The Master-Slave Stochastic Knapsack Modelling for Fully Dynamic Spectrum Allocation

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Abstract. Scarcity problem of radio spectrum resource stimulates the research on cognitive radio technology, in which dynamic spectrum allocation attracts lots of attention. For higher access efficiency in cognitive radio context, we suggest a fully dynamic resource allocation scheme for primary and secondary users, which is modelled by a master-slave stochastic knapsack process. Equilibrium behavior is analyzed, and expressions of blocking probability of both slave and master classes are derived as performance criterion and verified by numeric simulation, as well as forced termination probability of the secondary users. Compared to traditional opportunistic spectrum access (OSA), which can be regarded as half dynamic, our scheme leads to less termination events for the slaves while keeping the same behavior for the master class, promoting the system access performance.

Keywords: Blocking, forced termination probability, master-slave stochastic knapsack, full dynamic spectrum allocation, cognitive radio.

1 Introduction

Modern technology of wireless communications faces a severe problem of spectrum scarcity. New technique such as cognitive radio (CR) is involved to make the spectrum management more flexible [1], compared to traditional allocation scheme which leads to inefficient utilization [2]. In a CR system, there are two classes of users, called primary user (PU) and secondary user (SU). The former are licensed users who have preemption over the latter who are not. With the admission of SUs when PUs do not make full use of the spectrum, more customers may be served with the same bandwidth, and allocation becomes more efficient. In [3][4], brief overviews on CR were introduced, where some major challenges were proposed as well, among which is the forced termination of SU when PU seizes its channel.

P. Ren et al. (Eds.): WICON 2011, LNICST 98, pp. 480–489, 2012.

Generally speaking there are two kinds of processing for the SU's forced termination. Some literatures [5][6] consider a handoff (or called handover) method, in which terminated SU is transferred to another idle channel and continues its transmission. It may be a little ideal as the transferring operation is not specified, handoff duration are ignored, and transferring is regarded surely successful. It has no doubt that handoff scheme needs complicated implementation for SU terminals. In a handoff-free manner [5][6][7], terminated SUs are just dropped. It may be suitable for the best-effort services, and asks for no additional modification to original devices. The two methods have the same termination probability, since newcome PU have not attempted to evade the channels occupied by SUs. This may be not reasonable, as there are still idle channels PU can access to, termination should have been evitable.

In the above schemes, PUs' privileges emerge in two types: when the spectrum is not all-occupied, a new PU may seize a channel from a SU if it just chooses that SU's channel; or when the spectrum is exhausted, the new PU drives away a SU if there is SU in the system. However, in this paper, we propose a novel "fully" dynamic scheme for the CR access process, which allows PUs only the second privilege. PUs monitor channels and prevent to interrupt SUs as long as the channels are not exhausted. Thus, it can be expected to have advantages over the half dynamic scheme, for it avoids unnecessary terminations of SUs. We modelled the access procedure as a stochastic knapsack of masters and slaves, where master represents for PU, and slave for SU. Major analysis includes:

- An elaborate transition diagram for the master-slave knapsack process is proposed, as well as equilibrium analysis. Blocking probability is directly derived from equilibrium distribution for both master and slave classes.
- Probability of forced termination of SUs is also derived analytically, which is not as obvious as the blocking case. It is validated by the simulation results.

Our analysis is especially important to the termination behavior of SUs, which reveals some characteristic of fully dynamic spectrum allocation, and may help to develop effective access policy for cognitive radio system.

The rest of this paper is organized as follows. In Section 2 some related works are reviewed in Section 2. We model the full dynamic spectrum allocation problem with a master-slave stochastic knapsack process, and the equilibrium state transition is given in Section 3. Analytic result of blocking and termination probability is derived in Section 4. Section 4 gives numerical results and related discussions, and the conclusion is made in Section 5.

2 Related Work

Spectrum sensing is the basis and precondition for cognitive radio and much work have been done, from sensing architecture [8], distributed sensing scheme [9] and capacity limits of cognitive radio [10]. Although there is no efficient and viable solution to this problem up to now, we may expect such spectrum sensing will in the future work efficiently.

Loss model is first studied systematically by the Denmark mathematician and engineer A.K. Erlang during 1909-1920 who published a series of papers to solve basic problems in this telephone communication field using probabilistic theory [11]. The Erlang loss model is the simplest of all loss systems, consisting of a collection of resources, for example, C circuits, to which calls, each with an associated *holdingtime* and *class*, arrive at random instances. An arriving call can either be admitted into the system or blocked and lost [12].

As a mathematical model to such problem in dynamic spectrum allocation for wireless communication, the dynamic and random *knapsack* model has been well-studied [13,14,15]: "Dynamic" requests for the resources arrive in time according to a stochastic process, while "Random" means the demands for the resource and their associated rewards are random and unknown until their arrival.

As for the performance analysis in the coexistence of primary and secondary communication networks, Watanabe considers the cognitive radio performance when coexisting with primary communication system and shows that cognitive radio technology can not avoid the interference to primary system [16]. J. Neel proposes techniques to model and analyze the interactions of cognitive radio whose purpose is to improve the design of cognitive radio and distributed radio resource management algorithms[17].

Recently, performance analysis on the secondary user's behavior includes the cognitive research in spectrum access with optimal channel reservation[18], cognitive Ad Hoc networks[7], and cognitive radio network[19,20]. The contribution of this paper is to propose a novel fully dynamic spectrum access patter and derive the solutions using Markov chain state transition equations, which differs from current work.

3 System Model

Consider a CR system model in which there are N parallel channels shared by primary and secondary users. We use terms "master" and "slave" to denote primary user and secondary user. A user (either master or slave) asks for a channel if it tries to access the system spectrum. Poisson processes with rates λ_m λ_s are assumed for traffic generation of master and slave class, and their service durations distribute exponentially with expectation $1/\mu_m$ and $1/\mu_s$, respectively. μ_m, μ_s are means of service rate for master and slave.

The access manner is illustrated in Fig.1. It is modelled as a knapsack of masters and slaves. Masters have higher priority over slaves. That is, when the channels are all busy, an arrival of slave is just rejected; only if all channels are occupied by masters, new master is excluded from the spectrum; otherwise it seizes a channel from a slave (which may be picked out randomly or based on some rules, making no difference to our analysis). Masters do not seize channels from slaves as long as idle channels exist, which avoids some unnecessary termination events of the slave class. This assumption does not impair masters' performance of access at all (it is verified later), and intuitively reduces slaves' blocking probability.

System state is defined as (n_m, n_s), in which n_m denotes current number of masters, and n_s slaves. The feasible state set is:

$$S = \{(n_m, n_s)|n_m \geq 0, n_s \geq 0, n_m + n_s \leq N\}, \quad (1)$$

and $p(n_m, n_s)$ is used to denote the probability of state (n_m, n_s).

The set of full states (blocking) is

$$S_b = \{(n_m, n_s)|n_m + n_s = N\}, \quad (2)$$

as well as the set of states that slave termination may happen:

$$S_t = \{(n_m, n_s)|n_s > 0, n_m + n_s = N\}. \quad (3)$$

These sets are useful when discussing blocking and termination probabilities.

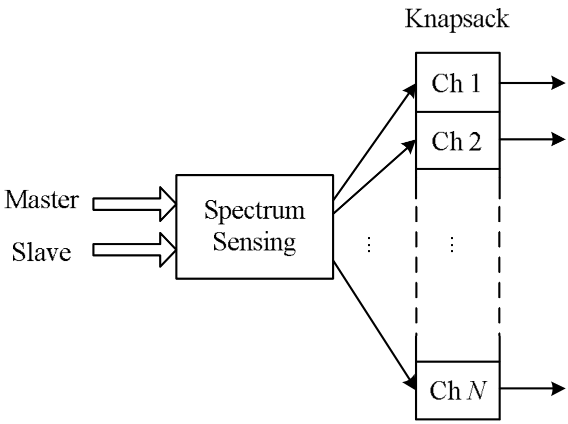

Fig. 1. System model for master and slave knapsack

4 Equilibrium State Transition Analysis

The knapsack manner leads to a 2-Dimension state transition diagram as shown in Fig. 2. Transitions marked by red dash lines are the differences from original stochastic knapsack without master-slave priority [12]. The red dotted arrow lines indicate the system behaviors when a master comes and knapsack is in its full state. Due to these transitions, the entire problem is no longer a reversible Markov process, and simple detailed balance conditions never stand. Hence solution to equilibrium distribution becomes complex to calculate the flow conservation equations for each state:

$$\sum_{v \in S} [p(v) \cdot t_{v,u}] - p(u) \cdot \sum_{v \in S} t_{u,v} = 0, \quad \forall u \in S, \quad (4)$$

where $t_{u,v}$ is the transition rate from state u to v .

The linear equations form a system of linear equations, with respect to $(N + 1)(N + 2)/2$ variables of equilibrium distribution. For details, these equations are categorized as following:

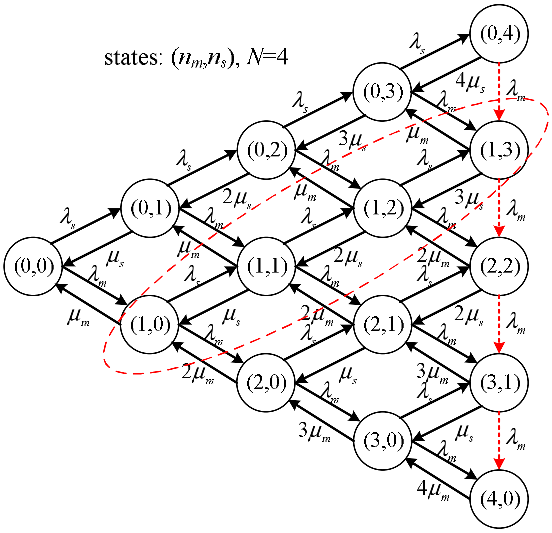

Fig. 2. States transition diagram for master-slave stochastic knapsack model

4.1 Non-full State

The states are $S \setminus S_b = \{(n_m, n_s) | n_m + n_s < N\}$. The corresponding balance equations are:

$$\begin{aligned}
&\lambda_m \cdot p(n_m - 1, n_s) + (n_m + 1)\mu_m \cdot p(n_m + 1, n_s) \\
&\quad + \lambda_s \cdot p(n_m, n_s - 1) + (n_s + 1)\mu_s \cdot p(n_m, n_s + 1) \\
&\quad - (\lambda_m + \lambda_s + n_m\mu_m + n_s\mu_s) \cdot p(n_m, n_s) = 0
\end{aligned}$$

4.2 Full State

The states are $S_b = \{(n_m, n_s) | n_m + n_s = N\}$. The corresponding balance equations are:

$$\begin{aligned}
&\lambda_m \cdot p(n_m - 1, n_s) + \lambda_m \cdot p(n_m - 1, n_s + 1) + \lambda_s \cdot p(n_m, n_s - 1) \\
&\quad - (\lambda_m + \lambda_s + n_m\mu_m + n_s\mu_s) \cdot p(n_m, n_s) = 0
\end{aligned}$$

5 Performance Analysis

5.1 Blocking Probability

Blocking probability of the master class is the probability that all N channels are occupied by master, which is:

$$P_b^m = \Pr(n_m = N) = p(N, 0). \tag{5}$$

Slaves' blocking events correspond to situations that channels are full, either with masters or slaves. So the blocking behavior of slave class is:

$$P_b^s = \Pr\{S_b\} = \sum_{i=0}^{N} p(i, N-i). \tag{6}$$

5.2 Forced Termination Probability

If all N channels are full, a new arrival of master will drive away a slave and grab its channel, which indicates a forced termination event of the slave class. The probability is calculated by:

$$P_t = c\Pr\{S_t\} = \frac{\lambda_m}{\lambda_s}\sum_{i=0}^{N-1} p(i, N-i), \tag{7}$$

This formula of forced termination probability (7) is a modification to that in [7]. It has a practical meaning: within the duration between two arrivals of slaves, there are $c = \lambda_m/\lambda_s$ master arrivals on average, inducing c termination events in condition that the knapsack is full and there is at least one slave in it.

Probability of forced termination is related to system parameters N, λ_m, μ_m, λ_s and μ_s, but not a simply monotonic function, like the blocking probability. An intuitive demonstration is, terminations of slaves may happen more frequent as masters get more; but when the master load $\rho_m = \frac{\lambda_m}{\mu_m}$ goes sufficient large, the channels are all occupied by masters with high probability. As a result, slaves are hardly let into the resource poll (which corresponds to a high blocking probability), to say nothing of terminations.

6 Numerical Result and Discussion

Parameters of system load are listed in Table.1. Linear system obtained in Section 4 with N equations has a rank of $(N-1)$, so the solution is unique, considering the feature of probability distribution. MATLAB is used to directly calculate the linear equations. The simulations are executed by C program: channel amount is $N = 10$; arrival time and holding time are independently generated exponential random variables. A total number of 2×10^8 is set as the upper bound of iteration, both master and slave classes are counted in. A window is used to detect the standard deviation of blocking times, when the normalized deviation (i.e., deviation divided by mean value) gets below a certain threshold, the system is believed to have reached equilibrium. The window size is set 100, threshold 0.05, and the deviation is calculated from the amount of blocked cases out of 2000 arrivals. 100 realizations are averaged to obtain the final results. Fig.3, 4 and 5 are based on the results of case 1, 2, 3 in Table.1, respectively.

Table 1. Simulation parameter setting for three cases

	C	λ_m/μ_m	λ_s/μ_s
Case1	10	a	a
Case2	10	$2a$	a
Case3	10	a	$2a$

6.1 Comparison of Analytic and Numeric Results

The analytic results of blocking and termination performance with our scheme match the simulation results excellently in all cases illustrated by Fig.3, 4 and 5, affirming the analysis to be rational and correct. As far as we know, this is a pioneer study on the fully dynamic spectrum allocation, which extends the our understanding on OSA. Our mathematical analysis inducts a guideline for this new direction and provides basis for further research. Following our work, performance criterion can be evaluated for network design.

6.2 Discussions on Termination Probability

It is necessary to clarify the curves of the forced termination probability of slaves, as the red solid lines shown in all the Fig.3, 4, 5. Within the first ascending of traffic load, approximately in the region $0 < a < 10$, the probability goes up, because heavier load leads to more termination. However, as the value a continues to increase, corresponding to greater arrival rate of both masters and slaves, the system is overloaded. There are more masters occupying the spectrum and less slaves, and as a result few termination events happen. That's why the termination probability decreases when a gets very large.

6.3 Comparison with OSA

We also give the simulation performance of OSA as a comparison. Master performance exhibits no difference whether OSA or our scheme is adopted, which guarantees the masters' priority as expected. Considerable reduction of slaves' termination probability is exhibited by the fully dynamic scheme, and this is the main contribution of our innovation. However, the slave blocking probability with our scheme is a little higher than that with OSA, which is revealed by the figures, and can be explained in this way: in OSA, masters drive away slaves more frequently, leaving more idle channels, which allows easier access of new slaves. But this phenomenon is obvious only in light system load (a is small), because in heavy load the knapsack is almost exhausted by the masters.

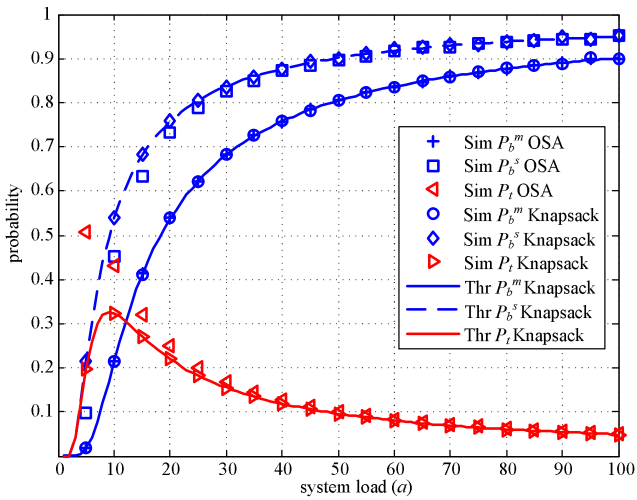

Fig. 3. Blocking and termination probability in Case1

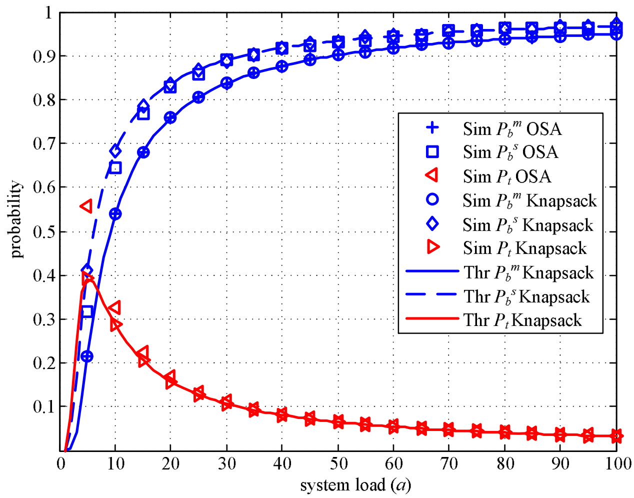

Fig. 4. Blocking and termination probability in Case2

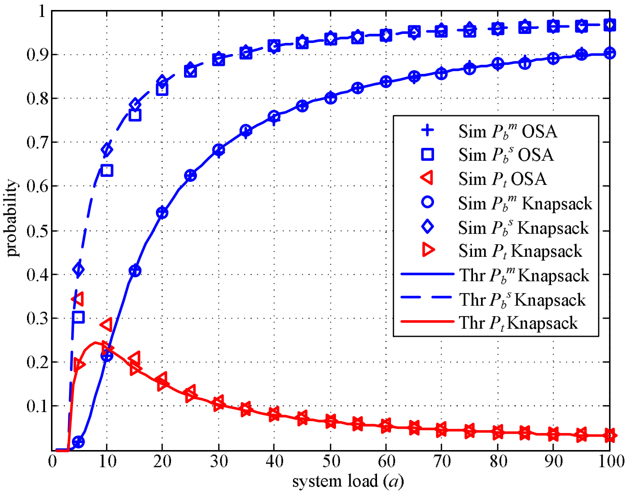

Fig. 5. Blocking and termination probability in Case3

7 Conclusion

In this paper, a fully dynamic spectrum allocation scheme for cognitive radio access is introduced and modelled by a master-slave stochastic knapsack process. We analyzed the equilibrium state transition, and derived the probability of blocking and forced termination. Our analysis is verified by simulation results. As a more flexible access manner, it keeps the same behavior of PUs as traditional OSA, while reduces the termination probability for the SUs, as shown in our simulation results. The fully dynamic access scheme is proved to have an encouraging promotion for the cognitive radio system. Our novel scheme introduces a framework for future access algorithm design, despite of some practical issues ignored.

Acknowledgment. This work was supported by the China High-Tech 863 Plan under Grant 2009AA011506, the National Major Special Projects in Science and Technology of China under Grant 2009ZX03003-009, 2010ZX03003-001, 2010ZX03005-003, and 2011ZX03003-003-04, National Key Technology R&D Program under Grant 2008BAH30B12, and National Basic Research Program of China under Grant 2007CB310602.

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