# Solids - A Combinatorial Auction for a Housing Corporation

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**Abstract.** On May 7, 2011, over one hundred bidders took part in a combinatorial auction for housing space in a newly erected building in Amsterdam (the Netherlands). This paper describes the development of this auction. We sketch our collaboration with the housing corporation that resulted in design choices with respect to first/second price, feedback, number of rounds, and tractability of the combinatorial auction. Furthermore, the winner determination problem is complicated by various municipal and building regulations that the allocation needs to satisfy. We show how these regulations can be included in an integer program that is used to solve the winner determination problem. Finally, computational experiments illustrate the tractability of this model.

**Keywords:** combinatorial auction, housing, auction design, integer program.

## 1 Introduction

To raise a building that will last for at least 200 years, with its tenants deciding how to use the building. The Dutch housing corporation Stadgenoot aims to make this happen in Amsterdam with so-called "solids". A solid is a sustainable building without predefined purpose, that offers a lot of flexibility. The Solids concept is inspired by historic canalside mansions in Amsterdam [2]. The main idea is that it is up to the tenants to decide on the use, the size, the configuration and even the rent of the space in the solid that they occupy. Stadgenoot sees solids as highly suitable for a large variety of tenants: for (large) families, with everyone getting their own area, all linked up to a shared family room and kitchen, for entrepreneurs who are looking for a living area with a work space, for students, restaurants, etc. It is Stadgenoot's position that solids should be open for everyone, including people with a small budget.

Solid spaces are delivered as shells. This means that within the building, there are walls between the solid spaces, and each solid space has access to a shaft with

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ventilation provision, drainage, electricity, etc. However, within a solid space, it is the tenant who decides where to place partition walls, interior doors, etc. This enables the rented space to be used for a whole range of purposes: living, working, culture, or a combination of all these. Stadgenoot remains the owner of the shell; the tenants rent the solid space, and own the interior. If a tenant leaves, he can sell the interior to a next tenant (throughout this paper, he can be replaced by she, and his by her). Over time, solid spaces can grow (when they are merged with another solid space) or shrink (when they are split up), and be used in very different ways.

Apart from the functional freedom, solids are also novel for the way they will be allocated to tenants, namely using a combinatorial auction. As far as we are aware, this is the first time that a combinatorial auction will be used for allocating real estate. Indeed, traditionally, residences, business accommodations, shops, and offices are treated as different markets, each with their own rules and target groups. Typically, the characteristics of the property are announced, owners set a price and wait for an interested buyer to show up. For solids, however, a combinatorial auction seems particularly suited due to the varying preferences of the different tenants. For instance, some tenants may want sun in the evening (west side), some want a small but practical working space on the first floor, others want spacious room on the top floor. A combinatorial auction is an ideal way to take all these preferences (including different budgets) into account.

The construction of the first solid, called Solid 11, was finished half 2011. Two other solids are already under construction, and there are plans to build many others in Amsterdam, for up to 50,000 square meters in total. We refer to [1] for more information about the various solids, written from the perspective of an interested tenant. In this paper, we will focus on Solid 11, located in the west part of Amsterdam, and featuring a spacious roof terrace with a splendid view over the city. In fact, on May 7, 2011, Solid 11 was allocated to bidders using the combinatorial auction described in this contribution (see [1]). On that day, over one hundred bidders participated in the auction. We, the authors of this paper, were involved in the design of this auction. We report here which considerations were taken into account, and which precise auction rules were selected. We also discuss how we determine which bidders wins which part of the solid.

The next section is devoted to the problem description. Section 3 shows how we decided upon the auction design. The winner determination problem is formally discussed in section 4, followed by computational experiments in section 5. The paper ends with conclusions in section 6.

## 2 Problem Description

In essence, the problem is to develop a combinatorial auction for Stadgenoot to allocate their solids to interested bidders. A combinatorial auction is an auction where multiple items are auctioned simultaneously, and bidders are allowed to bid on one or more subsets of the items. To this end, the solid is divided into 125 lots (i.e. items), distributed over 7 floors. Any interested tenant can specify solid

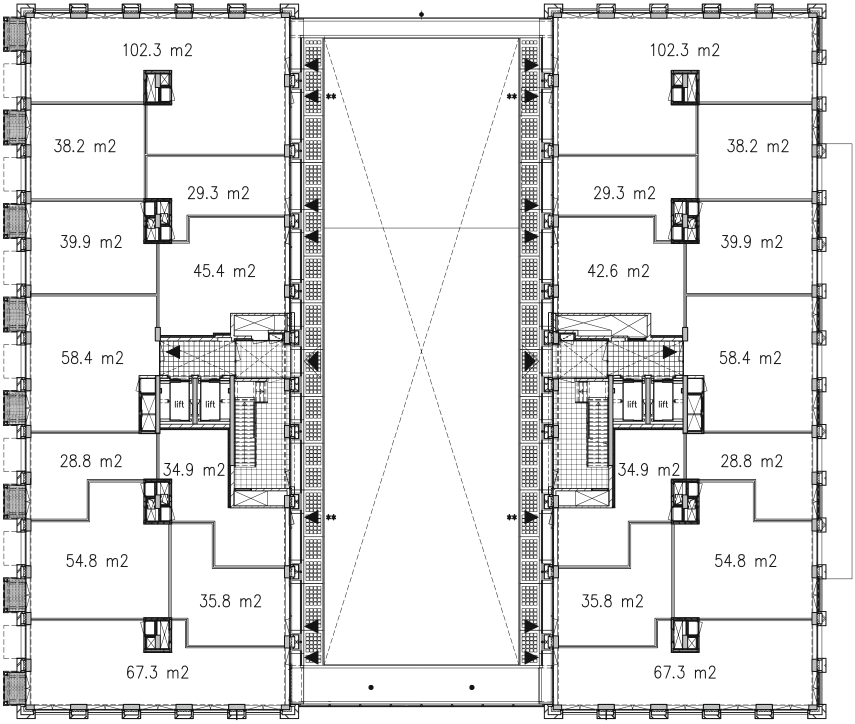


**Fig. 1.** Allotment of the second floor of Solid 11

spaces (i.e. sets of lots), together with the price he is willing to pay as a monthly rent (paying rent by month is the usual setting in the Netherlands). Figure 1 shows how the second floor of Solid 11 is divided into 22 lots; the surface area of each lot is given. By choosing an appropriate set of lots (on a bidder's favorite floor, according to a preferred orientation), a bidder specifies the resulting solid space.

A combinatorial auction is useful in this case, because typically, bidders are interested in multiple lots, and may value some sets of lots higher than the sum of the values of the lots individually. Moreover, some lots have no value at all as a single item, because e.g. they are not directly accessible from the hallway. These so-called complementarity effects may be bidder-specific, since bidders have different needs and preferences with respect to the space they want to rent. A combinatorial auction is an excellent way to make use of these synergies. Indeed, it allows a bidder to express his preferences to a greater extent than for individual items only, and it allows Stadgenoot to collect a higher total rent. For a thorough discussion on combinatorial auctions, integrating contributions on many interesting aspects from both theory and practice, we refer to the book edited by Cramton et al. [3].

Stadgenoot distinguishes three types of bidders: residential, commercial, and social bidders. The first group consists of people who plan to live in the solid, the second group plans to open a business in the solid, and people with a low income make up the third group. In a combinatorial auction in its most general form, bidders can bid whatever amount they please on any subset of the items. In the solid auction, this is not the case. First of all, there is a lower bound of 6 euro per square meter on each bid. Stadgenoot rather sees the space empty, than renting it out for less than 6 euro per square meter. Furthermore, for social bidders, Dutch law imposes an upper bound on the monthly rent of 650 euro. Second, and most important, is that bidders cannot bid on just any subset of lots. A valid subset consists solely of adjacent lots (on a single floor). Figure 1 shows that floors above ground level consist of two wings, separated by an open space in the middle, such that lots in different wings are not adjacent. On the ground level, the wings are connected by additional lots, and consequently, this is the only place where valid solid spaces spanning both wings are possible. A bid on a set of lots on different floors is not allowed. Furthermore, a valid subset of lots needs to have access to at least one of 10 available utility shafts, have enough doors to the central hallway (denoted by black triangles in Figure 1), and have enough incidence of light through the windows. For the remainder of this paper, when we use the term "solid space", we mean in fact sets of lots satisfying these requirements. About 1000 solid spaces can be formed with the 125 lots available in Solid 11. There are also limits to the surface area of the solid space, depending on the type of bidder. Residential bidders should bid on solid spaces of at least 90 square meters; commercial bidders can only go for solid spaces larger than 60 square meters, but there is a limit of 180 square meters for restaurants. Social bidders can only bid on solid spaces with a surface area of at most 70 square meters.

Apart from restrictions on the sets of lots bidders can bid on, there are also constraints on the allocation itself, based mainly on municipal and building regulations. The allocation should reserve at least 14.25% of the surface to social bidders. Residential bidders should get at least 25.75% of the surface, whereas commercial bidders should be allocated at least 30% of the surface. At most 3 restaurants are allowed in the solid. Furthermore, each floor above ground level has a rescue capacity per wing that should not be exceeded. The rescue capacity needed for each bid depends on the surface area of the solid space and the function that the bidder has in mind. Each solid space has to get its entire utilities through a single shaft, however, if a solid space contains multiple shafts, the choice is open which one to use. This choice can be made by the bidder, but can also be left open for the auctioneer in case the bidder has no preference. Each shaft goes from the ground level to the top floor, and has a ventilation capacity. Each bid has some ventilation requirement (again depending on the surface area and the intended function), and bids can only be allocated insofar the ventilation capacity of the shaft that they use is not exceeded. Each shaft offers a standard gas and electricity connection, however, some bidders require a connection with a higher capacity. Therefore, some shafts offer a high-capacity gas connection, on at most two floors, and a high current electricity connection on at most one floor. Bids with special requirements for gas and/or electricity can only be accepted insofar they can be accommodated by the shafts.

A concern of Stadgenoot is the possibility that a lack of bidders, or unpopular lots, would cause some parts of the solid to be unoccupied after the auction. In that case, the unoccupied lots should also form one or more solid spaces, such that they can still be rented out in the future. If there are not enough social bidders, there should be enough unoccupied solid spaces with a surface area less than 70 meters, such that when they are rented to social bidders later, the required 14.25% of the total surface of the solid can still be met. Furthermore, an unoccupied solid space should have at least one shaft where enough ventilation capacity is available to support basic use. Similarly, in order to make sure that a vacant solid space can be occupied later, sufficient rescue capacity should be reserved on its wing and floor.

For Stadgenoot, the goal is to maximize the total rent, taking into account the constraints mentioned above. Given the novelty of this way of allocating real estate, Stadgenoot wants the auction to be perceived as fair by the participants. Therefore, it is important that the auction is as transparent as possible, and accessible to everyone. Indeed, Stadgenoot expects and encourages many interested bidders with few or no experience in auctions, let alone combinatorial auctions. Therefore, the auction rules should be as simple as possible, and the bidding process should be user-friendly. Consequently, Stadgenoot decided that each bid should remain valid until the end of the auction: withdrawing or lowering bids is not allowed. Finally, each bidder can win at most one solid space, and the auction will be completed within a single day.

## 3 Auction Design

Every auction needs a set of rules that determines the course of the auction, the actions the bidders can take, and the feedback they will get. The design of the solid auction is based on a series of experiments with human bidders and a computer simulation [7], in which we studied the effect of various design settings on the following evaluation criteria:

- Total revenue
- Total surface of unallocated space
- Efficiency
- Auction stability
- User-friendliness
- Tractability.

The total revenue simply corresponds to the total monthly rent for Stadgenoot. Even though the total surface of unallocated space is related to the total revenue, Stadgenoot wants to avoid empty spaces, especially in the first of a series of solids that are to be auctioned. Efficiency refers to the total bidder value generated by the auction, relative to the highest possible value. This is an important measure, since an inefficient outcome implies that non-winning bidders have good reason to complain: at the final auction prices they are still willing to buy from winning bidders. The stability of the auction refers to how stable the winning allocation is over the rounds of the auction. Bidders might distrust the auction outcome if the set of winning bidders changes substantially from one round to the next, particularly in the final rounds of the auction. Indeed, we have to keep in mind that the vast majority of the bidders are not familiar with auctions. Therefore, user-friendliness of the auction and the user interface is even more important than in an auction with professional bidders. Finally, the auction needs to be tractable, given the fact that it has to be completed in a single day.

A user interface has been developed by an IT company for the bidders to express their preferences. This system shows them the allotment of the solid, including the positioning of the shafts, doors, and windows, and allows them to select lots to form a solid space. If the selected lots do not form a valid solid space, the system immediately provides feedback. Furthermore, a user can provide the user interface with a number of desired characteristics (e.g. surface area, orientation, etc.) and receive a list of solid spaces that satisfy them. This can be done weeks before the start of the auction; interested bidders also have the opportunity to visit the solid beforehand to actually see the space(s) that they intend to bid on. At any time, bidders can bid on at most 8 distinct solid spaces. We found that allowing more different solid space for a bidder to bid on, not only increases the computation time to solve the auction, but also makes it difficult for the bidders to keep track of their bids during the auction. Moreover, since tenants need to provide the interior for their solid space themselves, Stadgenoot is convinced a bidder will not have many different solid spaces in his mindset anyway.

We use a first price system: a bidder pays exactly the amount he bids for the solid space he wins. This is not an obvious choice, since e.g. second price auctions (i.e. the highest bidder pays the price of the second highest bid) have been shown to give each bidder the incentive to bid his true valuation [13]. Nevertheless, the simplicity of the first price concept, taking into account the inexperience of the bidders in the solid auction, and the computational complexity of the Vickrey-Clarke-Groves (VCG) auction (i.e. the combinatorial variant of the second price auction) lead us to adopt a first price rule.

Ideally, an auction ends when each winning bidder is happy with what he wins, and each losing bidder realizes that he is unwilling or unable to pay the amount needed to obtain what he wants. In a first price auction, it suffices that no bidder bids more than he is willing to pay to realize the first part; the second part, however, cannot be guaranteed if the number of rounds is determined before the start of the auction. Nevertheless, for practical reasons, announcing the number of rounds beforehand seems inevitable in our case. Moreover, since Stadgenoot wants the auction to take at most one day, 8 rounds seems to be a maximum (taking into account one hour per round to collect bids and compute the (provisional) winning allocation). We opted for 5 rounds; this choice is backed by our experiments which show that beyond 5 rounds, the total revenue does not rise drastically, provided that bidders have the incentive to start bidding early in the auction.

Because of the limited number of rounds, and Stadgenoot's aspiration for auction stability, it is important that bidders do not wait until the final round to reveal their preferences. We use two rules that encourage the bidders to bid seriously early in the auction. Firstly, a bidder is only allowed to bid on new solid spaces in the first two rounds of the auction; after that, he can only raise his existing bids. Since bidders rent empty shells, and given the fact that it takes a considerable effort for the bidders to plan and budget how they will provide the interior, Stadgenoot is convinced that most bidders have a strict and limited list of solid spaces in which they are interested. Since this list, for most bidders, does not depend on the course of the auction, Stadgenoot believes that it should force the bidders to disclose this information early in the auction. This forces the bidder to disclose the solid spaces in which he is interested already in the first half of the auction, but still gives him some opportunity to focus on solid spaces that turn out to be unpopular after the first round. Secondly, non-winning bidders should raise their bids by a minimum of 0.5 euro per square meter. This rule does not apply to bidders that won a solid space in the previous round: in the current round, they are allowed to wait and see what happens. As stated in section 2, there is a reserve price of 6 euro per square meter. Further, no bidder is allowed to lower or cancel his bid.

In an auction over multiple rounds, it is crucial that bidders receive feedback after each round. With the right feedback, bidders know whether it makes sense to raise their bid, and by how much, whether they should focus on other solid spaces, or whether the auction is beyond their budget. On the other hand, feedback may also encourage collusion or violate the bidder's privacy. Stadgenoot chose to report the winning allocation after each round, together with for each lot the number of bids in which it is involved. The former shows which lots are unallocated, how the solid is divided into solid spaces, and the prices; the latter makes it easy to find out for which lots the competition is likely to be stiff. Although there are ways to provide non-winning bidders with advice about a successful bid raise, Stadgenoot will not offer this kind of feedback. It was felt that this information could be interpreted by the bidder as holding some guarantee of becoming a winner in the next round, which could make Stadgenoot vulnerable to legal claims afterwards. Furthermore, tractability becomes an issue when for each bid a problem at least as hard as the winner determination problem itself needs to be solved.

## 4 Winner Determination Problem

The problem of deciding which bidders should get what items in order to maximize the auctioneer's revenue is called the winner determination problem. In general, this problem is $NP$-hard (van Hoesel and Müller [12]) and does not allow good approximation results (Sandholm [10]). However, for some special cases that involve an ordering of the items, the winner determination problem becomes easier.

Rothkopf et al. [9] found that the winner determination problem can be solved in polynomial time if the subsets on which a bidder can bid is limited to hierarchical subsets, i.e. if every two subsets are disjoint or one is a subset of the other. The same goes for settings where a linear order exists among the items and bidders can only bid on subsets of consecutive items, even when the first item in the ordering is considered the successor of the last (i.e. a circular order). This setting is somewhat similar to ours, where the lots are not on a line, but nevertheless arranged in a two-dimensional pattern, and where bidders can only bid on sets of adjacent lots. However, the results of Rothkopf et al. [9] assume a combinatorial auction where a bidder is allowed to win multiple subsets, which is clearly not the case in the solid auction. Nisan [8] elaborates on some of the results of Rothkopf et al. [9] by stating explicitly that the LP-relaxation of a set packing formulation results in an integral solution for the special cases with hierarchical subsets and linearly ordered items. Tennenholtz [11] presents a combinatorial network auction, which he proves is computationally tractable. In this auction, the items are assumed to be arranged in a tree, where every node corresponds to an item. The idea is that bids can be submitted only on subsets of items that form a path in the network. If the items are structured in a directed acyclic graph and bids are allowed on any directed subtree, Sandholm shows that the winner determination problem already becomes $NP$-hard again [10]. Finally, Day and Raghavan [4] describe the so-called *matrix bid auction*, where each bidder must submit a strict ordering (or ranking) of the items in which he is interested. Goossens et al. [6] show that the winner determination problem for this auction can be solved in polynomial time, given a fixed number of bidders, and provided that all bidders have the same ordering of the items (see also [5]).

We now develop a mathematical formulation for our winner determination problem, based on a set partitioning formulation. We use the following notation. The solid, with a total surface area of $A$ square meters, is divided into a set of lots denoted by $K$, and has a number of utility shafts $s \in S$. Each shaft $s$ has a ventilation capacity of $V_s$ m$^3$/h, and offers $G_s$ high capacity gas connections, and $E_s$ high current electricity connections. Per floor $f \in F$ above the ground level, there are two wings $w \in W$, which have a rescue capacity for $O_{f,w}$ persons. At most $R$ restaurants are allowed. We use $B_r$, $B_c$, and $B_s$ to represent the set of residential, commercial and social bidders, and they should be awarded at least a fraction $f^r$, $f^c$, and $f^s$ of the surface area of the solid respectively. $B = B_r \cup B_c \cup B_s$ denotes the set of all bidders. Each bid $t \in T$ belongs to one bidder $b(t)$, and is characterized by the following parameters. We use $L(t)$ to represent the set of lots included in bid $t$, $a_t$ for the surface area of this solid space, and $p_t$ for the price that the bidder is willing to pay for it as monthly rent. The solid space is situated on wing $w(t)$ and floor $f(t)$, and the utility shaft that is to be used is given by $s(t)$. If the bidder does not explicitly mention which shaft he wishes to use, we duplicate his bid for each shaft that is contained in his solid space. We use $o_t$ to denote the number of persons for which rescue capacity is needed for bid $t$, and $v_t$ for the required ventilation capacity. We define $T^{\mathrm{R}}$ as a subset of the set of bids $T$ containing those bids where the solid space is to serve as a restaurant. We also have subsets of $T$ for those bids requiring a high-capacity gas ($T_s^{\mathrm{G}}$), and/or electricity ($T_s^{\mathrm{E}}$) connection on shaft $s \in S$. We also added two dummy bidders $d_s, d_r \notin B$, where the former bidder bids on every (valid) solid space small enough for social bidders, and the latter bids on all other (valid) solid spaces. Both bidders always bid a price of zero, and require some minimal ventilation and rescue capacity. We use the decision variable $x_t$ which is 1 if bid $t$ is allocated, and 0 otherwise.

maximize

$$\sum_{t \in T} p_t x_t \tag{1}$$

subject to

$$\sum_{t \in T : k \in L(t)} x_t = 1 \qquad \forall k \in K \tag{2}$$

$$\sum_{t \in T : b(t) = b} x_t \leqslant 1 \qquad \forall b \in B \tag{3}$$

$$\sum_{t \in T : b(t) \in B_r} a_t x_t \geqslant f^r A \tag{4}$$

$$\sum_{t \in T : b(t) \in B_c} a_t x_t \geqslant f^c A \tag{5}$$

$$\sum_{t \in T : b(t) \in B_s \cup \{d_s\}} a_t x_t \geqslant f^s A \tag{6}$$

$$\sum_{t \in T^{\mathrm{R}}} x_t \leqslant R \tag{7}$$

$$\sum_{t \in T : w(t) = w, f(t) = f} o_t x_t \leqslant O_{f,w} \qquad \forall w \in W, f \in F \tag{8}$$

$$\sum_{t \in T : s(t) = s} v_t x_t \leqslant V_s \qquad \forall s \in S \tag{9}$$

$$\sum_{t \in T_s^{\mathrm{G}}} x_t \leqslant G_s \qquad \forall s \in S \tag{10}$$

$$\sum_{t \in T_s^{\mathrm{E}}} x_t \leqslant E_s \qquad \forall s \in S \tag{11}$$

$$x_t \in \{0, 1\} \qquad \forall t \in T \tag{12}$$

The objective function (1) states that the total rent should be maximized. The first set of constraints (2) enforces that each lot needs to be allocated. Indeed, this is necessary to ensure that unoccupied lots (i.e. lots allocated to a dummy bidder) form a valid solid space. The second set of constraints (3) ensures that each bidder wins at most one solid space (except for the dummy bidders). Constraints (4) - (6) make sure that each type of bidder acquires at least a given percentage of the total surface area, the next constraint prevents that more than $R$ restaurants get a place in the solid. Constraints (8) guarantees that the rescue requirement is available for each wing, and constraints (9) enforce that the ventilation capacity is respected for each shaft. Constraints (10) - (11) enforce that per shaft, no more than the available high capacity gas and electricity connections are used. The final set of constraints makes sure that bids are fully accepted or not at all.

Since we have dummy bids on all valid solid spaces, constraints (3) can always be satisfied by allocating the entire building to dummy bidders. Similarly, by including the social dummy bidder $d_s$ in constraint (6), this constraint will not cause infeasibility. However, due to constraints (4) and (5), there may not be a feasible solution. If this happens in the first round, the round is recomputed with $f^r$ lowered to 15%, and if necessary, again with both $f^r$ and $f^c$ lowered to 0%. In the latter case, the model will produce a feasible solution (e.g. leaving the whole building empty), but it will be announced to the bidders that in the current setting, the conditions for a valid auction result are not satisfied. Bidders can use the second round to bid on other solid spaces, such that after the second round, constraints (4) and (5) are satisfied. Otherwise, the model returns infeasibility, and the auction is canceled. In the next section, we implement this formulation and we discuss a number of computational experiments.

## 5 Computational Experiments

We implemented the formulation described in section 4 and solved it using IBM ILOG Cplex, version 12. In order to evaluate the performance of our algorithm, we carried out a number of computational experiments on randomly generated instances for the auction of Solid 11. Stadgenoot performed several studies with the intention of gaining information about interested tenants, e.g. in which solid spaces they are interested, and what special needs they have (ventilation, electricity, gas, etc.). They expect 55% social bidders, 30% residential bidders, and 15% commercial bidders. In order to have an idea of the amounts they would be willing to bid, Stadgenoot looked at prices for similar apartment spaces nearby. We used this information to generate realistic instances with 50, 150, 500, 1000, and 2000 bidders, where each bidder expressed between 4 and 8 bids. The main goal of these experiments is to evaluate whether the computation times are reasonable. Therefore, only one single round was considered. All instances were solved on a Windows XP based system, with 2 Intel Core 2.8GHz processors. The results are summarized in Table 1; each line gives values that are averaged over 10 similar instances.

The first column shows the number of bidders; the second gives the total number of bids (without the dummy bids). Stadgenoot was unsure about the number of participants, however, having over 1000 bidders was not considered unlikely. The total rent (in euros) is given in the third column. The increase in total rent gradually diminishes, suggesting that as soon as 500 bidders are participating,

**Table 1.** Overview of computational experiments

| Bidders | Bids | Total rent | Unallocated space | Avg. comp. time | Max comp. time |
|---|---|---|---|---|---|
| 50 | 299 | 69,593 | 21.2% | 0.5 | 1.6 |
| 150 | 913 | 87,966 | 5.0% | 0.9 | 1.8 |
| 500 | 2,995 | 101,824 | 0.0% | 4.0 | 8.8 |
| 1000 | 6,007 | 105,838 | 0.0% | 26.1 | 172.7 |
| 2000 | 11,981 | 107,018 | 0.0% | 100.3 | 451.9 |

the auction will not yield much excess value from extra bidders. Furthermore, the fourth column shows that as soon as 500 bidders can be reached, the solid is fully allocated. With only 50 bidders, as much as one fifth of the building remains empty. Nevertheless, we should take into account that only one round was considered in these experiments. We may expect the final percentage of unallocated space to be lower (as bidders will no doubt focus on unallocated solid spaces in the next round) and the total rent to be higher (due to competition in later rounds). The next column gives the average computation time (in seconds), followed by the maximal computation time in the final column. Clearly, these instances can be solved very efficiently, although there are considerable differences in computation time between instances with the same number of bidders. From these experiments, we conclude that we may expect our model to compute an optimal allocation within the range of 15 minutes (900 seconds) that was postulated in section 3.

## 6 Conclusions

We described a real-life combinatorial auction for allocating real estate. Due to the concept of solids, a variety of preferences and usages has to be accommodated. Also, different restrictions need to be taken into account in the winner determination problem. We developed a formulation that allows to allocate solid spaces to bidders efficiently. Based on computer simulations and experiments with human bidders, a multi-round auction format with limited feedback was chosen, where bidders are encouraged to start bidding early in the auction.

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