Joint Subcarrier and Power Allocation Considering Fairness in OFDM-Based Cognitive Radio Systems

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Abstract. Orthogonal frequency division multiplexing (OFDM) is an attractive modulation candidate for cognitive radio networks. In OFDM-based cognitive radio (CR) networks, effective and reliable subcarrier and power allocation is a challenging problem. And the fairness of resource allocation is another important problem in this network.In this paper, We present a joint subcarrier and power allocation algorithm considering fairness (JSPACF) among secondary users (SUs) for OFDM-based CR networks. In JSPACF, we allocate the subcarriers to SUs in the first step, not only considering the channel gain and the interference introduced to primary users (PUs) by SUs, but also considering proportional fairness among SUs. Then in the second step, we allocate the power to the subcarriers to maximize sum capacity of all SUs with total power constraint and interference constraint, considering proportional fairness among SUs too. Theory analysis and simulation results show that JSPACF can offer the beneficial tradeoff between system performance and fairness, while largely reducing complexity compared to the optimal solution.

Keywords: OFDM, cognitive radio, subcarrier and power allocation, fairness.

With the increasing explosion of wireless communications, available spectrum resource is becoming more and more scarce, which seriously hindered the development of new technologies. However, one of the FCC documents has indicated that many licensed frequency bands are severely underutilized in both the time domain and the spatial domain [1]. The spectrum is extremely under-utilized mostly due to the unreasonable command-and-control spectrum regulation, but not the physical scarcity of spectrum. Cognitive Radio (CR) [2][3] is a promising technology for dynamic spectrum access with the ability of observing the surrounding environment and adapting itself to the change of network environment. In the cognitive radio systems, secondary users (SU) can use the spectrum of primary users (PU) as long as the interference introduced to the PU by SU remains within a tolerable range.

Orthogonal frequency division multiplexing (OFDM) has already been recognized as a potential transmission technology for CR systems, since it has the reconfigurable subcarrier structure that can facilitate adaptive adjustment of

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parameters, and the Fast Fourier Transform module of its receivers can also be used for the spectrum sensing [4].

In CR system, available resource for SUs include spectrum, power, bit and so on. Resource allocation is a very important problem, because it can Seriously affect the performance of cognitive radio systems. All of the classical algorithms that was proposed to solve the problem in conventional multicarrier systems cannot be applied to the CR systems due to the existence of the two different types of users (PU and SU) where the interference introduced to the PU by SU should be taken into consideration. Recently, there has been a flurry of literatures addressing difference approaches on resource allocation for cognitive networks.The authors in [5] proposed an optimal and two suboptimal power loading algorithms for a downlink transmission scenario using the Lagrange formulation to maximize the downlink capacity of the CR system while keeping the interference induced to only one PU below a pre-specified threshold without the consideration of the total power constraint, and showed that the amount of interference introduced to the PU's band by a CR user's subcarrier depends on the power allocated in that subcarrier as well as the spectral distance between that particular subcarrier and the PU's band. An energy-efficient power allocation scheme is proposed based on a risk-return model in [6]. The authors in [7][8]present two-step resource allocation solution for multiuser based CR systems employing OFDM, separating subcarrier and power allocation, thus reducing the number of variables in the objective function of the optimization problem by half, is a promising method to reduce the complexity.

However these resource allocation algorithms do not consider the fairness among SUs. it is important to maintain fairness among users to avoid severe QoS degradation for users with unfavorable channel conditions. The The author in [9] proposes a joint channel and power allocation algorithm based on fair sharing, and introduces the fairness utility based on the definition of poverty line (PL) to guarantee fairness among SUs. In [10], a two-step resource allocation in multiuser OFDM-based CR systems is proposed, which has similar process with [7][8], but it considers proportional fairness among SUs. In [11], the authors proposed a power loading algorithms that guarantee the fairness of multiple SUs.

In this paper, we propose a joint subcarrier and power allocation algorithm considering fairness among secondary users (SUs) for OFDM-based Cognitive radio networks. In JSPACF, we allocate the subcarriers to SUs first, not only considering the channel gain and the interference introduced to primary users (PUs) by SUs, but also considering proportional fairness among SUs. Then for a given subcarrier assignment, we allocate the power to the subcarriers to maximize sum capacity of all SUs with total power constraint and interference constraint, considering proportional fairness among SUs too. JSPACF can offer the beneficial tradeoff between system performance and fairness, while largely reducing complexity compared to the optimal solution.

The rest of this paper is organized as follows: Section 2 presents the system model. Section 3 formulates JSPACF algorithm that we propose. In Section 4, the simulation result is presented. Finally, we conclude this paper in Section 5.

1 System Model

We consider a typical cellular transmission scenario with a single cell. In this transmission scenario, the CR system coexist with the PUs radio in the same geographical location, and PUs allow SUs to transmit while keeping the interference level low. it is assumed that The available bandwidth for CR transmission is divided into N subcarrier based OFDM system, and the bandwidth for each subcarrier is Δf Hz. there are M SUs, and SUs can use the non-active PU bands provided that the interference introduced to the PU by SU is within the interference threshold. The frequency band has been occupied by the PU (active PU band) is B Hz.

In the transmission scenario considered by us, there are three instantaneous fading gains: between the m^{th} SU's transmitter and receiver for the n^{th} subcarrier denoted as $h_{m,n}$; between the m^{th} SU's transmitter and PU receiver for the n^{th} subcarrier denoted as $g_{m,n}^{sp}$; between PU's transmitter and the m^{th} SU's receiver for the n^{th} subcarrier denoted as $g_{m,n}^{ps}$. In this paper, we assume that these instantaneous fading gains are perfectly known at the SU's transmitter.

In cognitive radio systems, due to the coexistence of PUs and SUs, there are two types of interference. One is introduced by the PUs into the SU's band, and the other is introduced by the SU into the PU's band. Now, we briefly describe the mathematical models for interference between SUs and PUs.

We assume that the signal transmitted on the subcarrier is an ideal Nyquist pulse, according to [12], the power density spectrum of the n^{th} subcarrier can be written as

$$\varphi_{m,n}\left(f\right) = p_{m,n}T_s\left(\frac{\sin\pi fT_s}{\pi fT_s}\right)^2 \tag{1}$$

where $p_{m,n}$ is the transmit power in the n^{th} subcarrier for the m^{th} SU and T_s is the symbol duration. Then the interference introduced to the PU band by the the m^{th} SU in the n^{th} subcarrier is

$$I_{m,n}\left(p_{m,n}\right) = \left|g_{m,n}^{sp}\right|^2 p_{m,n}T_s\int_{d_n-B/2}^{d_n+B/2}\left(\frac{\sin\pi fT_s}{\pi fT_s}\right)^2 df \tag{2}$$

where d_n is the distance in frequency between the n^{th} subcarrier and the PU band, $K_{m,n}$ denotes the interference factor for the m^{th} SU in the n^{th} subcarrier.

According to [12], the power density spectrum of the PU signal after M-fast Fourier transform (FFT) processing can be expressed as

$$E\left\{I_N\left(\omega\right)\right\} = \frac{1}{2\pi M}\int_{-\pi}^{\pi}\varphi_{PU}\left(e^{j\omega}\right)\left(\frac{\sin(\omega-\psi)M/2}{\sin(\omega-\psi)/2}\right)^2 d\psi \tag{3}$$

where $\varphi_{PU}\left(e^{j\omega}\right)$ is the power density spectrum of the PU signal, the PU signal has been taken to be an elliptically filtered white noise process with an amplitude P_{PU} [12].

According to [5], the interference introduced to the n^{th} subcarrier by the PU band can be written as

$$J_{m,n}\left(P_{PU}\right) = \left|g_{m,n}^{ps}\right|^2\int_{d_n+\Delta f/2}^{d_n-\Delta f/2}E\left\{I_N\left(\omega\right)\right\}d\omega \tag{4}$$

According to Shannon capacity formula, the transmission rate for the m^{th} SU in the n^{th} subcarrier is given by

$$r_{m,n} = \Delta f \log_2 \left(1 + \frac{|h_{m,n}|^2 p_{m,n}}{\sigma^2 + J_{m,n}}\right) \tag{5}$$

where σ^2 denotes the additive white Gaussian noise (AWGN) variance (we assume that the noise of each subcarrier is AWGN).

Let $a_{m,n}$ to be a subcarrier allocation indicator, and $a_{m,n} \in \{0,1\}$. if and only if the n^{th} subcarrier is allocated to the m^{th} user, $a_{m,n} = 1$, else $a_{m,n} = 0$. It is assumed that each subcarrier can be used for transmission to at most one user at any given time, so $\sum_{m=1}^{M} a_{m,n} \leq 1$.

Our objective is to maximize the total transmission rate of SUs with total transmit power constraint and interference constraint. Therefore, the optimization problem can be formulated as

$$\max_{a_{m,n}, p_{m,n}} \sum_{m=1}^{M} \sum_{n=1}^{N} a_{m,n} \Delta f \log_2 \left(1 + \frac{|h_{m,n}|^2 p_{m,n}}{\sigma^2 + J_{m,n}}\right) \tag{6}$$

subject to:

$$\begin{array}{l} a_{m,n} \in \{0,1\} \\ \sum\limits_{m=1}^{M} a_{m,n} \leq 1, for\ \forall n \in \{1, 2, \cdots, N\} \\ \sum\limits_{m=1}^{M} \sum\limits_{n=1}^{N} a_{m,n} p_{m,n} \leq P_T \\ p_{m,n} \geq 0 \\ \sum\limits_{m=1}^{M} \sum\limits_{n=1}^{N} a_{m,n} p_{m,n} K_{m,n} \leq I_{th} \\ R_1 : R_2 : \cdots\cdots : R_M = \gamma_1 : \gamma_2 : \cdots\cdots : \gamma_M \end{array} \tag{7}$$

where P_T is the total power constraint, I_{th} is the interference threshold of PU, R_m is the total transmission rate of the m^{th} SU, and $R_m = \sum_{n=1}^{N} a_{m,n} r_{m,n}$, $\{\gamma_1, \gamma_2, \cdots\cdots, \gamma_M\}$ is a set of predetermined constants to ensure proportional fairness [13] amongst SUs.

The fairness index is defined as

$$\zeta = \left(\sum_{m=1}^{M} \frac{R_m}{\gamma_m}\right)^2 \Big/ \left(M \sum_{m=1}^{M} \left(\frac{R_m}{\gamma_m}\right)^2\right) \tag{8}$$

Note that ζ with maximum value of 1 is the greatest fairness case in which all users would achieve the same proportional data rate.

The optimization problem in (6)(7) under multiple constraints is generally very hard to solve because of the uncertain variables $a_{m,n}$ and the continuous variables $p_{m,n}$. Therefore, it is computationally very costly to find the optimal schemes. Moreover, there is always a trade-off between the optimal schemes and the constraints.

2 The JSPACF Algorithm

The author in [7][8][10] have proposed some classic algorithms to solve this optimization problem, they separately find the subcarrier allocation and power allocation solution. Similarly, in JSPACF, we first solve the subcarrier allocation problem, not only considering the channel gain and the interference introduced to primary users (PUs) by SUs, but also considering proportional fairness among SUs. Then for a given subcarrier allocation, we present a suboptimal scheme to solve power allocation problem, considering proportional fairness among SUs too. In the next subsection, we first present the algorithm for subcarrier allocation in JSPACF.

2.1 Subcarrier Allocation

Since the proportion of rates are hardly guaranteed, a rough proportionality is acceptable as long as the capacity is maximized and the algorithm complexity is low. We use the reasonable assumption in [14] that the number of subcarriers assigned to each CR is approximately the same as their rates after power allocation, and thus would roughly satisfy the proportionality constraints. Based on this assumption, the number of allocated subcarriers per CR is accomplished by

$$N_1^{\max} : N_2^{\max} : \cdots\cdots : N_M^{\max} = \gamma_1 : \gamma_2 : \cdots\cdots : \gamma_M \tag{9}$$

where $N_m^{\max}$ is the maximal number of subcarriers allocated to the m^{th} SU. Since $N_1^{\max} + N_2^{\max} + \cdots\cdots + N_M^{\max} = N$, we can know that

$$N_m^{\max} = \frac{\gamma_m}{\gamma_1 + \gamma_2 + \cdots\cdots + \gamma_M} N \tag{10}$$

In the subcarrier allocation, we assume that equal power is in all subcarriers. To satisfy the interference constraint and power constraint, the power in all subcarriers is described as

$$p_{eq} = \min\left\{ \frac{P_T}{N}, \frac{I_{th}}{\sum_{m=1}^{M} \sum_{n=1}^{N} K_{m,n}} \right\} \tag{11}$$

The classical algorithm in many literatures allocate subcarriers according to channel gain, such that the subcarriers are allocated to the SU who has the best channel gain. But in the subcarrier allocation algorithm we propose, the subcarriers are allocated to the SU who has the best channel gain and produces least interference to PU. We define Ω_N as the set of the subcarriers that have not been allocated to SU, Ω_M as the set of SU who requires subcarriers, N_m is the number of subcarriers allocated to the m^{th} SU, and $N_m \le N_m^{\max}$, Φ_m is the set of the subcarriers allocated to the m^{th} SU. The proposed subcarrier allocation algorithm is as follows.

(a) Initialization
Set $\Omega_N = \{1, 2, \cdots, N\}, \Omega_M = \{1, 2, \cdots, M\}, N_m = 0, a_{m,n} = 0$, and $\Phi_m = \varphi, R_m = 0, \forall m, \forall n$

(b) For $m = 1$ to M
$n = \arg\max_{n \in \Omega_N} \frac{|h_{m,n}|}{K_{m,n}}$
$a_{m,n} = 1$
$\Omega_N = \Omega_N - \{n\}$
$N_m = N_m + 1$
$\Phi_m = \Phi_m + \{n\}$
$\mathrm{R}_m = r_{m,n}$
(c) While $\Omega_N \neq \phi$
$m = \arg\min_{m \in \Omega_M} \frac{R_m}{\gamma_m}$
$if\ N_m < N_m^{\max}$
$n = \arg\max_{n \in \Omega_N} \frac{|h_{m,n}|}{K_{m,n}}$
$a_{m,n} = 1$
$\Omega_N = \Omega_N - \{n\}$
$N_m = N_m + 1$
$\Phi_m = \Phi_m + \{n\}$
$\mathrm{R}_m = \mathrm{R}_m + r_{m,n}$
$else\ \ \Omega_M = \Omega_M - \{m\}$

From this algorithm, we can know $\Phi_1 \cup \Phi_2 \cup \cdots \cup \Phi_M = \{1, 2, \cdots, N\}$. our subcarrier allocation algorithm assigns roughly the proportional number of subcarriers to each CR according to the proportional fairness, thus improving fairness amongst SUs. Furthermore, this algorithm is suboptimal in a sense that equal power has been assumed in all subcarriers, however the complexity of the algorithm is low. Now, in the next subsection we introduce the power allocation scheme in JSPACF for a given subcarrier assignment.

2.2 Power Allocation

The power allocation in JSPACF consider interference constraint and total power constraint. Using Lagrange multiplier, we can get

$$\begin{aligned} G = &\sum_{m=1}^{M} \sum_{n=1}^{N} a_{m,n} \log\left(1 + H_{m,n} p_{m,n}\right) - \alpha \left(\sum_{m=1}^{M} \sum_{n=1}^{N} a_{m,n} p_{m,n} - P_T \right) \\ &- \beta \left(\sum_{m=1}^{M} \sum_{n=1}^{N} a_{m,n} p_{m,n} K_{m,n} - I_{th} \right) \end{aligned} \tag{12}$$

where α and β are Lagrangian multipliers, $H_{m,n} = \frac{|h_{m,n}|^2}{\sigma^2 + J_{m,n}}$. We differentiate (12) with respect to $p_{m,n}$ and set each derivative to zero to obtain

$$\frac{a_{m,n} H_{m,n}}{1 + H_{m,n} p_{m,n}} - \alpha a_{m,n} - \beta a_{m,n} K_{m,n} = 0 \tag{13}$$

Then, it can be derived that

$$p_{m,n} = \left[\frac{1}{\alpha + \beta K_{m,n}} - \frac{1}{H_{m,n}} \right]^{+} \tag{14}$$

where $[x]^{+} = \max\left(x, 0\right)$, α and β are determined by

$$\alpha\left(\sum_{m=1}^{M}\sum_{n=1}^{N}a_{m,n}p_{m,n}-P_T\right)=0 \tag{15}$$

$$\beta\left(\sum_{m=1}^{M}\sum_{n=1}^{N}a_{m,n}p_{m,n}K_{m,n}-I_{th}\right)=0 \tag{16}$$

Solving for the more than one Lagrangian multiplier is computational complex. Of course, these multipliers can be found numerically using ellipsoid or interior point method, but its complexity is very high. The high computational complexity makes this solution unsuitable for practical application, so we propose a low complexity power allocation algorithm.

If the interference constraint is ignored in (7). Similarly, we use Lagrange multiplier, when the total transmission capacity is maximized, the power of the m^{th} SU in the n^{th} subcarrier is given by

$$p_{m,n}^{1}=\left[\frac{1}{\alpha}-\frac{1}{H_{m,n}}\right]^{+} \tag{17}$$

where α is determined by

$$\sum_{m=1}^{M}\sum_{n=1}^{N}a_{m,n}p_{m,n}^{1}=\sum_{m=1}^{M}\sum_{n\in\Phi_m}\left[\frac{1}{\alpha}-\frac{1}{H_{m,n}}\right]^{+}=P_T \tag{18}$$

Consequently,we can get

$$\alpha=\frac{N}{P_T+\sum_{m=1}^{M}\sum_{n\in\Phi_m}\frac{1}{H_{m,n}}} \tag{19}$$

It is obvious that if the summation of the interference to PU under only the total power constraint is lower than or equal the interference constraint, i.e. $\sum\limits_{m=1}^{M}\sum\limits_{n=1}^{N}a_{m,n}p_{m,n}^{1}K_{m,n}\le I_{th}$, then (17) (19) will be the optimal solution with a given subcarrier assignment under the given total transmit power constraint and interference constraint.

Similarly, If the total power constraint is ignored in (7). we use Lagrange multiplier, the power of the m^{th} SU in the n^{th} subcarrier is given by

$$p_{m,n}^{2}=\left[\frac{1}{\beta K_{m,n}}-\frac{1}{H_{m,n}}\right]^{+} \tag{20}$$

where β is determined by

$$\sum_{m=1}^{M}\sum_{n=1}^{N}a_{m,n}K_{m,n}\left[\frac{1}{\beta K_{m,n}}-\frac{1}{H_{m,n}}\right]^{+}=I_{th} \tag{21}$$

Therefore,we can get

$$\beta=\frac{N}{I_{th}+\sum_{m=1}^{M}\sum_{n\in\Phi_m}\frac{K_{m,n}}{H_{m,n}}} \tag{22}$$

Similarly, if the summation of the allocated power under only the interference constraint is lower than or equal the available total power budget,, i.e. $\sum_{m=1}^{M}\sum_{n=1}^{N} a_{m,n}p_{m,n}^2 \leq P_T$, then (20)- (22) will also be the optimal solution with a given subcarrier assignment under the given total transmit power constraint and interference constraint.

It is assumed that P_T^{re} is the left available total power, I_{th}^{re} is the left interference constraint, Ω is the set of SU who need to be allocated power again, M^{re} is the number of SU who need to be allocated power again. The power allocation in JSPACF is described as follows.

1. Initialization
   Set $\Omega = \{1, 2, \cdots, M\}, M^{re} = M, P_T^{re} = P_T, I_{th}^{re} = I_{th}, p_{m,n} = 0, \forall m, \forall n$
2. while $M^{re} > 0$
   (a) Ignore the interference constraint, there are M^{re} SUs, through equations (17)(19), we get the power of the m^{th} SU in the n^{th} subcarrier $p_{m,n}^1$ only with the total power constraint P_T^{re}, if $\sum_{m\in\Omega}\sum_{n\in\Phi_m} a_{m,n}p_{m,n}^1 K_{m,n} \leq I_{th}^{re}$, the solution is found and $p_{m,n} = p_{m,n}^1$, else continue.
   (b) Ignore the total power constraint, there are M^{re} SUs, through equations (20)(22), we get the power of the m^{th} SU in the n^{th} subcarrier $p_{m,n}^2$ only with the interference constraint I_{th}^{re}, if $\sum_{m\in\Omega}\sum_{n\in\Phi_m} a_{m,n}p_{m,n}^2 \leq P_T^{re}$, the solution is found and $p_{m,n} = p_{m,n}^2$, else continue.
   (c) Set $p_{m,n} = \min\left(p_{m,n}^1, p_{m,n}^2\right)$, calculate the transmission rate of the m^{th} SU R_m through $R_m = \sum_{n=1}^{N} a_{m,n}r_{m,n}$.
   (d) Find m that satisfies $\frac{R_m}{\gamma_m} \geq \frac{R_i}{\gamma_i}$ for all $i \in \Omega$, for the found m, assign:
   $\Omega = \Omega - \{m\}$
   $I_{th}^{re} = I_{th}^{re} - \sum_{n\in\Phi_m} a_{m,n}p_{m,n}K_{m,n}$
   $P_T^{re} = P_T^{re} - \sum_{n\in\Phi_m} a_{m,n}p_{m,n}$
   $M^{re} = M^{re} - 1$

2.3 Complexity Analysis

For the optimization problem in (6)(7), if we use exhaustive search algorithm to find the optimal solution, there are M^N methods for subcarrier allocation. For a given subcarrier assignment, the complexity of the optimal power allocation algorithm is $O\left(N^3\right)$, so the complexity of the optimal solution in the optimization problem (6)(7) is $O\left(M^N N^3\right)$, which is very high. For JSPACF, the complexity of subcarrier allocation is $O(N)$, the complexity of power allocation is $O(MN\log N)$, so the total complexity of JSPACF is $O(MN\log N + N)$, which is much lower than the complexity of the optimal scheme.

3 Simulation Results

In the numerical results presented in this section, we assume the value of M and N to be 5 and 20 respectively, i.e., there are 5 SUs and 20 subcarriers. We assume the value of T_s to be 4 μs, and Δf ,B have been assigned the value of 0.3125MHz, 5MHz respectively. The channel noise is assumed to be AWGN, the value of σ^2 is assumed to be 10^{-6}. The value of amplitude P_{PU} is assumed to be 0.01W. The channel gains $h_{m,n}$,$g_{m,n}^{sp}$, and $g_{m,n}^{ps}$ are assumed to be Rayleigh fading with an average channel power gain equal to 1. For the proportional fairness,we assume that $\gamma_1 : \gamma_2 : \gamma_3 : \gamma_4 : \gamma_5 = 3 : 4 : 4 : 4 : 5$, so $R_1 : R_2 : R_3 : R_4 : R_5 \approx 3 : 4 : 4 : 4 : 5$. As is shown in table 1.

Table 1. Parameter Values

Parameter	M	N	T_s	Δf	B	σ^2	P_{PU}
Values	5	20	4 μs	$0.3125MHz$	$5MHz$	10^{-6}	$0.01W$

3.1 Subcarrier Allocation

The results of Subcarrier allocation is shown in Fig. 1. In simulation, it is assumed that the interference constraint of PU is 4mW, the total power constraint of SUs is 1W. there are 5 SUs and 20 subcarrier, $N_1^{\max} : N_2^{\max} : N_3^{\max} : N_4^{\max} : N_5^{\max} = \gamma_1 : \gamma_2 : \gamma_3 : \gamma_4 : \gamma_5$, so we can know that $N_1^{\max} = 3$, $N_2^{\max} = 4$, $N_3^{\max} = 4$, $N_4^{\max} = 4$, $N_5^{\max} = 5$. As is shown in Fig. 1, the subcarrier allocated to SU1 is 3, the subcarrier allocated to SU2 is 4, the subcarrier allocated to SU3 is 4, the subcarrier allocated to SU4 is 4, the subcarrier allocated to SU5 is 5, this is the same as the theory and it approximately guarantees proportional fairness among SUs, which is also shown in the simulation of fairness.

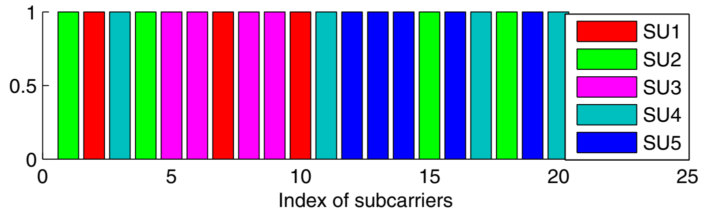

Fig. 1. The results of subcarrier allocation among SUs in JSPACF

3.2 Comparisons of Achievable Maximum Transmission Data Rates

In Fig. 2, we plot the achievable transmission rate of SUs versus the total power constraint of SUs for the optimal algorithm, JSPACF algorithm , and the classical resource allocation algorithm. In Fig. 2, the interference constraint of PU is fixed and is equal to 1mW. The relationship between the achievable transmission rate of SUs and interference constraint for the optimal algorithm, JSPACF algorithm , and the classical resource allocation algorithm, is shown in Fig. 3. In Fig. 3, the total power constraint of SUs is fixed and is equal to 2W. Here, by the classical resource allocation algorithm we mean the algorithm described in [7]. the classical resource allocation algorithm has not guaranteed the fairness among SUs. there is two steps in the classical resource allocation algorithm, the first step is subcarrier allocation for SUs, the second step is power allocation, which is optimal power allocation algorithm for a given subcarrier assignment.

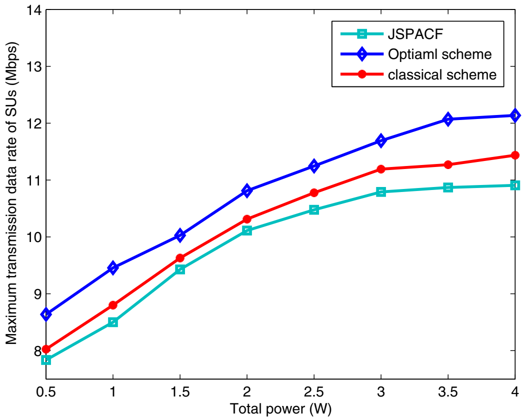

Fig. 2. The relationship between transmission rates of SUs and total power constraint

As shown in Fig. 2, In the same interference constraint of PU, the total transmission rate of SUs increases with the total power constraint of SUs increasing for different schemes under consideration, which is the same with the theory. The power allocated to every subcarrier is more when the total power constraint is larger, so the total transmission rates of SUs is larger. But the total transmission rates of SUs don't increase and are almost unchanged when the total power constraint increase to certain range. This is because with such a given interference constraint, the total power constraint is the main factor affecting the total transmission rates of SUs when the total power constraint is small, but when the total power constraint is increasing to certain range, the total transmission rates of SUs are mainly limited by interference threshold of primary users, and almost have nothing to do with the total power constraint. The system reach to the maximum total power that can be used to keep the interference to the primary user below the prescribed threshold. In the same interference threshold, the optimal scheme achieves the highest transmission rate for secondary users. It can be noted that the capacity achieved using JSPACF is close to that achieved using the optimal algorithm with a good reduction in the computational complexity. At the same time, the transmission rate that can be achieved using JSPACF is also close to that achieve by the classical algorithm. As mentioned before, there is two steps in the classical resource allocation algorithm, the complexity of the first step is $O\left(N\right)$, the complexity of the second step is $O\left(N^3\right)$, so the

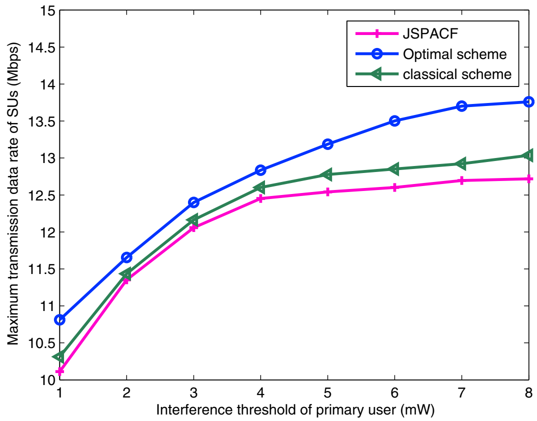

Fig. 3. The relationship between transmission rates of SUs and interference constraint

complexity of the classical resource allocation algorithm is $O\left(N^3+N\right)$, which is higher than the complexity of JSPACF.

As shown in Fig. 3, In the same total power constraint of SUs, the total transmission rate of SUs increases with the interference constraint of SUs increasing for different schemes under consideration. The power allocated to every subcarrier is more when the interference constraint is larger as long as the total power of SUs is not beyond total power constraint, so the total transmission rates of SUs is larger. But the total transmission rates of SUs don't increase and are almost unchanged when the interference constraint increase to certain range. This is because the interference constraint is the main factor affecting the total transmission rates of SUs when the interference threshold is small, but the total transmission rates of SUs are mainly limited by the total power constraint, and almost have nothing to do with the interference threshold when the interference threshold is increasing to certain range.

3.3 Comparisons of Fairness

The comparison of fairness index versus two algorithms (JSPACF and the classical algorithm) can be seen from Fig. 4. From Fig. 4, we can see that JSPACF significantly improves fairness compared to the the classical algorithm, and the fairness index of JSPACF is very close to 1 (maximum value of 1 is the greatest fairness), so all SUs can almost achieve the same proportional data rate in JSPACF. This is due to the fact that the subcarrier allocation and the power allocation in JSPACF are all considering the proportional fairness, so as to improve fairness among the SUs. As mentioned before, the transmission rate that can be achieved using JSPACF is a little lower than that achieve by the classical algorithm, so we can know that there is always a trade-off between the performance and the fairness among SUs.

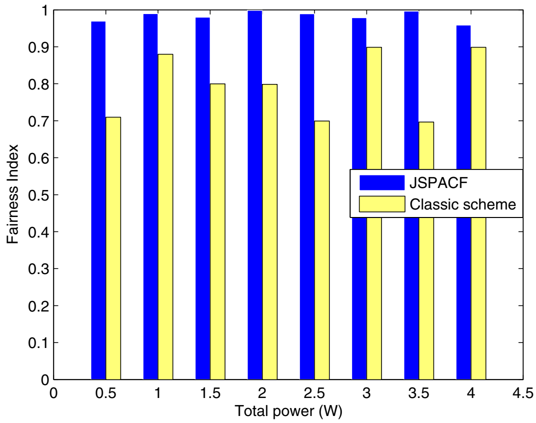

Fig. 4. Fairness comparison of 2 schemes among SUs

4 Conclusion

In this paper, we investigate the resource (subcarrier and power) allocation algorithm in the cellular transmission scenario with a single cell for OFDM-based cognitive radio systems, our objective is maximize the transmission data rate of secondary users, and We have augmented the optimization formulation of this problem by taking into account the fairness among SUs. we propose a joint subcarrier and power allocation algorithm considering fairness among secondary users (SUs) for OFDM-based Cognitive radio networks. In JSPACF, we allocate the subcarriers to SUs in the first step, not only considering the channel gain and the interference introduced to primary users (PUs) by SUs, but also considering proportional fairness among SUs. Then in the second step, we allocate the power to the subcarriers to maximize sum capacity of all SUs with total power constraint and interference constraint, considering proportional fairness among SUs too. Theory analysis and simulation results show that JSPACF can offer the beneficial tradeoff between system performance and fairness, while largely reducing complexity compared to the optimal solution.

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