

# Peak-to-Average Power-Ratio Reduction Scheme Employing Fountain Codes

Juan Zhou, Ying Shen, and Youxi Tang

National Key Lab of Science and Technol. on Commun.,  
University of Electronic Science and Technology of China  
Chengdu, China  
{zhoujuan06, shenyingshen, tangyx}@uestc.edu.cn

**Abstract.** New peak-to-average power-ratio (PAPR) reduction scheme for multiuser CDMA systems are developed by employing fountain codes. In the proposed scheme, the transmission data is encoded by the fountain code. The encoded data is discarded at the transmitter if the PAPR exceeds the specified threshold, and at the same time another encoded data will be created. In consequence, the PAPR is dominated below the threshold. From the simulation methods, we find that the proposed scheme can reduce the PAPR with no degradation on the performance.

**Keywords:** PAPR, fountain codes, reduction.

## 1 Introduction

The multiuser DS/CDMA (Direct Sequence Code Division Multiple Access) wireless systems are known to show high capacities and good performances against the time dispersion effects of multipath propagation [1][2]. However, envelope fluctuations and peak-to-average power-ratio (PAPR) of multiuser CDMA signals can be very high when we combine a large number of signals with different spreading codes. A number of approaches have been proposed to deal with the PAPR problem. These techniques include amplitude clipping [3], clipping and filtering [4], coding[5], and so on. These techniques achieve PAPR reduction at the expense of transmit signal power increase, bit error rate (BER) increase, data rate loss, and so on. In this paper, we propose a new scheme to reduce the PAPR without loss in receiver performance, transmit signal power or data rate.

In the proposed scheme, the fountain code is employed to implement the PAPR reduction. As mentioned in [6][7], the key feature of fountain codes is their so-called rateless property: an arbitrary number of code symbols can be generated from a given set of information symbols. The rateless property means that a transmitter can simply transmit code symbols until each receiver has enough unerased symbols to recover the associated information symbols. Here in this paper, we make use of this feature to generate enough encoded data satisfying the PAPR requirement. We first determine the PAPR threshold and encode rate. If the PAPR of the encoded data exceed the

PAPR threshold, then the encoded data is discarded, otherwise, is reserved for transmission. This process persists until the reserved transmit data are enough to meet the specified encode rate. As a result of the application of fountain codes, the PAPR reduction is restricted under the PAPR threshold.

The rest of this paper is organized as follows. The system model is introduced in Section 2. The proposed PAPR reduction scheme employing fountain codes is studied in Section 3 and Section 4 provides numerical results. Finally in Section 5 we present the conclusion.

## 2 System Model

In this new PAPR reduction scheme, the system model contains two parts: the transmitter and receiver. Since the receiver is the same as the conventional receiver, as depicted in Fig. 1, we emphasis on the transmitter in this paper.

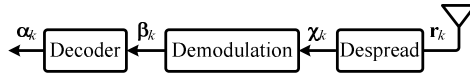


Fig. 1. Receiver of User  $k$

Consider a multiuser CDMA transmitter with  $K$  users data to be transmitted as illustrated in Fig. 2 where the FC represents the frame controller and CS denotes that the PAPR of the data is compared with the PAPR threshold and then saved if lower than the PAPR threshold.

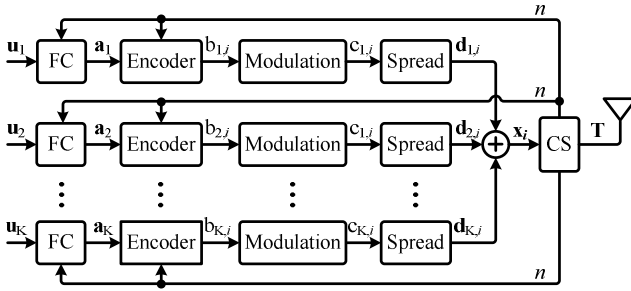


Fig. 2. Multiuser CDMA Transmitter. (FC represents frame controller. CS denotes that the PAPR of the data is compared with the PAPR threshold and then saved if lower than the PAPR threshold)

As for user  $k$  ( $k=1,2,\dots,K$ ), the  $p$ th frame from the FC is  $\mathbf{a}_{k,p} = [a_{k,p,1}, a_{k,p,2}, \dots, a_{k,p,q}]_{1 \times q}$ , where  $q$  is the number of bits in a frame. The frame data is first encoded by fountain coded, which is given as

$$b_{k,p,i} = \mathbf{a}_{k,p} \mathbf{G}_i. \quad (1)$$

where  $\mathbf{G}$  is the  $q \times w$  generation matrix of fountain codes, and  $\mathbf{G}_i$  is the  $i$ th row of  $\mathbf{G}$ . After BPSK modulation, we get

$$c_{k,p,i} = 1 - 2b_{k,p,i}. \quad (2)$$

The modulated data is then spread

$$\mathbf{d}_{k,p,i} = c_{k,p,i} \Psi_k. \quad (3)$$

where  $\Psi_k = [\psi_{k,1}, \psi_{k,2}, \dots, \psi_{k,N}]_{1 \times N}$  is the spreading code for user  $k$  and  $N$  is the length of the spreading code. Summing up the data of all the users, we have

$$\mathbf{x}_{p,i} = \sum_{k=1}^K \mathbf{d}_{k,p,i} = [x_{p,i,1}, x_{p,i,2}, \dots, x_{p,i,N}]_{1 \times N}. \quad (4)$$

The PAPR of  $\mathbf{x}_{p,i}$  is calculated and compared with the given PAPR threshold  $PAPR_t$ . If  $PAPR_{\mathbf{x}_{p,i}} \leq PAPR_t$ , the number of qualified data should be increased and  $\mathbf{T}_{p,m+1} = \mathbf{x}_{p,i}$  is saved for transmitting. Otherwise, keep the value of  $m$  and delete  $\mathbf{x}_{p,i}$ . For simplicity, we assume the delete position  $i$  will be broadcast to all the users. The change of  $m$  corresponding to each case can be expressed as

$$m = \begin{cases} m+1 & \text{if } PAPR_{\mathbf{x}_{p,i}} \leq PAPR_t \\ m & \text{if } PAPR_{\mathbf{x}_{p,i}} > PAPR_t \end{cases}. \quad (5)$$

$m$  is feedback to determine the work of the frame controller and fountain encoder. If the  $m$  reaches the specified number  $M$ , the frame controller will chose next frame  $\mathbf{a}_{k,p+1}$  data to process, meanwhile, the encoder will stop the encoding work on the present frame and all the saved data  $\mathbf{T} = [\mathbf{T}_{p,1}, \mathbf{T}_{p,2}, \dots, \mathbf{T}_{p,N}]$  are transmitted immediately. Here, the number  $M$  is determined by the coding rate  $r$  and frame length  $q$ , that is  $M=q/r$ . Otherwise, the encoder will do the fountain coding on the present frame  $\mathbf{a}_{k,p}$ . The detail on the implementation of PAPR reduction scheme will be described later in Section 3.

### 3 Proposed PAPR Reduction Scheme

In this section, the new PAPR reduction scheme employing fountain codes is presented. According to system requirement, we first predetermined the PAPR threshold  $PAPR_t$ , the coding rate  $r$ , the required number of qualified data  $M$ , and the

frame length  $q$ . Then under the system model described in Section 2, the proposed scheme is summarized as follows:

- 1) *Initialization.* Get new frames  $\mathbf{a}_{k,p}$  ( $k=1,2,\dots,K$ ) for all the users, and let the qualified data number counter  $m=0$ , the encoder time number counter  $i=0$ .
- 2) *Encoding of fountain codes, BPSK modulation, spread, and summing up to get  $\mathbf{x}_{p,i}$ , give by eqn. (4).*
- 3) *PAPR calculation.* The PAPR of  $\mathbf{x}_{p,i}$  can be calculated as

$$PAPR_{\mathbf{x}_{p,i}} (dB) = 10 \log_{10} \left( \frac{\max_{j=1,2,\dots,N} \left\{ |x_{p,i,j}|^2 \right\}}{E \left[ |\mathbf{x}_{p,i}|^2 \right]} \right). \quad (6)$$

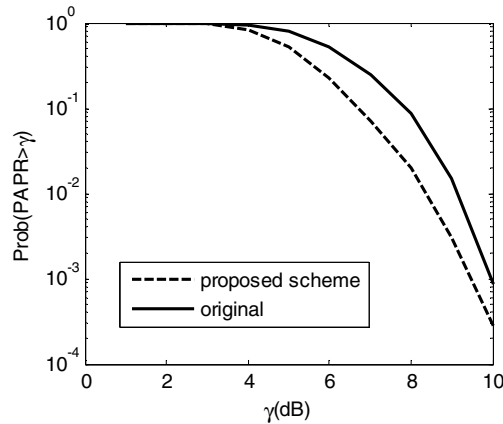
where  $E \left[ |\mathbf{x}_{p,i}|^2 \right] = \sum_{j=1}^N |x_{p,i,j}|^2 / N$

- 4) *PAPR comparison.* Compare  $PAPR_{\mathbf{x}_{p,i}}$  with the PAPR threshold  $PAPR_t$ , and
  - a) If  $PAPR_{\mathbf{x}_{p,i}} \leq PAPR_t$ : It denotes the PAPR of the data fulfilling the system requirement. We first let  $m=m+1$ , and then save the data  $\mathbf{T}_{p,m+1} = \mathbf{x}_{p,i}$ .
    - If  $m=M$ , meaning the saved data num is enough,  $\mathbf{T} = [\mathbf{T}_{p,1}, \mathbf{T}_{p,2}, \dots, \mathbf{T}_{p,N}]$  are transmitted immediately and the positions for the deletion data are broadcast. At the same time, frame controller and encoder get the information that the processing on the present frame is completed. The encoder stops the encoding work on the present frame. The frame controller gets the next frame, let  $p=p+1$  and return to step 1).
    - If  $m < M$ , meaning the saved data num is not enough, the encoding num  $i=i+1$  and the encoder continues the encoding work on the present frame. The frame controller still keeps the present frame. Return to step 2).
  - b) If  $PAPR_{\mathbf{x}_{p,i}} > PAPR_t$ : It denotes the PAPR is not meeting the requirement. We delete the  $\mathbf{x}_{p,i}$ , and record the delete position  $i$ . The encoding num  $i=i+1$  and the encoder continues the encoding work on the present frame. The frame controller still keeps the present frame. Return to step 2).

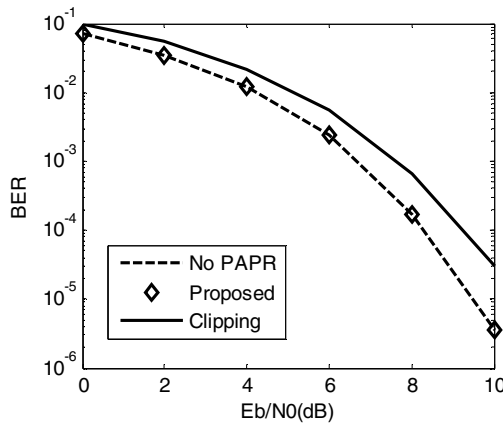
This proposed PAPR reduction scheme benefits the communication system in several ways: i) Reducing the PAPR effectively. ii) No performance degradation due to PAPR reduction. iii) The receiver of user is the same as before.

## 4 Simulation Results

In this section, we present simulation results of the proposed scheme. Consider a network of  $K=16$  users, the coding rate  $r=1/2$ , the required number of qualified data  $M=20$ , and the frame length  $q=10$ .



**Fig. 3.** CCDF plot of proposed PAPR reduction scheme



**Fig. 4.** BER of multiple CDMA signals. “No PAPR” represents no PAPR reduction on the signals. “Proposed” means that the proposed PAPR reduction scheme is performed on the signals. “Clipping” denotes the conventional clipping PAPR reduction is executed on the signals.

Fig. 3 depicts the complementary cumulative distribution function (CCDF) of the proposed PAPR reduction scheme. In the simulation, the baseband samples are deleted if the amplitude is larger than the threshold. Here the threshold is assumed to be 6.4. From this figure we can find that the scheme can reduce the PAPR effectively. The choice of the threshold is a matter of convenience, other thresholds can be applied according to the system requirement.

Fig. 4 depicts the bit error rate (BER) performance comparison between the proposed scheme and the conventional clipping scheme. In the simulation, the

amplitude threshold for clipping method is the same as that of the proposed scheme. From Fig. 4, we can see that the proposed scheme does not impact the system performance, whereas, the clipping method declines the performance.

## 5 Conclusion

In this paper, a new scheme employing fountain codes for PAPR reduction is proposed. The proposed scheme takes advantage of the rateless property of the fountain codes, and delete the samples if the amplitudes of which are larger than the threshold. In consequence, the PAPR is reduced with no performance degradation. Meanwhile, the proposed scheme will not increase the complexity of the receiver.

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