure 1 shows the PSNR comparison between simultaneous and independent registration and reconstruction steps in case of using estimated blurring operators. From this figure, it can be shown that performing simultaneous registration and reconstruction steps is more efficient than performing independent registration and reconstruction steps.

The visual results shown in Figs. 2 demonstrate the results obtained by the PSNR comparison. From this figure we can see that the reconstructed HR image using simultaneous registration and reconstruction steps is clearer than that obtained by independent registration and reconstruction steps. Figure 3 shows the efficiency of estimated blurring operator compared to the original blurring operator.

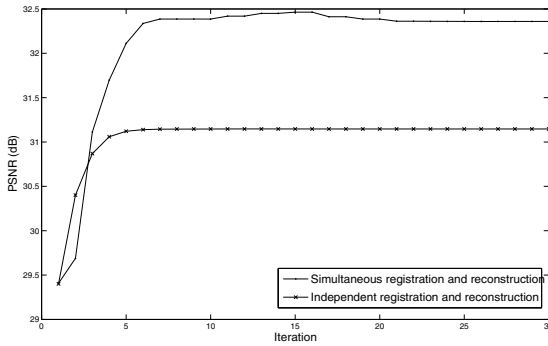


Fig. 1. Using estimated blurring operator for Lenna image sequence



Fig. 2. From left to right and from up to down, (a) Original HR Lenna image; estimated HR image using (b) Bicubic interpolation; (c) Independent registration and reconstruction with known blurring operator; (d) Simultaneous registration and reconstruction with known blurring operator; (e) Independent registration and reconstruction while estimating blurring operator; (f) Blind super-resolution

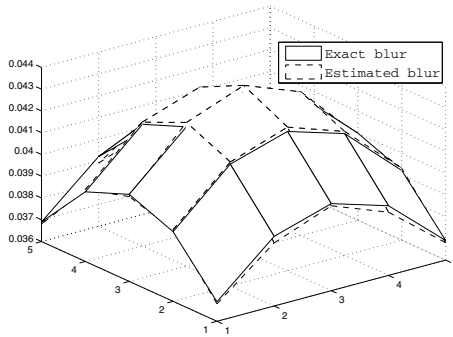


Fig. 3. Estimated blurring operator assuming known HR image for Lenna image sequence

5 Conclusions

We presented a new approach for image super-resolution based on simultaneous registration and reconstruction while estimating blurring operators. The proposed approach alternatively updates the registration parameters. Based on the simulation results, the proposed approach efficiently estimates the blurring operators in case of known HR image. We recommend using the proposed approach to estimate blurring operators while assuming known HR image and then use the estimated blurring operators to reconstruct HR image.

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